Nonlinear Optical Engineering

Pulse Propagation in Fibers (1)
(NFO 5th ed: 2.1 ~ 2.2)

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: yoonchan@snu.ac.kr

Maxwell's Equations (1)

Maxwell's equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \cdot \mathbf{B} = 0$$

For most cases:

$$\leftarrow \mathbf{J} = 0,\ \ \rho_f = 0,\ \ \mathbf{M} = 0,\ \ and\ \ \chi^{(2)} = 0$$

Constitutive relations:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P},$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

Wave equation:

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \qquad \leftarrow \mathbf{P}(r,t) = \mathbf{P}_L(\mathbf{r},t) + \mathbf{P}_{NL}(\mathbf{r},t)$$

$$\rightarrow \mathbf{P}_L(\mathbf{r},t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t-t') \cdot \mathbf{E}(\mathbf{r},t')dt'$$

"Small perturbation" $\rightarrow \mathbf{P}_{NL}(\mathbf{r},t) = \varepsilon_0 \int\int\int_{-\infty}^{\infty} \chi^{(3)}(t-t_1, t-t_2, t-t_3) \vdots \mathbf{E}(\mathbf{r},t_1)\mathbf{E}(\mathbf{r},t_2)\mathbf{E}(\mathbf{r},t_3)dt_1 dt_2 dt_3$

In the frequency domain (linear response only):

"Dielectric constant"

$$\nabla \times \nabla \times \tilde{\mathbf{E}}(\mathbf{r},\omega) - \varepsilon(\omega)\frac{\omega^2}{c^2}\tilde{\mathbf{E}}(\mathbf{r},\omega) = 0 \qquad \leftarrow \varepsilon(\omega) = 1 + \tilde{\chi}^{(1)}(\omega)$$

$$\leftarrow \tilde{\mathbf{E}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r},t)\exp(i\omega t)dt$$

Maxwell's Equations (2)

Dielectric constant: $\varepsilon(\omega) = 1 + \tilde{\chi}^{(1)}(\omega)$

$$\rightarrow \varepsilon(\omega) = (n + i\alpha c / 2\omega)^2$$

"Refractive index" $\rightarrow n(\omega) = 1 + \frac{1}{2}\mathrm{Re}[\tilde{\chi}^{(1)}(\omega)]$

"Absorption coefficient" $\rightarrow \alpha(\omega) = \frac{\omega}{nc}\mathrm{Im}[\tilde{\chi}^{(1)}(\omega)]$

Recall:

$$\nabla \times \nabla \times \tilde{\mathbf{E}}(\mathbf{r},\omega) - \varepsilon(\omega)\frac{\omega^2}{c^2}\tilde{\mathbf{E}}(\mathbf{r},\omega) = 0$$

$$\leftarrow k = \frac{\omega}{c} n + i\frac{\alpha}{2}$$

Lossless medium (or low loss):

$$\rightarrow \nabla^2 \tilde{\mathbf{E}} + n^2(\omega)\frac{\omega^2}{c^2}\tilde{\mathbf{E}} = 0$$

Fiber Modes

Wave equation:

"Beware!"

$$\nabla^2\tilde{\mathbf{E}} + n^2(\omega)\frac{\omega^2}{c^2}\tilde{\mathbf{E}} = 0$$

$$\rightarrow \mathbf{E}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{\mathbf{E}}(\mathbf{r},\omega)\exp(-i\omega t)d\omega$$

$$\rightarrow \tilde{E}_z(\mathbf{r},\omega) = A(\omega)F(\rho)\exp(\pm im\phi)\exp(i\beta z)$$

$$\rightarrow \frac{d^2F}{d\rho^2} + \frac{1}{\rho}\frac{dF}{d\rho} + (n^2k_0^2 - \beta^2 - \frac{m^2}{\rho^2})F = 0$$

$$\rightarrow F(\rho) = C_1J_m(\kappa\rho) + C_2N_m(\kappa\rho), \quad \rho \le a \quad \leftarrow \kappa = (n_1^2k_0^2 - \beta^2)^{1/2}$$

$$\rightarrow F(\rho) = D_1K_m(\gamma\rho) + D_2I_m(\gamma\rho), \quad \rho \ge a \quad \leftarrow \gamma = (\beta^2 - n_2^2k_0^2)^{1/2}$$

Eigenvalue equation:

$$\rightarrow \left[\frac{J_m'(\kappa a)}{\kappa J_m(\kappa a)} + \frac{K_m'(\gamma a)}{\gamma K_m(\gamma a)}\right]\left[\frac{J_m'(\kappa a)}{\kappa J_m(\kappa a)} + \frac{n_2^2}{n_1^2}\frac{K_m'(\gamma a)}{\gamma K_m(\gamma a)}\right] = \left(\frac{m\beta k_0(n_1^2 - n_2^2)}{an_1\kappa^2\gamma^2}\right)^2$$

Single-mode condition:

$$\rightarrow V = \kappa_c a = k_0 a(n_1^2 - n_2^2)^{1/2} \qquad \rightarrow V < 2.405$$