

Nonlinear Optical Engineering

Pulse Propagation in Fibers (2) (NFO 5th ed: 2.3 ~ 2.4)

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Pulse-Propagation Equation (1)

Nonlinear pulse propagation:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2} \quad \leftarrow \text{Perturbation, scalar, \& quasi-monochromatic approach}$$

Electric field:

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2} \hat{x} [E(\mathbf{r}, t) \exp(-i\omega_0 t) + c.c.]$$

Linear polarization component:

$$\mathbf{P}_L(\mathbf{r}, t) = \frac{1}{2} \hat{x} [P_L(\mathbf{r}, t) \exp(-i\omega_0 t) + c.c.] \rightarrow P_L(\mathbf{r}, t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi_{xx}^{(1)}(t-t') E(\mathbf{r}, t') \exp[i\omega_0(t-t')] dt'$$

Nonlinear polarization component:

$$\mathbf{P}_{NL}(\mathbf{r}, t) = \frac{1}{2} \hat{x} [P_{NL}(\mathbf{r}, t) \exp(-i\omega_0 t) + c.c.]$$

$$\rightarrow \mathbf{P}_{NL}(\mathbf{r}, t) = \varepsilon_0 \int \int \int_{-\infty}^{\infty} \chi^{(3)}(t-t_1, t-t_2, t-t_3) \mathbf{E}(\mathbf{r}, t_1) \mathbf{E}(\mathbf{r}, t_2) \mathbf{E}(\mathbf{r}, t_3) dt_1 dt_2 dt_3$$

\leftarrow Nearly instantaneous response

$$\rightarrow \mathbf{P}_{NL}(\mathbf{r}, t) = \varepsilon_0 \chi^{(3)} \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t)$$

$$\rightarrow P_{NL}(\mathbf{r}, t) \approx \varepsilon_0 \varepsilon_{NL} E(\mathbf{r}, t) \quad \leftarrow \varepsilon_{NL} = \frac{3}{4} \chi_{xxxx}^{(3)} |E(\mathbf{r}, t)|^2$$

Pulse-Propagation Equation (2)

Electric field in the Fourier domain:

$$\tilde{E}(\mathbf{r}, \omega - \omega_0) = \int_{-\infty}^{\infty} E(\mathbf{r}, t) \exp[i(\omega - \omega_0)t] dt$$

Helmholtz equation:

$$\nabla^2 \tilde{E} + \varepsilon(\omega) k_0^2 \tilde{E} = 0 \quad \leftarrow k_0 = \frac{\omega}{c} \quad \leftarrow \varepsilon(\omega) = 1 + \tilde{\chi}_{xx}^{(1)}(\omega) + \varepsilon_{NL}$$

Refractive index:

$$\tilde{n} = n + n_2 |E|^2$$
$$\rightarrow n_2 = \frac{3}{8n} \text{Re}(\chi_{xxxx}^{(3)})$$

Absorption coefficient:

$$\tilde{\alpha} = \alpha + \alpha_2 |E|^2$$
$$\rightarrow \alpha_2 = \frac{3\omega_0}{4nc} \text{Im}(\chi_{xxxx}^{(3)})$$

Recall: $\varepsilon(\omega) = (\tilde{n} + i\tilde{\alpha} / 2k_0)^2$

Solution form in the Fourier domain:

$$\tilde{E}(\mathbf{r}, \omega - \omega_0) = F(x, y) \tilde{A}(z, \omega - \omega_0) \exp(i\beta_0 z)$$

$$\rightarrow \frac{d^2 F}{dx^2} + \frac{d^2 F}{dy^2} + [\varepsilon(\omega) k_0^2 - \tilde{\beta}^2] F = 0$$

$$\rightarrow 2i\beta_0 \frac{\partial \tilde{A}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2) \tilde{A} = 0$$

Pulse-Propagation Equation (3)

For a small perturbation:

$$\begin{aligned}\varepsilon &= (n + \Delta n)^2 \approx n^2 + 2n\Delta n \\ &\rightarrow \Delta n = n_2 |E|^2 + \frac{i\tilde{\alpha}}{2k_0} \\ &\rightarrow \tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta \quad \rightarrow \Delta\beta = \frac{k_0 \int \int_{-\infty}^{\infty} \Delta n |F(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy}\end{aligned}$$

