Chapter 8. Fracture of Cracked Members

Mechanical Strengths and Behavior of Solids

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8.1 Introduction

- **Unexpected failures below the material's yield strength**

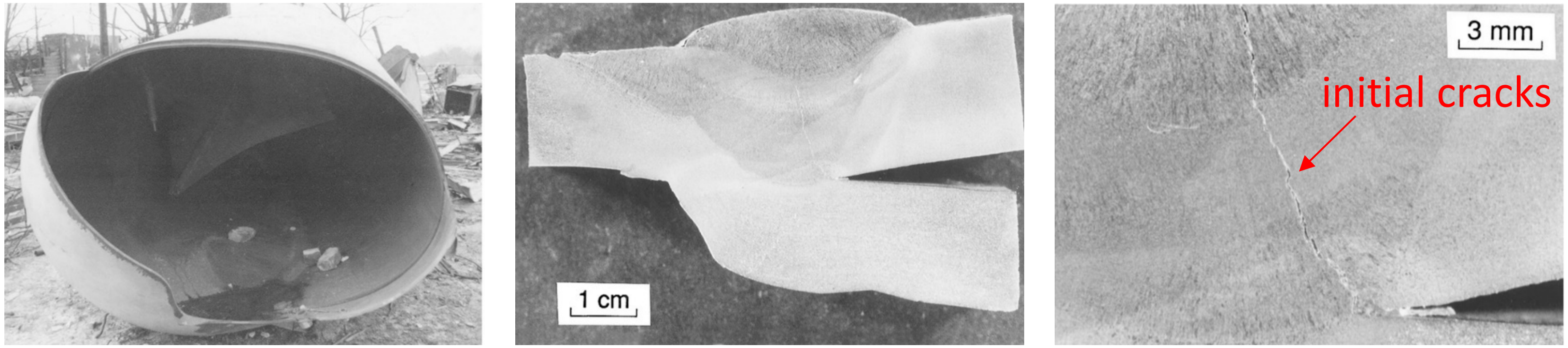
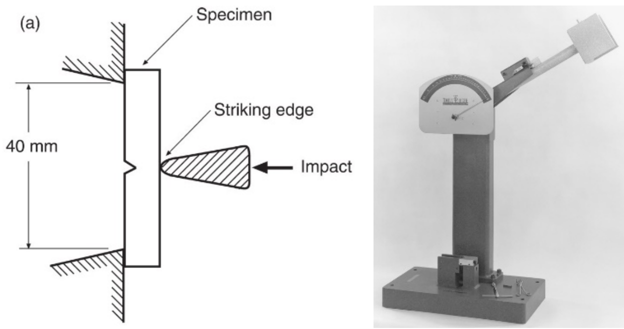


Figure 8.1 A propane tank truck explosion due to fracture from initial cracks in welds

- **Fracture mechanics**

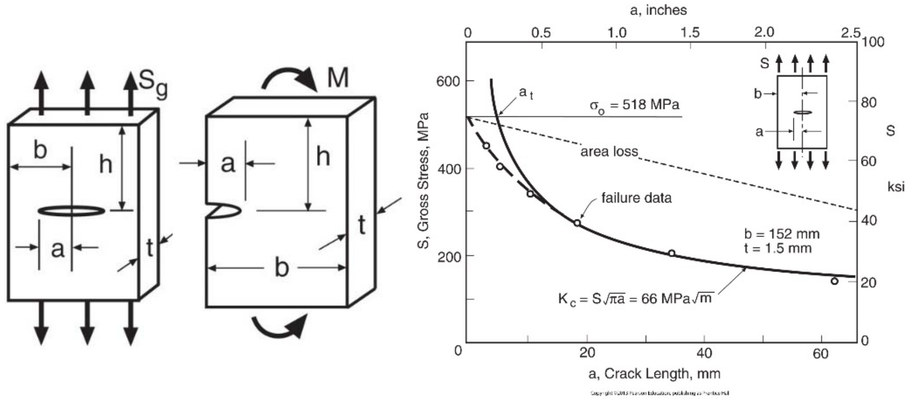
<Notch-impact test>

Rough guide for choosing materials



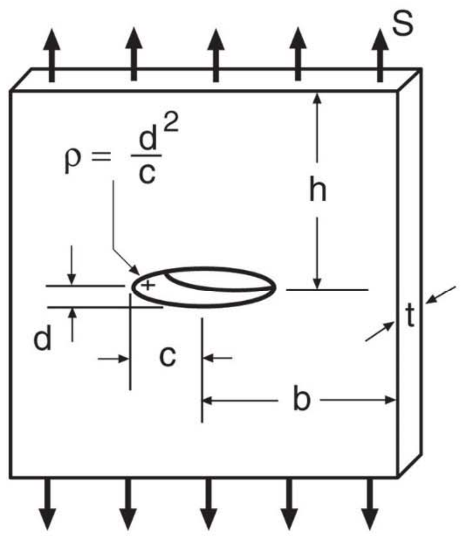
<Fracture mechanics>

Specific analysis of strength and life for various cracks

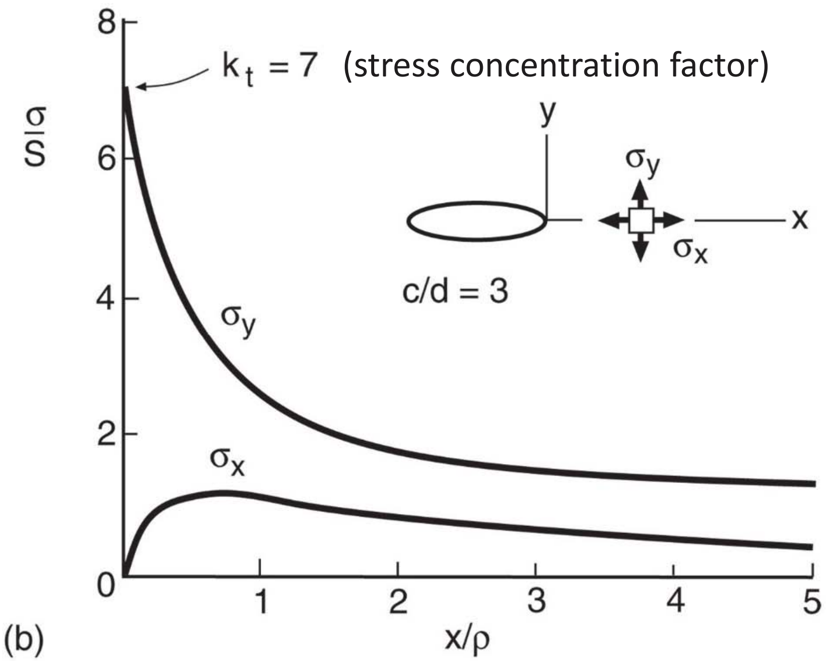


8.2 Preliminary Discussion

- **Cracks as Stress Raisers**



(a)

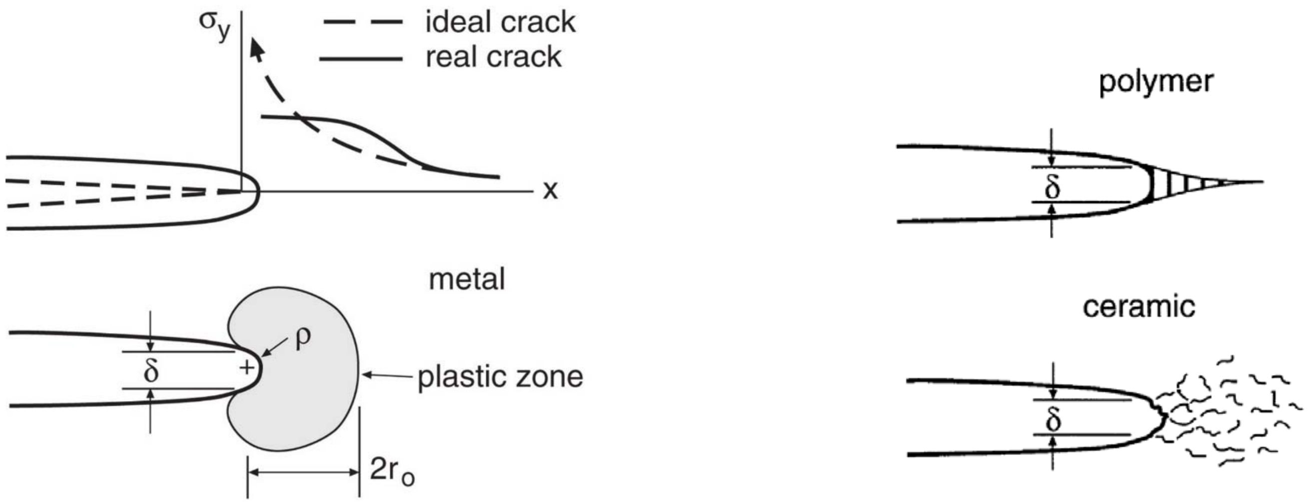


(b)

$$\sigma_y = S\left(1 + 2\frac{c}{d}\right) = S\left(1 + 2\sqrt{\frac{c}{\rho}}\right)$$

8.2 Preliminary Discussion

- **Behavior at Crack Tips in Real Materials**
  - Infinite stress cannot, of course, exist in a real material. If the applied load is not too high, the material can accommodate the presence of an initially sharp crack in such a way that the theoretically infinite stress is reduced to a finite value.
  - In ductile materials, large plastic deformations near the crack tip (*plastic zone*)
  - In some polymers, craze zone (fibrous structure bridging the crack faces)
  - In brittle materials, a high density of tiny cracks
  - High stress is spread over a region (*stress redistribution*)

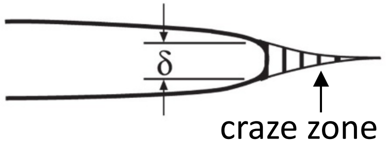


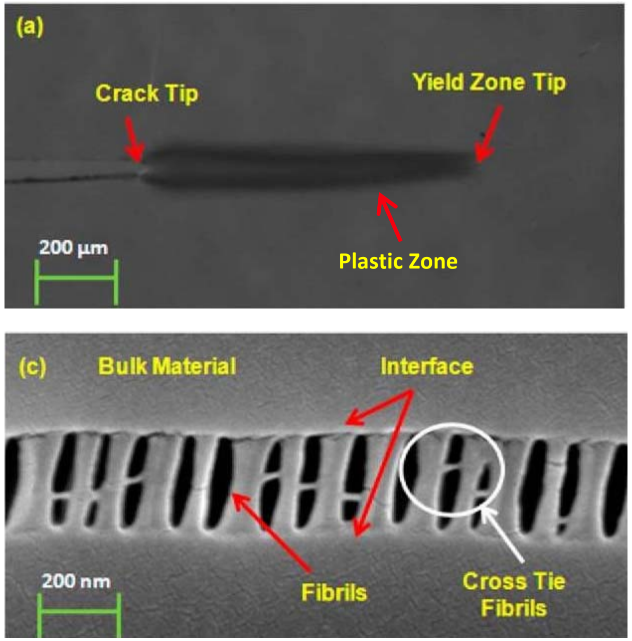
δ (crack-tip opening displacement, CTOD).

8.2 Preliminary Discussion

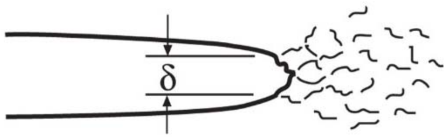
- **Behavior at Crack Tips in Real Materials**

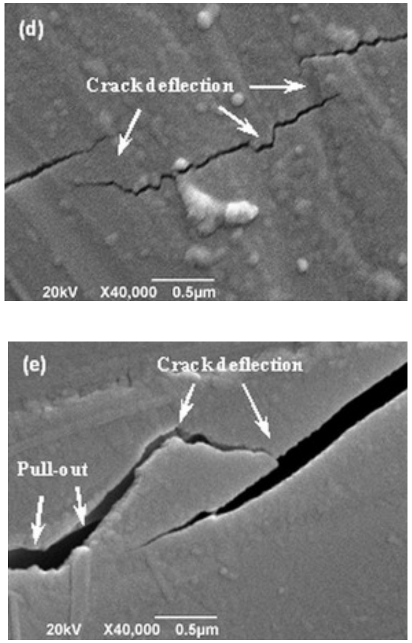
<Polymer>





<Ceramic>





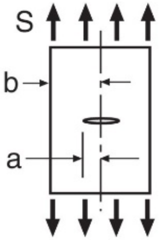
*Measurement of Cohesive Parameters of Crazes in Polystyrene Films (Experimental and Applied Mechanics) http://what-when-how.com/
**Enhancement mechanisms of graphene in nano-58S bioactive glass scaffold: mechanical and biological performance http://www.nature.com/srep/2014/140416/srep04712/full/srep04712.html

System Health & Risk Management

- **Effects of Cracks on Strength**

Stress intensity factor K
- a measure of crack severity
- affected by size, stress, and geom.
- linear-elastic assumption (LEFM)

$$K = S\sqrt{\pi a}\ (a \ll b)$$



Fracture toughness K_c
- criteria for brittle fracture
- affected by material, temperature, loading rate, thickness

$K < K_c$: elastic deformation
$K > K_c$: brittle fracure

Plane strain fracture toughness K_{Ic}
- thicker plate: a lower value of K_c
- a worst-case value of K_c
- material-dependent property

Material	Steel AISI 4130	Polymer ABS	Ceramic Concrete
Toughness K_{ic} $[\mathrm{MPa}\sqrt{\mathrm{m}}]$	110	3.0	1.19

8.2 Preliminary Discussion

- **Effects of Cracks on Strength**
  - Note that the failure data all far below the material's yield strength, σ_0.
  - The solid curve agrees well with most of the data, a degree of success for LEFM.
  - But, as the stress S approaches σ_0, the deviation occurs because of the assumption of LEFM.

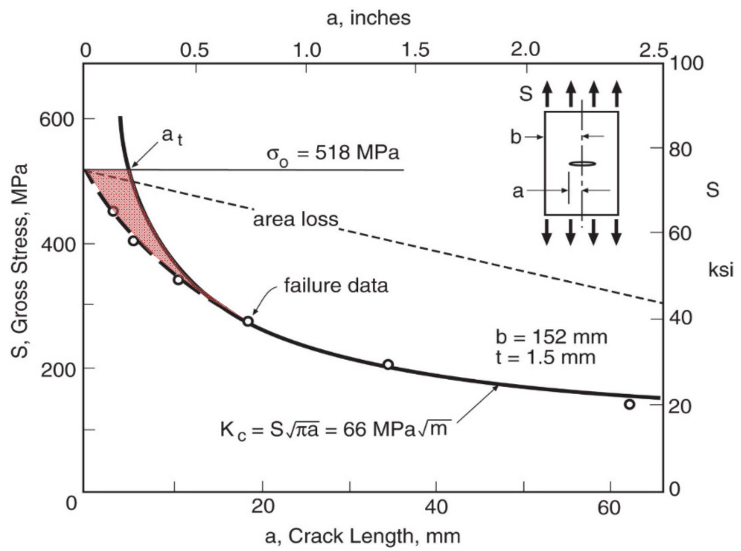


Figure 8.5 Failure data for cracked plates of 2014-T6 Al at −195°C.

Area loss **(·········)**
: area loss due to crack

$$S = \frac{P}{2t(b-a)} = \sigma_0(1 - a/b)$$

Critical stress **(———)**
: fracture due to stress intensity

$$S_c = K_c/\sqrt{\pi a}$$

Stress deviation **( )**
: due to plastic deformation violating LEFM assumption

8.2 Preliminary Discussion

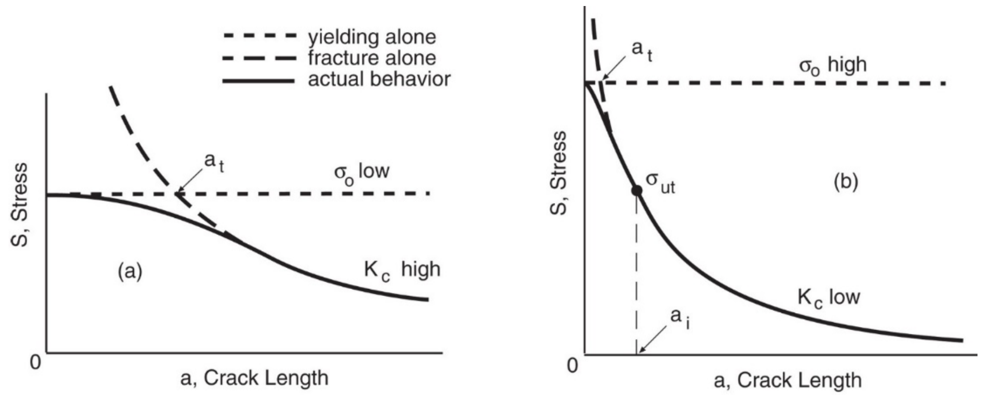
- **Effects of Cracks on Brittle vs. Ductile behavior**

  : Consider S_c=σ_0

  Transition crack length a_t

  : critical size b/w yielding and brittle fracture

$$a_t = \frac{1}{\pi}\left(\frac{K_c}{\sigma_0}\right)^2$$



<Ductile material> <Brittle material>

Material	Yield strength σ_0	Fracture toughness K_c	Transition crack length a_t
Ductile	↓	↑	↑
Brittle	↑	↓	↓

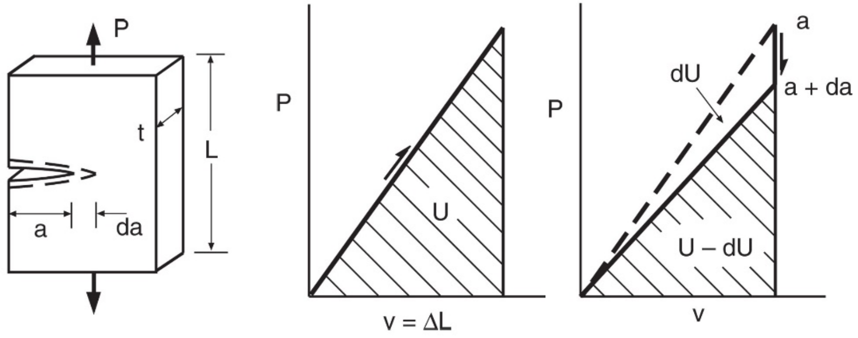
*above a_t is valid for wide and center-cracked plate

8.3 Relationship between G and K (1)

System Health & Risk Management

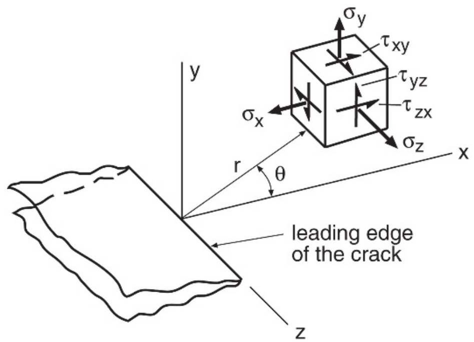
- **Strain Energy Release Rate** G (proposed by A. A. Griffith in 1920)
  : Energy per unit crack are to extend the crack (approximately true)

$$G = -\frac{1}{t}\frac{dU}{da}$$



- **Stress Intensity Factor** K_I
  : stresses near the ideal sharp crack (linear-elastic & isotropic)

$$\sigma_y \cong \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]+\ldots$$

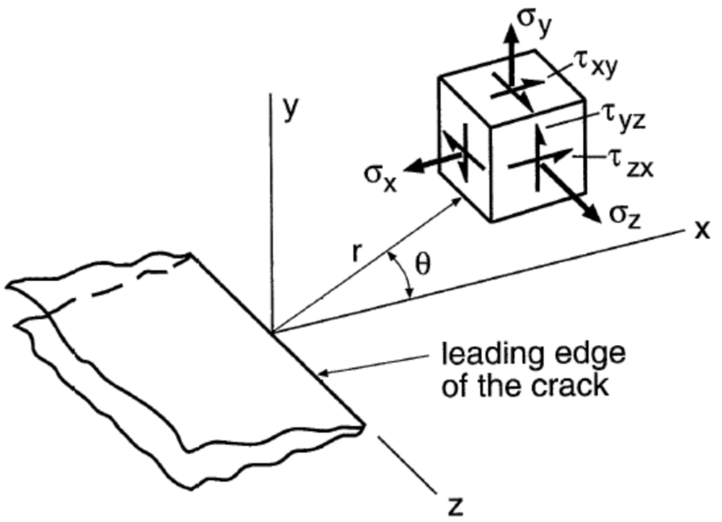


$$K_I = \lim_{r,\theta\to 0}\left(\sigma_y\sqrt{2\pi r}\right)$$ Mathematical sense

$$K_I = FS\sqrt{\pi a}$$ Practical sense

F : a dimensionless function that depends on the geometry and loading configuration

8.3 Relationship between G and K (1)



$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] + \cdots \quad \text{(a)}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] + \cdots \quad \text{(b)}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} + \cdots \quad \text{(c)}$$

$$\sigma_z = 0 \qquad \text{(plane stress)} \quad \text{(d)}$$

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right) \qquad (\text{plane strain; } \varepsilon_z = 0) \quad \text{(e)}$$

$$\tau_{yz} = \tau_{zx} = 0 \quad \text{(f)}$$

Figure 8.10 Three-dimensional coordinate system for the region of a crack tip. (Adapted from [Tada 85]; used with permission.)