Electric field:

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2} \hat{x} \{ F(x, y) A(z, t) \exp[i(\beta_0 z - \omega_0 t)] + c.c. \}$$

Slowly varying amplitude in the Fourier domain:

$$\rightarrow \frac{\partial \tilde{A}}{\partial z} = i[(\beta(\omega) + \Delta\beta - \beta_0)] \tilde{A}$$

Recall: $\rightarrow 2i\beta_0 \frac{\partial \tilde{A}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2) \tilde{A} = 0$

Propagation constant:

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2 \beta_2 + \frac{1}{6}(\omega - \omega_0)^3 \beta_3 + \dots$$

$$\leftarrow \beta_m = \left(\frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0} \quad (m = 1, 2, \dots)$$

Pulse-Propagation Equation (4)

Slowly varying amplitude:

$$A(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(z, \omega - \omega_0) \exp[-i(\omega - \omega_0)t] d\omega$$

Recall:

$$\frac{\partial \tilde{A}}{\partial z} = i[(\beta(\omega) + \Delta\beta - \beta_0)] \tilde{A} = 0$$

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2 \beta_2 + \dots$$

Recall: $\rightarrow \omega - \omega_0 \leftrightarrow i \frac{\partial}{\partial t}$

Nonlinear Schrödinger equation:

$$\rightarrow \frac{\partial A}{\partial z} = -\beta_1 \frac{\partial A}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i\Delta\beta A$$

$$\rightarrow \frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = i\gamma |A|^2 A$$

$$\leftarrow \gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}} \quad \leftarrow \text{"Nonlinear parameter"}$$

$$\rightarrow v_g = 1 / \beta_1$$

$$\rightarrow \beta_2 : \text{"GVD parameter"}$$

$$\rightarrow n_2 \approx 2.6 \times 10^{-20} \text{ m}^2 / \text{W}$$

$$\leftarrow A_{\text{eff}} = \frac{\left(\iint_{-\infty}^{\infty} |F(x, y)|^2 dx dy \right)^2}{\iint_{-\infty}^{\infty} |F(x, y)|^4 dx dy}$$

Higher-Order Nonlinear Effects (1)

Incoherent (intensity-dependent) nonlinear effects:

$$\rightarrow \chi^{(3)}(t-t_1, t-t_2, t-t_3) = \chi^{(3)} R(t-t_1) \delta(t-t_2) \delta(t-t_3) \quad \leftarrow \int_0^\infty R(t) dt = 1$$

\leftarrow "Nonlinear response function"

Recall:

$$\mathbf{P}_{NL}(\mathbf{r}, t) = \varepsilon_0 \int \int \int_{-\infty}^{\infty} \chi^{(3)}(t-t_1, t-t_2, t-t_3) \mathbf{E}(\mathbf{r}, t_1) \mathbf{E}(\mathbf{r}, t_2) \mathbf{E}(\mathbf{r}, t_3) dt_1 dt_2 dt_3$$

Nonlinear polarization:

$$\rightarrow \mathbf{P}_{NL}(\mathbf{r}, t) = \varepsilon_0 \chi^{(3)} \mathbf{E}(\mathbf{r}, t) \int_{-\infty}^t R(t-t_1) |\mathbf{E}(\mathbf{r}, t_1)|^2 dt_1$$

\leftarrow For "causality"

$$\rightarrow P_{NL}(\mathbf{r}, t) = \frac{3\varepsilon_0}{4} \chi_{xxxx}^{(3)} E(\mathbf{r}, t) \int_{-\infty}^t R(t-t_1) |E(\mathbf{r}, t_1)|^2 dt_1$$

$$\rightarrow \int_{-\infty}^{\infty} R(t) dt = 1$$

$$\leftarrow R(t) = 0 \text{ if } t < 0$$

In the Fourier domain:

$$\rightarrow \Delta n(\omega) = n_2(\omega) \int_0^\infty R(t_1) |E(\mathbf{r}, t-t_1)|^2 dt_1 + \frac{i\alpha(\omega)}{2k_0}$$

$$\rightarrow \gamma(\omega) = \gamma(\omega_0) + \gamma_1(\omega - \omega_0) \quad \leftarrow \gamma_1 = \left(\frac{d\gamma}{d\omega} \right)_{\omega=\omega_0}$$

$$\rightarrow \alpha(\omega) = \alpha(\omega_0) + \alpha_1(\omega - \omega_0) \quad \leftarrow \alpha_1 = \left(\frac{d\alpha}{d\omega} \right)_{\omega=\omega_0}$$