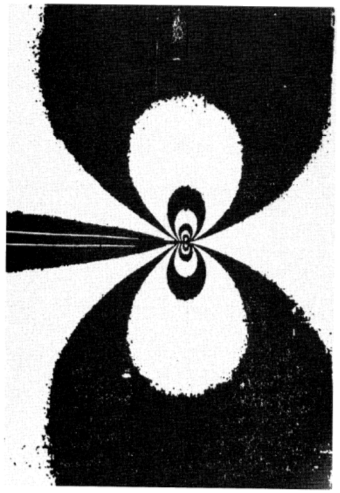


Figure 8.11 Contours of maximum in-plane shear stress around a crack tip. These were formed by the photoelastic effect in a clear plastic material. The two thin white lines entering from the left are the edges of the crack, and its tip is the point of convergence of the contours. (Photo courtesy of C. W. Smith, Virginia Tech, Blacksburg, VA.)

System Health & Risk Management

- **Energy-balance approach**

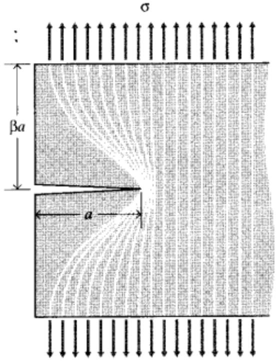
Fracture energy = released strain energy U + bond-breaking energy S

Released strain energy U

$$U^* = \frac{1}{V}\int f\,dx = \frac{E\epsilon^2}{2} = \frac{\sigma^2}{2E}$$

$$U = -2\left(\frac{\beta a * a}{2}\right)U^* = -\frac{\sigma^2}{2E}\pi a^2$$

(for plane stress loading $\beta = \pi$)



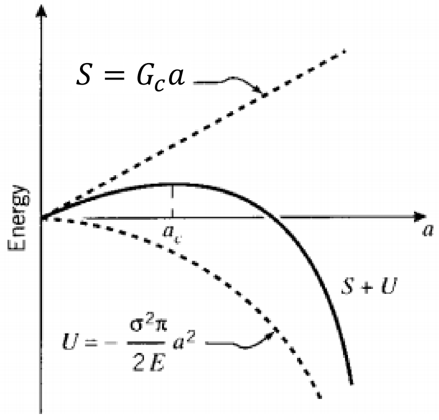
Bond-breaking energy S

$$S = G_c * a$$

Energy-balance

For $a = a_c$, $\frac{\partial(U+S)}{\partial a} = -\frac{\sigma_f^2}{E}\pi a_c + G_c = 0$

$$\therefore \sigma_f = \sqrt{\frac{EG_c}{\pi a_c}} \quad ①$$



*Introduction to Fracture Mechanics, David Roylance, 2001
http://ocw.mit.edu/courses/materials-science-and-engineering/3-11-mechanics-of-materials-fall-1999/modules/frac.pdf

8.3 Relationship between G and K(3)

System Health & Risk Management

- **Stress criteria for Rupture**

$$K_c = \sigma_f \sqrt{\pi a_c} \quad ②$$

- **Relationship b/w G and K**

From ① & ②,

$$K_c = \sqrt{\frac{EG_c}{\pi a_c}} * \sqrt{\pi a_c} = \sqrt{EG_c}$$

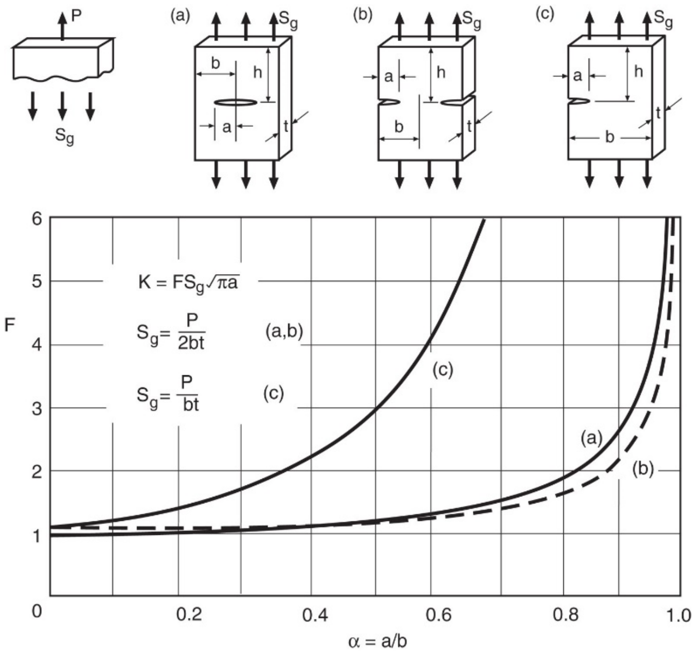
$$K_c^2 = G_c E \quad \text{for plane stress } (\sigma_z = 0)$$

$$K_c^2 = G_c E/(1 - \nu^2) \quad \text{for plane strain } (\epsilon_z = 0)$$

*Introduction to Fracture Mechanics, David Roylance, 2001
http://ocw.mit.edu/courses/materials-science-and-engineering/3-11-mechanics-of-materials-fall-1999/modules/frac.pdf

8.4 K for various cases (1)

- **Cracked plates under tension**



Values for small a/b and limits for 10% accuracy:

(a) $K = S_g\sqrt{\pi a}$ $(a/b \le 0.4)$

(b) $K = 1.12 S_g\sqrt{\pi a}$ $(a/b \le 0.6)$

(c) $K = 1.12 S_g\sqrt{\pi a}$ $(a/b \le 0.13)$

Expressions for any $\alpha = a/b$:

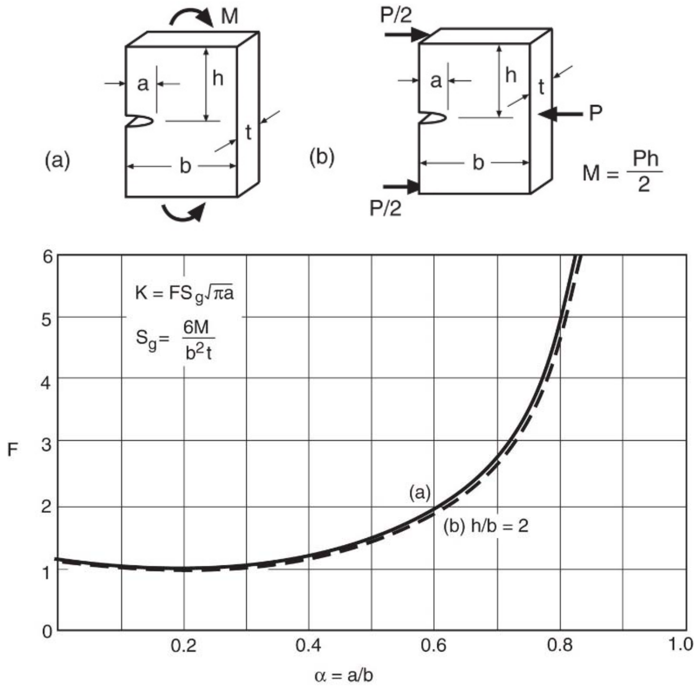
(a) $F = \dfrac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}}$ $(h/b \ge 1.5)$

(b) $F = \left(1 + 0.122\cos^4\dfrac{\pi\alpha}{2}\right)\sqrt{\dfrac{2}{\pi\alpha}\tan\dfrac{\pi\alpha}{2}}$ $(h/b \ge 2)$

(c) $F = 0.265\,(1-\alpha)^4 + \dfrac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}}$ $(h/b \ge 1)$

8.4 K for various cases (2)

- **Cracked plates under bending**



Values for small a/b and limits for 10% accuracy:

$$\text{(a, b)} \quad K = 1.12 S_g \sqrt{\pi a} \qquad (a/b \leq 0.4)$$

Expressions for any $\alpha = a/b$:

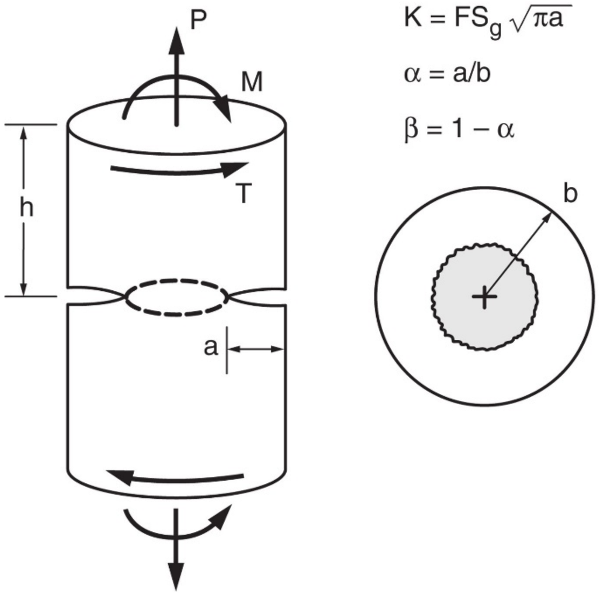
$$\text{(a)} \quad F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1 - \sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right] \qquad (\text{large } h/b)$$

(b) F is within 3% of (a) for $h/b = 4$, and within 6% for $h/b = 2$, at any a/b:

$$F = \frac{1.99 - \alpha(1-\alpha)\left(2.15 - 3.93\alpha + 2.7\alpha^2\right)}{\sqrt{\pi}\,(1+2\alpha)(1-\alpha)^{3/2}} \qquad (h/b = 2)$$

8.4 K for various cases (3)

- **Round shaft with circumferential crack**



$K = FS_g\sqrt{\pi a}$

$\alpha = a/b$

$\beta = 1 - \alpha$

(a) Axial load P: $S_g = \dfrac{P}{\pi b^2}$, $F = 1.12$ (10%, $a/b \le 0.21$)

$$F = \frac{1}{2\beta^{1.5}}\left[1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 - 0.363\beta^3 + 0.731\beta^4\right]$$

(b) Bending moment M: $S_g = \dfrac{4M}{\pi b^3}$, $F = 1.12$ (10%, $a/b \le 0.12$)

$$F = \frac{3}{8\beta^{2.5}}\left[1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 + \frac{5}{16}\beta^3 + \frac{35}{128}\beta^4 + 0.537\beta^5\right]$$

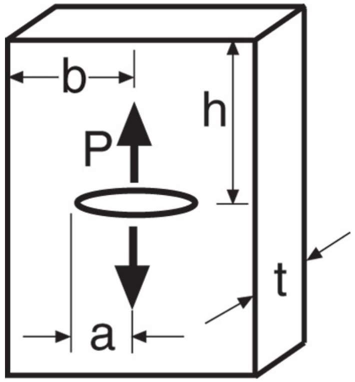
(c) Torsion T, $K = K_{III}$: $S_g = \dfrac{2T}{\pi b^3}$, $F = 1.00$ (10%, $a/b \le 0.09$)

$$F = \frac{3}{8\beta^{2.5}}\left[1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 + \frac{5}{16}\beta^3 + \frac{35}{128}\beta^4 + 0.208\beta^5\right]$$

8.4 K for various cases (4)

System Health & Risk Management

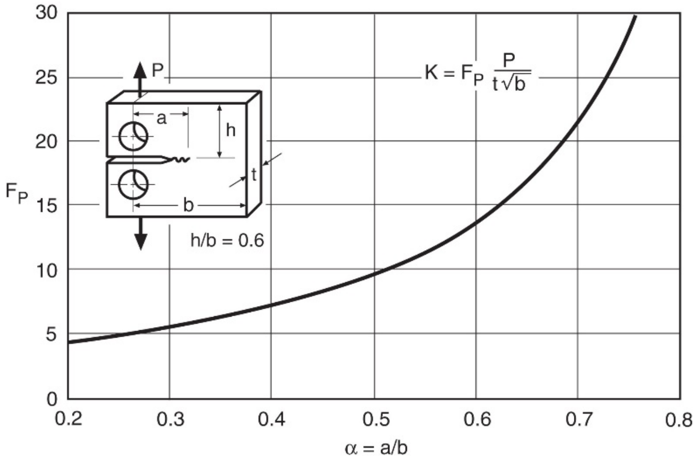
- **Plate w/ forces to the crack faces**



$$K = F_P \frac{P}{t\sqrt{b}}, \qquad \alpha = \frac{a}{b}, \qquad F_P = \frac{1}{\sqrt{\pi\alpha}} \qquad \left(10\%,\ \frac{a}{b} \leq 0.3\right)$$

$$F_P = \frac{1.297 - 0.297\cos\frac{\pi\alpha}{2}}{\sqrt{\sin\pi\alpha}} \qquad \left(0 \leq \frac{a}{b} \leq 1\right)$$

- **ASTM standard compact specimen**



$$F_P = \frac{2+\alpha}{(1-\alpha)^{3/2}}(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4) \qquad (a/b \geq 0.2)$$

8.4 K for various cases (5)

System Health & Risk Management

- **Safety factors against brittle fracture**

$$X_K = \frac{K_{Ic}}{K} = \frac{K_{Ic}}{F S_g \sqrt{\pi a}}$$

$$K_{Ic} = F_c S_g \sqrt{\pi a_c}$$

$$X_a = \frac{a_c}{a} = \left(\frac{F}{F_c} X_K\right)^2$$

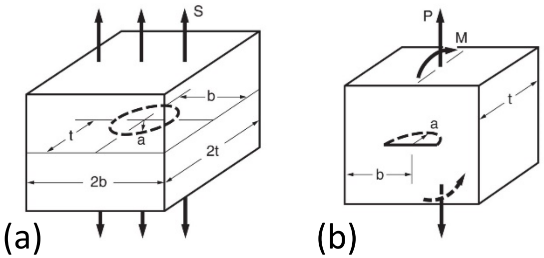
X_K : safety factor for fracture toughness
X_a : safety factor for crack size
S_g : applied stress
a : crack size
a_c : critical crack length
K : fracture toughness
K_{Ic} : plane strain fracture toughness
F_c : fracture toughness at a_c

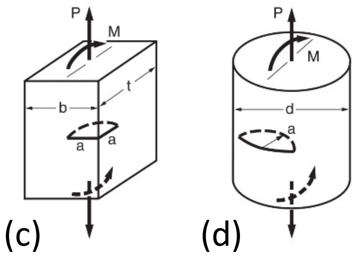
- Elements for assigning safety factors
  1) statistical information of crack shape, stress, material prop.
  2) safety factor set by design code, company policy, government regulation
- Crack size a should be quite smaller than a_c to satisfy reasonable X_K
- In general, X_K is set to be large due to great variance of K_{Ic}
- Safety factors on crack length must be rather large to achieve reasonable safety factors on K and stress

8.5 Additional topics on *K* (1)

System Health & Risk Management

- **Practical applications: Complex 3-D crack cases**
  - Useful cases include cracked plates, shafts, cracked tubes, discs, stiffened panels, etc., including three-dimensional cases
  - F values are elevated for points where the crack front intersects the surface and max. K (b)-(d).





$$K = FS\sqrt{\pi a}$$

S_t : tension, P

S_b : bending, M

Case	S_t	S_b	F for small a	Limits for $\pm10\%$ on F
(a)	$\frac{P}{4bt}$	—	$\frac{2}{\pi} = 0.637$	$\frac{a}{t}, \ \frac{a}{b} < 0.5$
(b)	$\frac{P}{2bt}$	$\frac{3M}{bt^2}$	0.728	$\frac{a}{t} < 0.4, \ \frac{a}{b} < 0.3$
(c)	$\frac{P}{bt}$	$\frac{6M}{bt^2}$	0.722	$\frac{a}{t} < 0.35, \ \frac{a}{b} < 0.2$
(d)	$\frac{4P}{\pi d^2}$	$\frac{32M}{\pi d^3}$	0.728	$\frac{a}{d} < 0.2$ or 0.35[1]

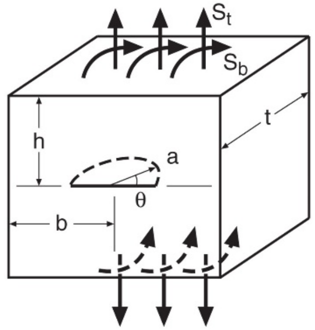
Note: [1]Different limits for tension or bending, respectively.

Figure 8.17 Stress intensity factors for (a) an embedded circular crack, (b) half-circular surface crack, (c) quarter-circular corner crack, and (d) half-circular surface crack in a shaft

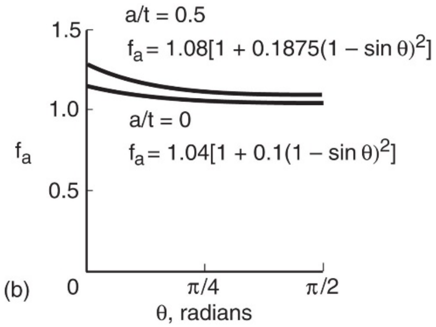
8.5 Additional topics on *K* (2)

- **Half-circular surface crack**

(a)



(b)



Functional forms for $a/b < 0.5$, $h/b > 1$:

$$K = f_a f_w \frac{2}{\pi}(S_t + f_b S_b)\sqrt{\pi a}, \qquad f_w = \sqrt{\sec\left(\frac{\pi a}{2b}\sqrt{\frac{a}{t}}\right)}$$

where $f_a = f_a(a/t,\ \theta), \qquad f_b = f_b(a/t)$

Expressions for $\theta = 0$ and $180°$ (surface) for any $\alpha = a/t$:

$$f_a = (1.04 + 0.2017\alpha^2 - 0.1061\alpha^4)(1.1 + 0.35\alpha^2)\ , \qquad f_b = 1 - 0.45\alpha$$

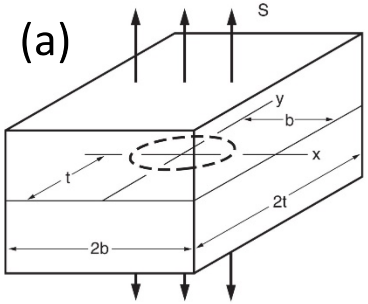
Expressions for $\theta = 90°$ (deepest point) for any $\alpha = a/t$:

$$f_a = 1.04 + 0.2017\alpha^2 - 0.1061\alpha^4\ , \qquad f_b = 1 - 1.34\alpha - 0.03\alpha^2$$

8.5 Additional topics on *K* (3)

- **Elliptical crack**

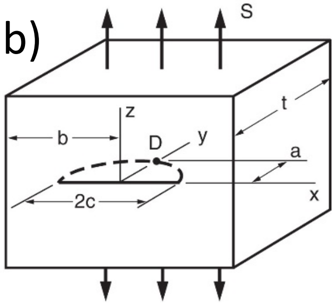
(a)



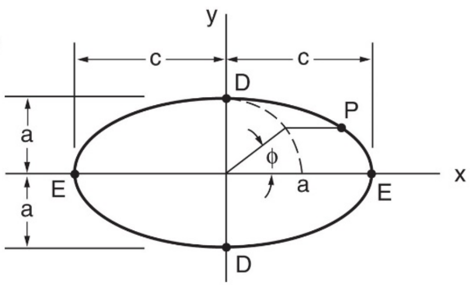
$$K = S\sqrt{\frac{\pi a}{Q}} f_\phi, \qquad f_\phi = \left[\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right]^{1/4} \quad (a/c \le 1)$$

$$\sqrt{Q} = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\beta}\, d\beta, \qquad k^2 = 1 - \left(\frac{a}{c}\right)^2 \quad (Q\text{: flow shape factor})$$

(b)



$$K_D = F_D S\sqrt{\frac{\pi a}{Q}}, \qquad Q \approx 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad (a/c \le 1)$$

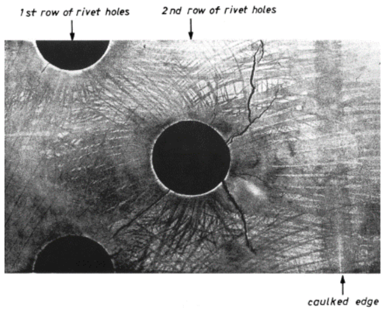


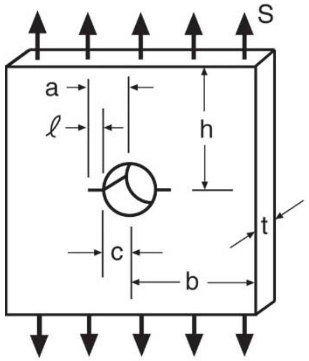
Case	Values for small a/t, c/b	Limits for 10% accuracy
(a)	$F_D = 1$	$a/t < 0.4,\ c/b < 0.2$
(b)	$F_D \approx 1.12$	$a/t < 0.3$,[1] $c/b < 0.2$

Note: [1]Except limit to $a/t < 0.16$ if $a/c < 0.25$.