Higher-Order Nonlinear Effects (2)

Nonlinear Schrödinger equation:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = i\gamma |A|^2 A$$

Recall:

$$\rightarrow \gamma(\omega) = \gamma(\omega_0) + \gamma_1(\omega - \omega_0)$$

$$\rightarrow \alpha(\omega) = \alpha(\omega_0) + \alpha_1(\omega - \omega_0)$$

$$\rightarrow \frac{\partial A}{\partial z} + \frac{1}{2} \left(\alpha(\omega_0) + i\alpha_1 \frac{\partial}{\partial t} \right) A - i \sum_{n=1}^{\infty} \frac{i^n \beta_n}{n!} \frac{\partial^n A}{\partial t^n} = i \left(\gamma(\omega_0) + i\gamma_1 \frac{\partial}{\partial t} \right) \left(A(z, t) \int_0^{\infty} R(t') |A(z, t - t')|^2 dt' \right)$$

Nonlinear response function:

$$R(t) = (1 - f_R)\delta(t) + f_R h_R(t) \quad \leftarrow \text{“Raman response function”}$$

Raman gain: $\rightarrow g_R(\Delta\omega) = \frac{\omega_0}{cn_0} f_R \chi^{(3)} \text{Im}[\tilde{h}_R(\Delta\omega)]$ $\leftarrow \text{“Kramers-Kronig relation”}$
 for the real part of \tilde{h}_R
 $\leftarrow \Delta\omega = \omega - \omega_0$

Raman response function
in view of the damped oscillation:

$$\rightarrow h_R(t) = (\tau_1^{-2} + \tau_2^{-2})\tau_1 \exp(-t/\tau_2) \sin(t/\tau_1)$$

For silica fibers: $\leftarrow \tau_1 = 12.2 \text{ fs}$ & $\tau_2 = 32 \text{ fs}$
 $\leftarrow f_R \approx 0.18$

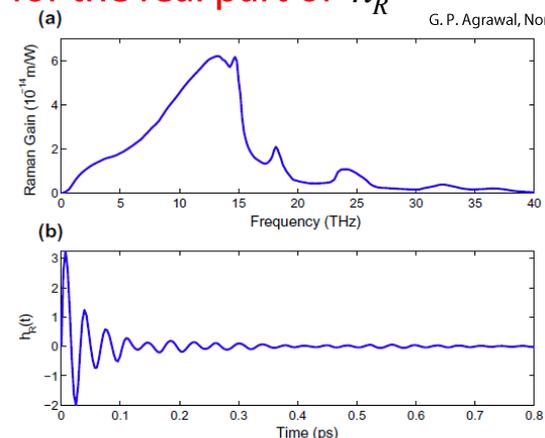


Figure 2.2 (a) Measured Raman-gain spectrum of silica fibers; (b) temporal form of the Raman response function deduced from the gain data. (Based on the Raman-gain data provided by R.H. Stolen.)

Higher-Order Nonlinear Effects (3)

Nonlinear parameter:

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}} \rightarrow \gamma(\omega_0) = \frac{n_2 \omega_0}{c A_{eff}} \rightarrow \gamma_1(\omega_0) = \left(\frac{d\gamma}{d\omega} \right)_{\omega=\omega_0} = \frac{n_2}{c A_{eff}} + \frac{\omega_0}{c A_{eff}} \left(\frac{dn_2}{d\omega} \right)_{\omega=\omega_0} - \frac{n_2 \omega_0}{c A_{eff}^2} \left(\frac{dA_{eff}}{d\omega} \right)_{\omega=\omega_0}$$

$$\rightarrow \frac{\gamma_1(\omega_0)}{\gamma(\omega_0)} = \frac{1}{\omega_0} + \frac{1}{n_2} \left(\frac{dn_2}{d\omega} \right)_{\omega=\omega_0} - \frac{1}{A_{eff}} \left(\frac{dA_{eff}}{d\omega} \right)_{\omega=\omega_0}$$