8.5 Additional topics on *K* (4)

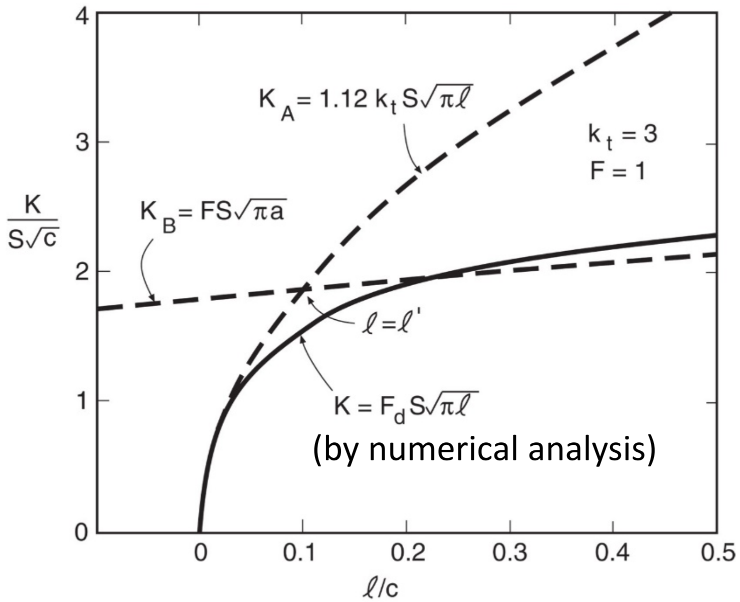
- **Crack growing from notches (holes, fillets, rivets, etc.)**

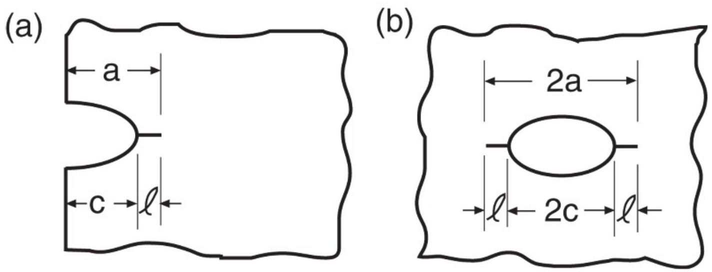




$$K = F_d S\sqrt{\pi \ell}, \qquad d = \frac{\ell}{a} = \frac{\ell}{c+\ell}$$

$$F_d = 0.5(3 - d)[1 + 1.243(1 - d)^3]$$





<Short crack>

For small l/c,

$K_A = 1.12 S' \sqrt{\pi l}$

$S' = k_t S$

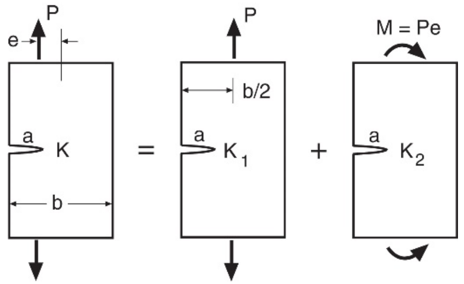
<Long crack>

For large l/c, $w_{\mathrm{hole}} \approx a_{\mathrm{crack}}$.

$K_B = FS\sqrt{\pi a}$

8.5 Additional topics on *K* (5)

- **Superposition for combined loading**

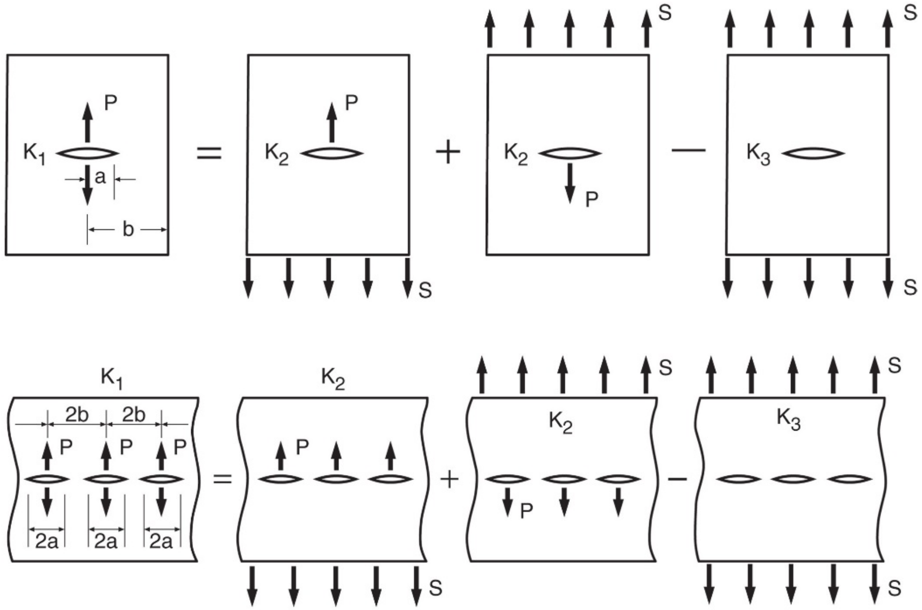


<Eccentric loading of a plate>

$$K_1 = F_1 S_1 \sqrt{\pi a}, \qquad S_1 = \frac{P}{bt}$$

$$K_2 = F_2 S_2 \sqrt{\pi a}, \qquad S_2 = \frac{6M}{b^2 t} = \frac{6Pe}{b^2 t}$$

$$K = K_1 + K_2 = \frac{P}{bt}\left(F_1 + \frac{6F_2 e}{b}\right)\sqrt{\pi a}$$



<Single/row of a cracks (bolt, rivet)>

$$K_1 = F_{P1}\frac{P}{t\sqrt{b}}$$

$$K_3 = F_3 S\sqrt{\pi a}$$

$$K_2 = \frac{1}{2}(K_1 + K_3) = \frac{P}{2t\sqrt{b}}\left(F_{P1} + \frac{F_3\sqrt{\pi a}}{2}\right)$$

$$K_1 = \frac{P}{t\sqrt{b}}\frac{1}{\sqrt{\sin \pi \alpha}}$$

$$K_3 = S\sqrt{2b\tan\frac{\pi\alpha}{2}}$$

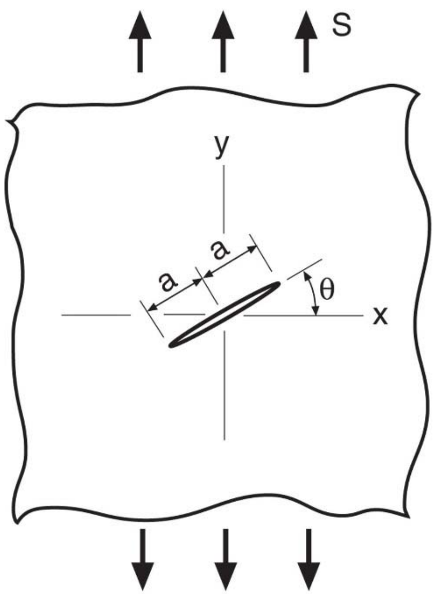
$$K_2 = \frac{1}{2}(K_1 + K_3) = \frac{P}{2t\sqrt{b}}\left(\frac{1}{\sqrt{\sin \pi\alpha}} + \sqrt{\frac{1}{2}\tan\frac{\pi\alpha}{2}}\right)$$

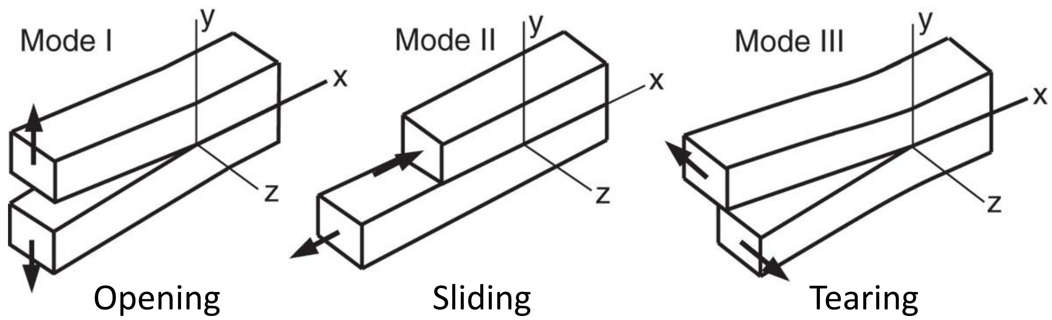
8.5 Additional topics on *K* (6)

- **Inclined or parallel cracks to an stress**
  - *Alternating crack direction*
    : It does not grow in its original plane.
  - *Interactive stresses*
    : Fracture modes are not independent.

  Toughness for mixed-mode are generally unknown.

  - *Possible approach*
    : Projection of crack normal to the stress direction





$$K_I = S(\cos^2\theta)\sqrt{\pi a}$$
$$K_{II} = S(\cos\theta)(\sin\theta)\sqrt{\pi a}$$
$$\approx \quad K = S\sqrt{\pi\, \mathrm{a} \cos\theta}$$

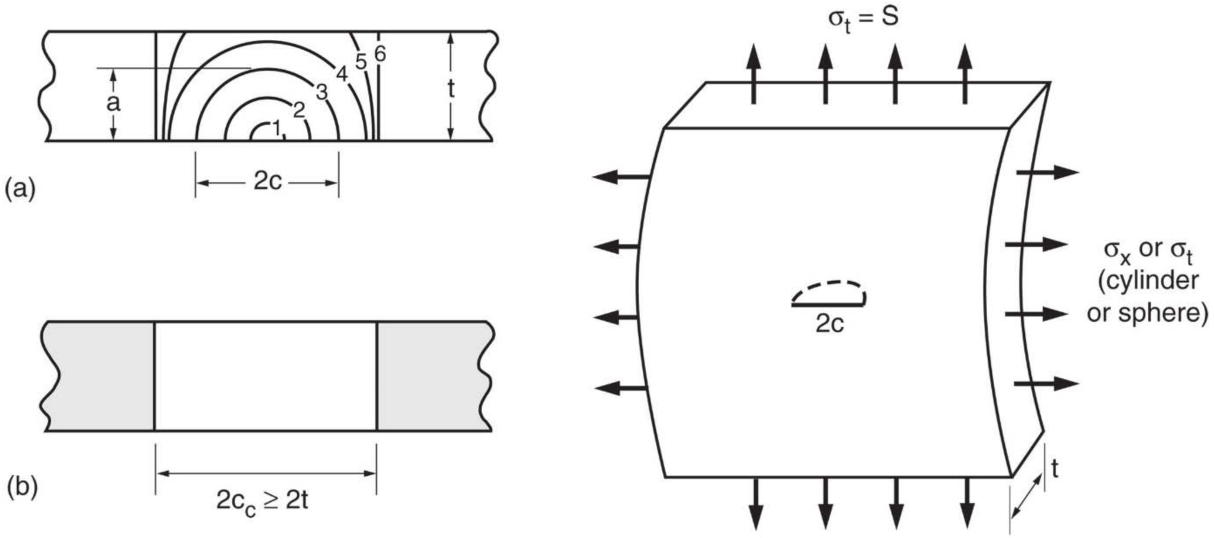
8.5 Additional topics on *K* (7)

- **Leak-Before-Break (LBB) design of pressure vessels**
  - Pressure vessels should be designed to leak before fracture.
  - A through-wall crack length $2c \approx 2t$
  - Critical crack size c_c

$$K_{I_c} = FS\sqrt{\pi c_c}$$
$$F = 1\ (\because \text{ wide plate})$$

$$\Rightarrow \quad c_c = \frac{1}{\pi}\left(\frac{K_{I_c}}{\sigma_t}\right)^2$$

$t < c_c$: leak before break
$t > c_c$: brittle fracture



8.6 Trends of Fracture Toughness K_{I_c} (1)

- **Fixtures for a fracture toughness with crack growth bend specimen**

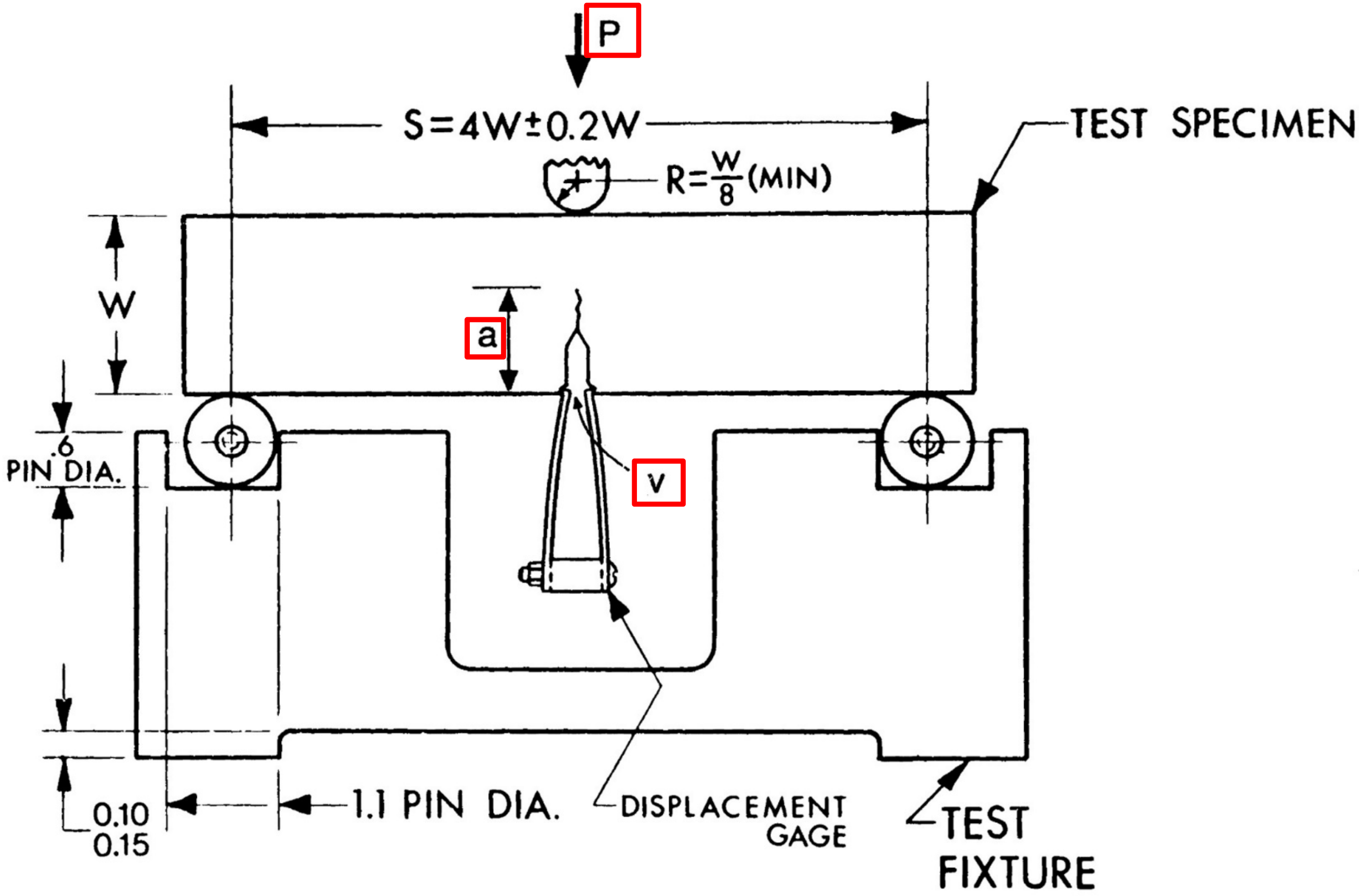
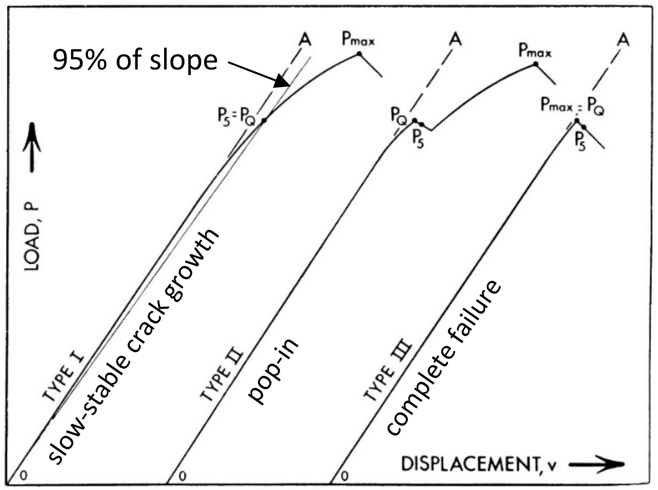
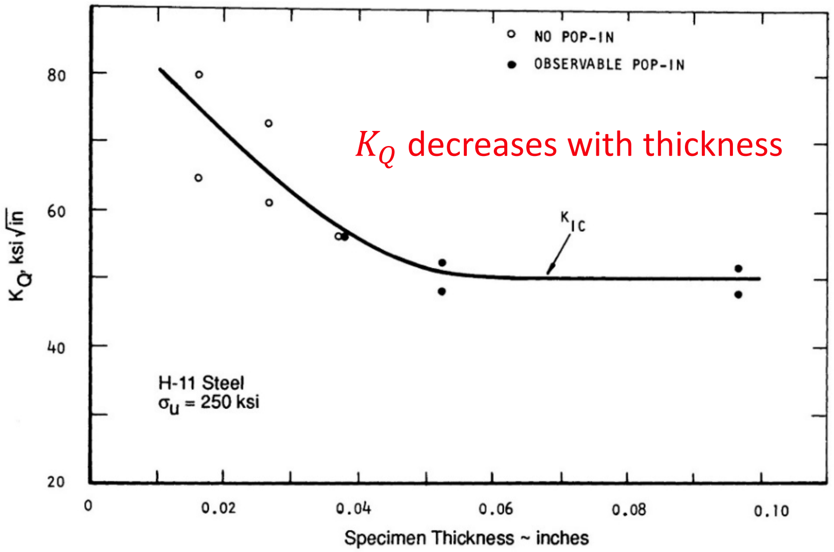


Figure 8.27 Fixtures for a fracture toughness test on a bend specimen. The dimension *W* corresponds to our *b*. (Adapted from [ASTM 97] Std. E399; copyright © ASTM; reprinted with permission.)

8.6 Trends of Fracture Toughness K_{I_c} (2)

- **Fracture Toughness**
  - A deviation from linearity on the *P-v* plot, or a sudden drop in force due to rapid cracking, identifies a point P_Q corresponding to an early stage of cracking
  - The value of K, denoted K_Q, is the stress intensity factor corresponding to P_Q
  - K_Q may be somewhat lower than the value K_c corresponding to the final fracture of the specimen.
  - Fracture toughness testing of metals based on LEFM principles governed by several ASTM standards, notably Standard Nos. E399 and E1820.
  - Standards No. D5045 (polymers) and No. C1421 (ceramics)





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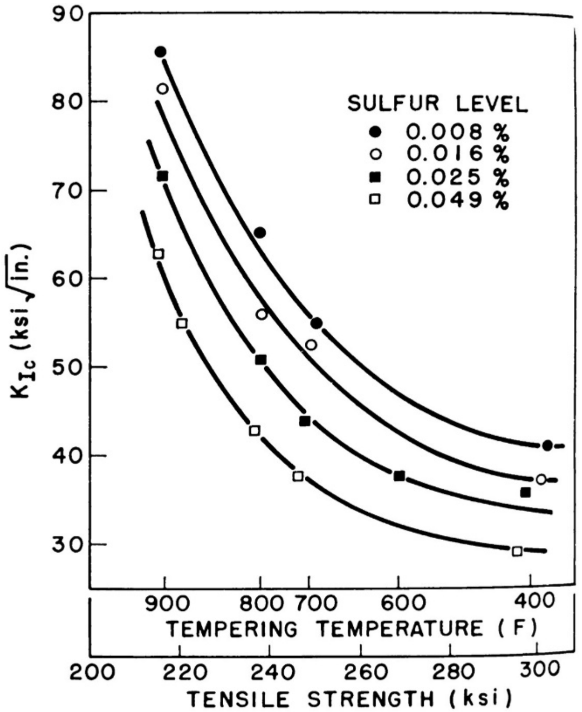
- **Material**
  - Material-dependent K_{I_c}

Material	Metal	Polymer	Ceramic
Toughness $[\text{MPa}\sqrt{\text{m}}]$	20~200	1~5	1~5

  - Large CoV of K_{I_c}: 10~20%

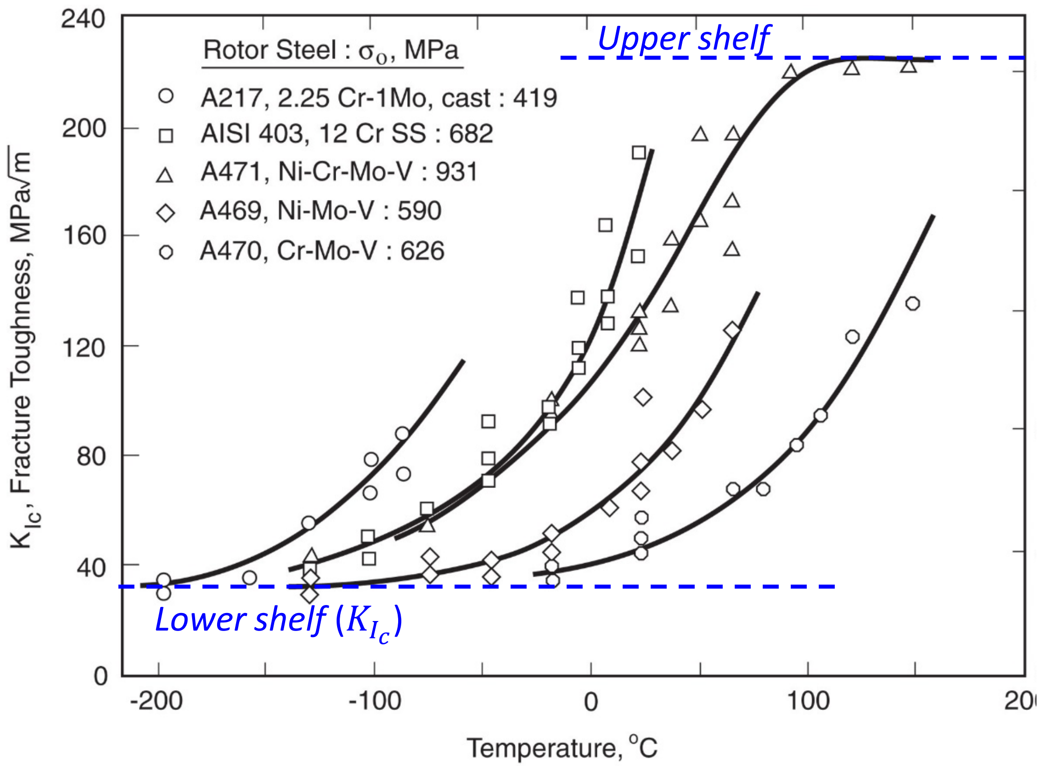
- **Microstructural influences**
  - Chemical composition : sulfide inclusions facilitate fracture.
  - Processing (forging, rolling, extruding) : anisotropy and planes of the flattened grains
  - Neutron radiation (radiation embrittlement) : large numbers of point defects