Further approximation (pulse width > 100 fs):

$$|A(z, t - t')|^2 \approx |A(z, t)|^2 - t' \frac{\partial}{\partial t} |A(z, t)|^2$$

The first moment of the Raman response function: $\leftarrow R(t) = (1 - f_R) \delta(t) + f_R h_R(t)$

$$T_R \equiv \int_0^\infty t R(t) dt \approx f_R \int_0^\infty t h_R(t) dt = f_R \left. \frac{d\{\text{Im}[\tilde{h}_R(\Delta\omega)]\}}{d(\Delta\omega)} \right|_{\Delta\omega=0}$$

Approximated nonlinear Schrödinger equation:

$$\rightarrow \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} = i\gamma \left(|A|^2 A + \frac{i}{\omega_0} \frac{\partial}{\partial T} (|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial T} \right)$$

$\leftarrow T = t - z/v_g \equiv t - \beta_1 z$ \leftarrow "Retarded frame"

Numerical Methods (1)

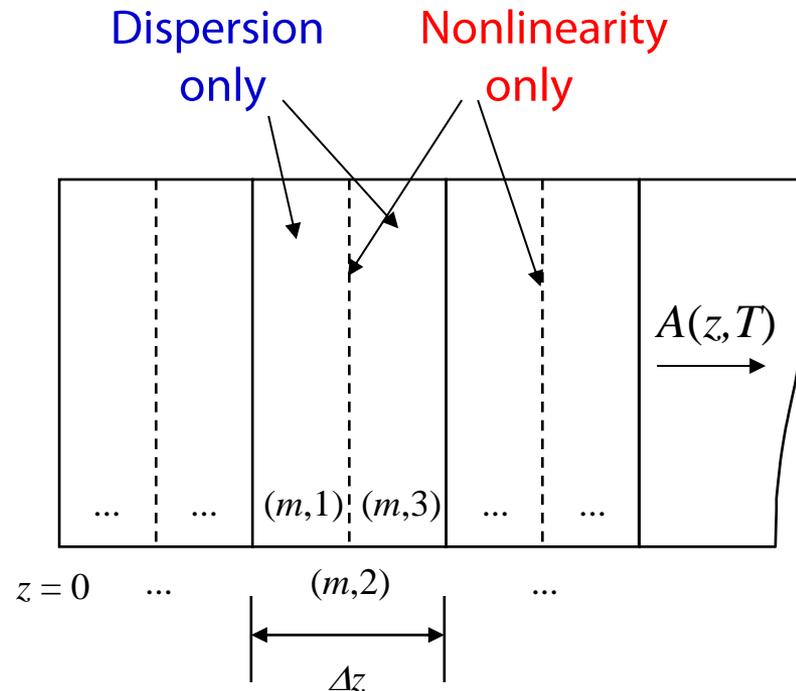
Split-step Fourier method:

$$\rightarrow \frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A$$

$$\leftarrow \hat{D} = -\frac{i\beta_2}{2} \frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3}{\partial T^3} - \frac{\alpha}{2}$$

$$\leftarrow \hat{N} = i\gamma \left(|A|^2 + \frac{i}{\omega_0} \frac{1}{A} \frac{\partial}{\partial T} (|A|^2 A) - T_R \frac{\partial |A|^2}{\partial T} \right)$$

Schematic illustration:



Numerical Methods (2)

Predictor-corrector scheme:

$$\frac{dy}{dz} = f(z, y)$$

$$\rightarrow y_p(z + \Delta z) = y(z) + f[z, y(z)]\Delta z$$

$$\rightarrow y(z + \Delta z) = y(z) + \frac{1}{2}\{f[z, y(z)] + f[z, y_p(z + \Delta z)]\}\Delta z$$

→ The value predicted by an initial rough estimation is to be corrected by iterations.