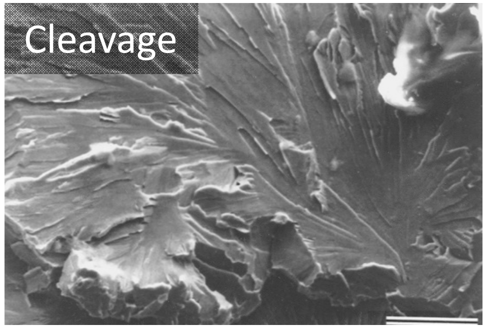


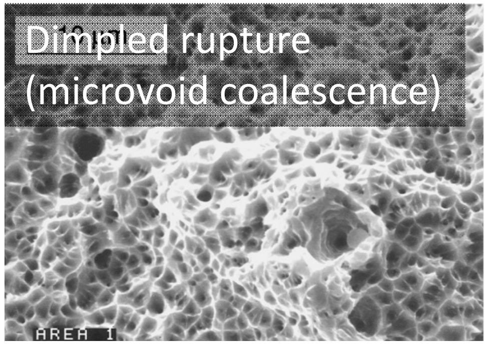
*CoV: coefficient of variation (CoV $= \sigma/\mu$)

8.6 Trends of Fracture Toughness K_{I_c} (4)

- **Temperature**
  - *Cleavage* @ low temp.
    : fracture with little plastic deform. along the crystal planes of low resistance
  - *Dimples rupture* @ high temp.
    : fracture with plasticity-induced formation, growth, and joining of tiny voids



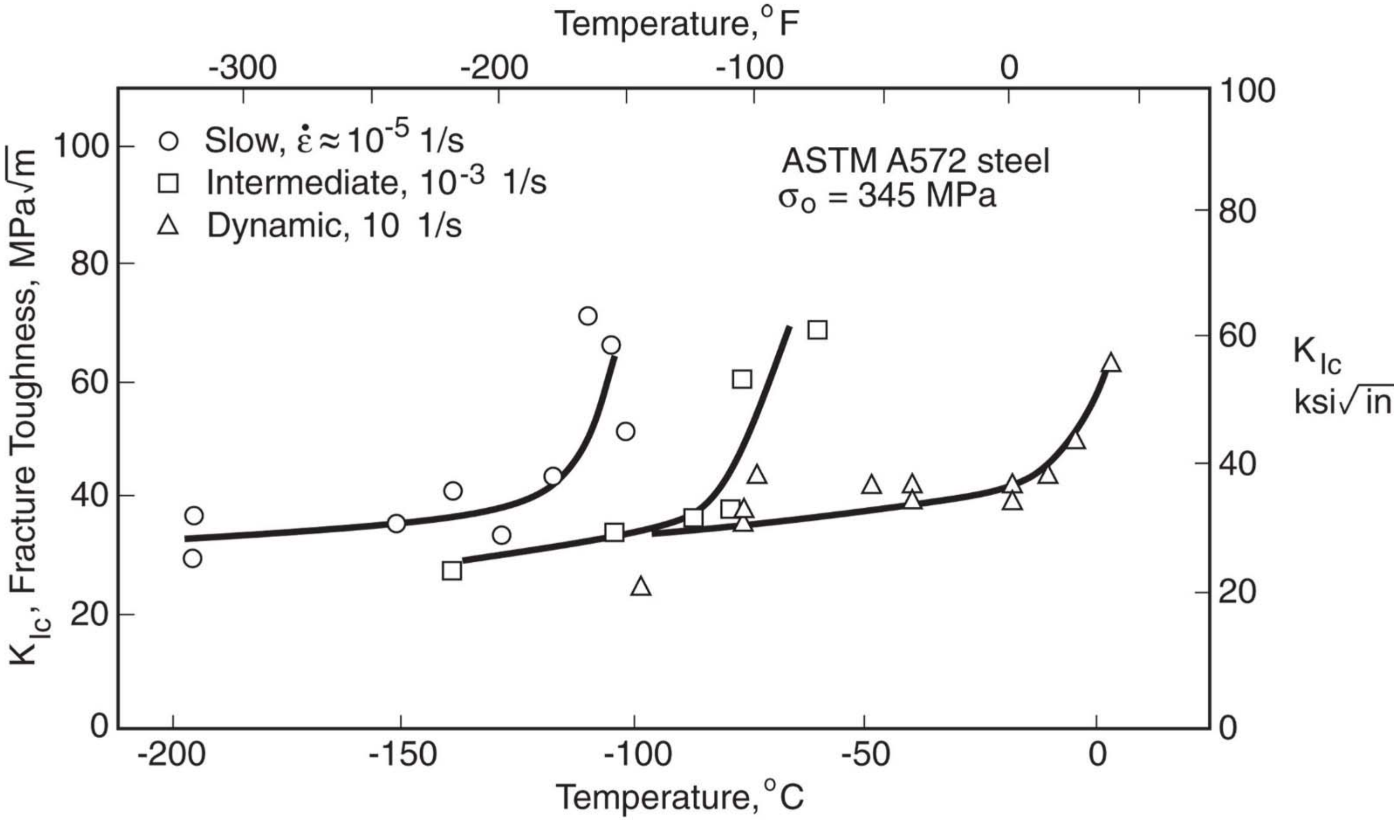




<Fracture mechanics shift>

8.6 Trends of Fracture Toughness K_{I_c} (5)

- **Temperature and Loading rate**
  - A higher rate of lading lowers the fracture toughness K_{I_c}. (temperature shift)
  - Statistical variation of K_{I_c} is especially large within the temperature transition.



8.7 Fracture mechanics under Plasticity (1)

- **The size of plastic zone r_o**
  - Yielding at crack tips (*plastic zone* ~ $2r_{o\sigma}$) will be studied.
  - Plastic zone may not be large if the LEFM theory is to be applied.
  - Plastic zone size increases if K is increased, but smaller for the same K for materials with higher σ_0

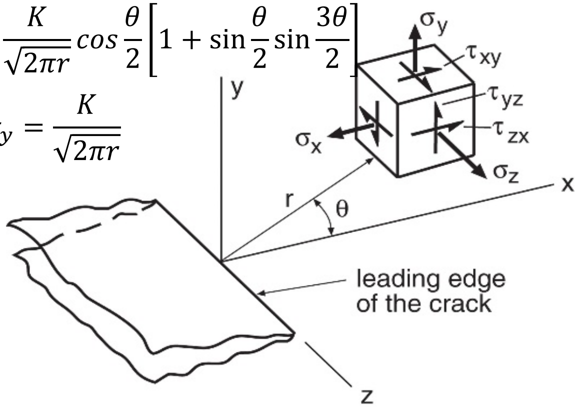
For ***plane stress*** $(\sigma_z = 0)$,

$$\sigma_x = \sigma_y = \frac{K}{\sqrt{2\pi r}}$$

$$r_{o\sigma} = \frac{1}{2\pi}\left(\frac{K}{\sigma_o}\right)^2 \rightarrow 2r_{o\sigma} = \frac{1}{\pi}\left(\frac{K}{\sigma_o}\right)^2$$

$$\sigma_y \cong \frac{K}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\lim_{\theta\to 0} \sigma_y = \frac{K}{\sqrt{2\pi r}}$$

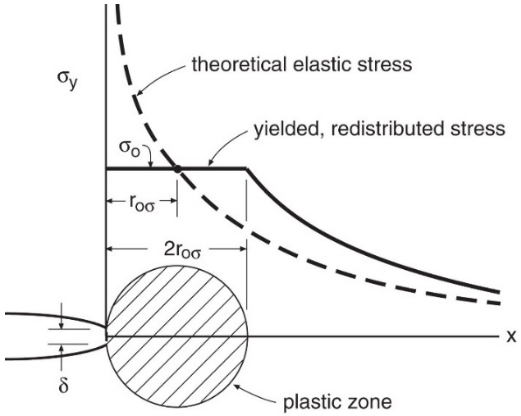


For ***plane strain*** $(\varepsilon_z = 0, \sigma_z = 2\nu\sigma_y)$,

For octa. or max. shear stress yield criterion,

$$\sigma_x = \sigma_y = \frac{\sigma_0}{1-2\nu} \approx 2.5\sigma_0 \rightarrow \sqrt{3}\sigma_0$$

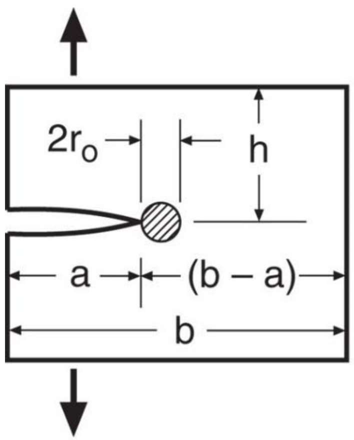
$$2r_{o\varepsilon} = \frac{1}{3\pi}\left(\frac{K}{\sigma_o}\right)^2$$



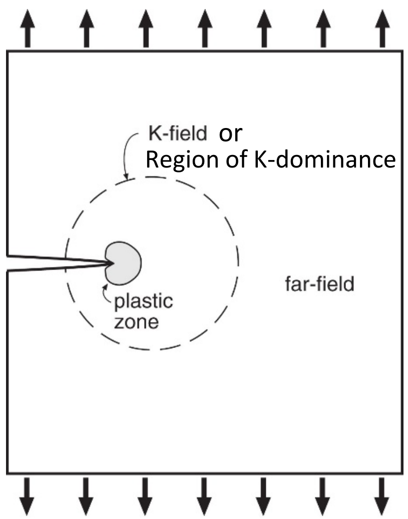
8.7 Fracture mechanics under Plasticity (3)

- **Plasticity limitations on LEFM**
  - LEFM is valid for small plastic zone compared with crack tip-to-boundary dist.
  - $8r_o$ is generally considered to be sufficient → 4 times of crack zone size
  - Since $r_{o\sigma}$ is larger than $r_{o\varepsilon}$, an overall limit of the use of LEFM is

$$a, (b-a), h \geq 8r_{o\sigma} = \frac{4}{\pi}\left(\frac{K}{\sigma_o}\right)^2 \qquad \text{(LEFM applicable)}$$



<Crack specimen geometry>



<LEFM applicable region, K-field>

- **Plasticity limitations on LEFM**
  - LEFM is valid for small plastic zone compared with crack tip-to-boundary dist.
  - $8r_o$ is generally considered to be sufficient → 4 times of crack zone size
  - Since $r_{o\sigma}$ is larger than $r_{o\varepsilon}$, an overall limit of the use of LEFM is

$$a, (b-a), h \geq 8r_{o\sigma} = \frac{4}{\pi}\left(\frac{K}{\sigma_o}\right)^2 \qquad \text{(LEFM applicable)}$$

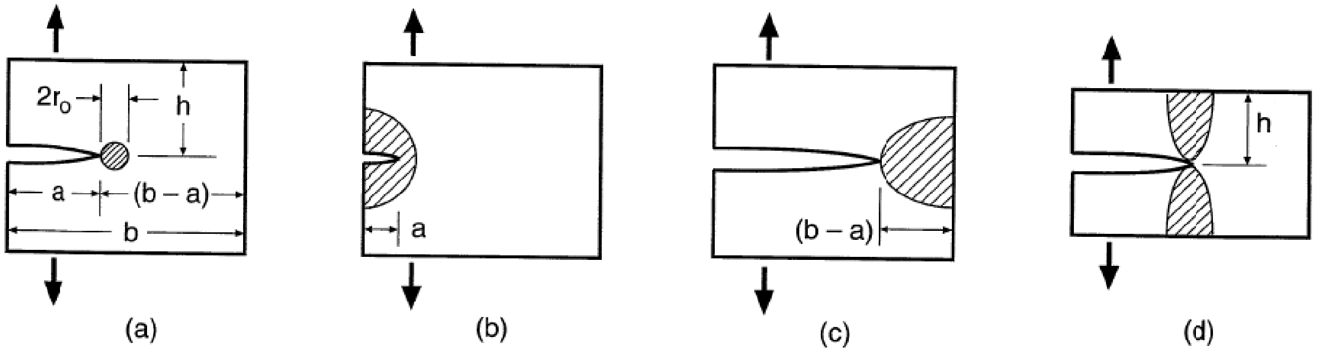
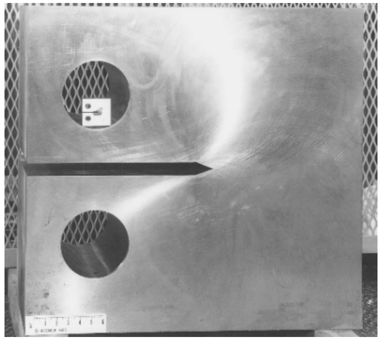
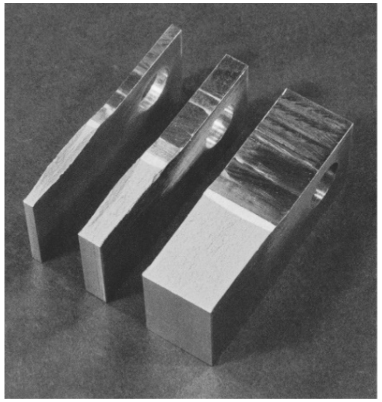
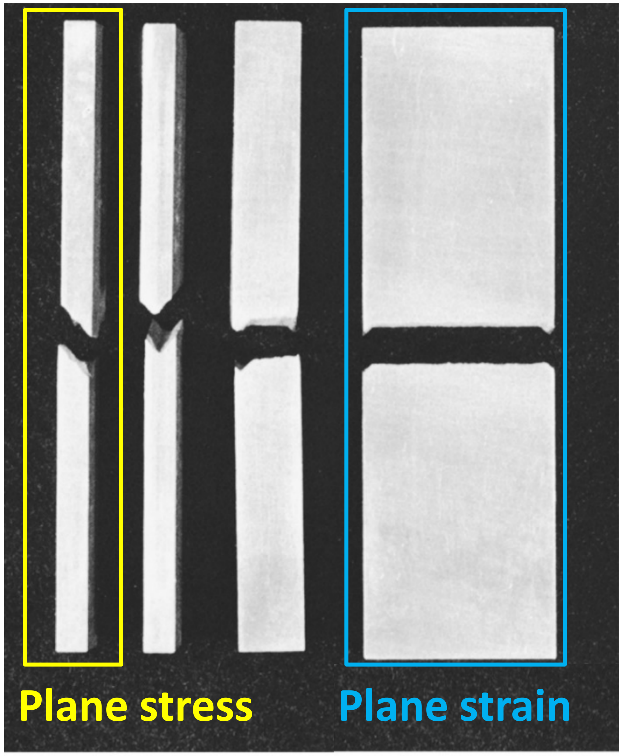


Figure 8.46 Small plastic zone compared with planar dimensions (a), and situations where LEFM is invalid due to the plastic zones being too large compared with (b) crack length, (c) uncracked ligament, and (d) member height.

- **The size of plastic zone r_o**
  - For thin specimen, poisson contraction in the thickness occurs freely around the crack tip, resulting in yielding on shear planes inclined through the thickness.
  - Fracture under plane stress also occurs along such inclined planes.







8.7 Fracture mechanics under Plasticity (4)

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- **Fracture mechanics beyond linear elasticity**
  - Condition of "$a, (b-a), h \geq 8r_{o\sigma}$" is not satisfied, (under excessive yielding)
  - LEFM and K are not applicable due to excessive yielding.
  - Following three approaches are available.

(1) Plastic zone adjustment

- The stress outside of the plastic zone is similar to elastic stress for a hypothetical crack ($a_e = a + r_{o\sigma}$) with its tip near the center of the plastic zone.
- Not applicable for large stress to cause yielding on whole section; 80% of the fully plastic force or moment.

$$K = FS\sqrt{\pi a}$$

$$K_e = F_e S\sqrt{\pi a_e} = F_e S\sqrt{\pi(a + r_{o\sigma})}$$

where

$$F_e = F(a_e/b)$$

$$r_{o\sigma} = \frac{1}{2\pi}\left(\frac{K_e}{\sigma_o}\right)^2$$

8.7 Fracture mechanics under Plasticity (5)

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(2) *J*-Integral

- J is the generalization of the strain energy release rate, G, to nonlinear-elastic.
- It retains significance as a measure of the intensity of the elasto-plastic (nonlinear) stress and strain fields around the crack tip.
- Two different P-v curves (*a* and *a*+*da*) need to be obtained from independent tests on two different members.
- Basis of fracture toughness tests with small specimen, ASTM Standard No. E1820.

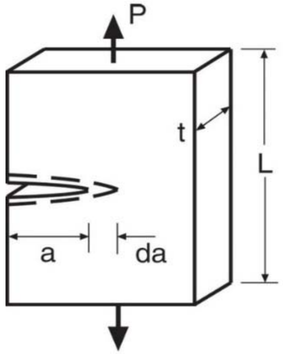
$$K_{I_c}^2 = G_{I_c}E'$$

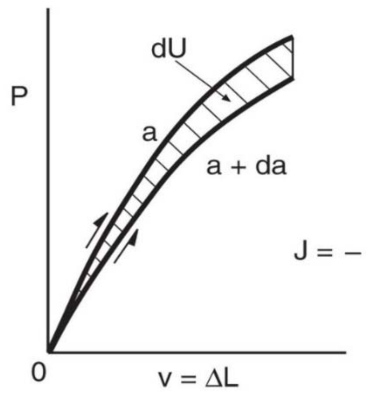
Eq. (8.10)

$$K_{I_cJ} = \sqrt{J_{I_c}E'}$$

$E' = E$ for plane stress ($\sigma_z = 0$)

$E' = E/(1-\nu^2)$ for plane strain ($\epsilon_z = 0$)





$$J = -\frac{1}{t}\frac{dU}{da}$$

$$K_J = \sqrt{JE}$$

$$K_J \approx K\sqrt{1 + \frac{\varepsilon_p}{\varepsilon_e\sqrt{n}}}$$

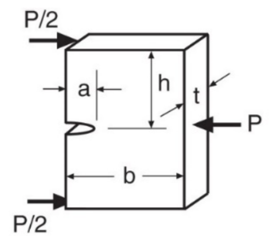
8.7 Fracture mechanics under Plasticity (6)

(2) *J*-Integral: Fracture toughness tests for J_{I_c}

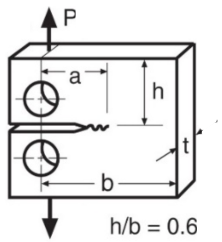
- Complexity encountered in J_{I_c} testing is that nonlinearity in P-v behavior is due to a combination of crack growth and plastic deformation
- J calculation for the standard bend and compact specimens

$$J = J_{el} + J_{pl} = \frac{K^2(1-\nu^2)}{E} + \frac{\eta A_{pl}}{t(b-a)}$$

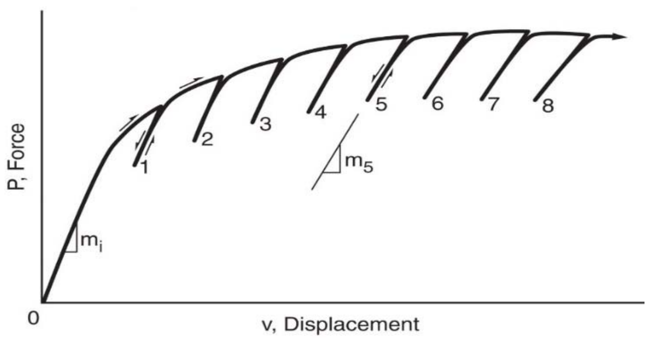
$\eta = 1.9$ for bend specimen
$\eta = 2 + 0.522(1 - a/b)$ for compactspecimen



<Bend specimen>



<Compact specimen>



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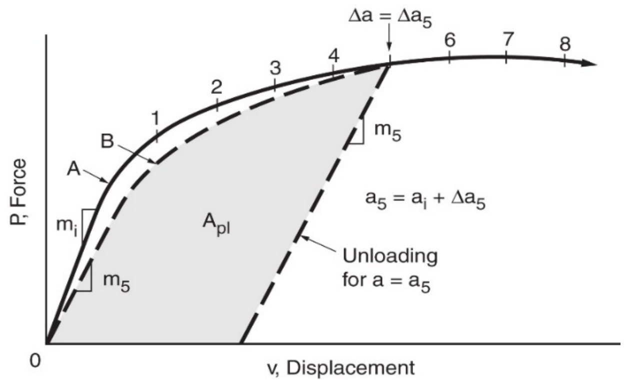

<Potential drop test>

(3) Crack-Tip Opening Displacement (CTOD; δ)

- K can be used to estimate the displacement separating the crack faces.
- CTOD is also used as the basis of fracture toughness tests; ASTM Standards E1290 and E1820

$$\delta \approx \frac{K^2}{E\sigma_o} \approx \frac{J}{\sigma_o}$$

- **Summary**