Nonlinear Optical Engineering

Group-Velocity Dispersion (1)
(NFO 5th ed: 3.1 ~ 3.2)

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Different Propagation Regimes

Nonlinear Schrödinger equation (pulse widths > 5 ps):

$$i\frac{\partial A}{\partial z} = -\frac{i\alpha}{2}A + \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \gamma|A|^2 A$$

Parameter change: $\rightarrow \tau = \dfrac{T}{T_0} = \dfrac{t - z/v_g}{T_0}$ ← Initial width

$$\rightarrow A(z,\tau) = \sqrt{P_0}\exp(-\alpha z/2)U(z,\tau)$$

← Initial peak power

$$\rightarrow i\frac{\partial U}{\partial z} = \frac{\mathrm{sgn}(\beta_2)}{2L_D}\frac{\partial^2 U}{\partial \tau^2} - \frac{\exp(-\alpha z)}{L_{NL}}|U|^2 U \qquad \leftarrow L_D = \frac{T_0^2}{|\beta_2|}, \quad L_{NL} = \frac{1}{\gamma P_0}$$

"Dispersion length" "Nonlinear length"

Dispersion-dominant regime:

$$\rightarrow \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} << 1$$

Nonlinear-dominant regime:

$$\rightarrow \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} >> 1$$

Dispersion-Induced Pulse Broadening

Propagation in a linear dispersive medium:

$$\rightarrow i\frac{\partial U}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} \qquad \leftarrow \gamma = 0$$

Recall: $\rightarrow U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{U}(z,\omega)\exp(-i\omega T)d\omega$

In the Fourier domain:

$$\rightarrow i\frac{\partial \tilde{U}}{\partial z} = -\frac{1}{2}\beta_2\omega^2\tilde{U}$$

$$\rightarrow \tilde{U}(z,\omega) = \tilde{U}(0,\omega)\exp\left(\frac{i}{2}\beta_2\omega^2 z\right)$$

In the time domain:

$$\rightarrow U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{U}(0,\omega)\exp\left(\frac{i}{2}\beta_2\omega^2 z - i\omega T\right)d\omega$$

$$\leftarrow \tilde{U}(0,\omega) = \int_{-\infty}^{\infty}U(0,T)\exp(i\omega T)dT$$

Gaussian Pulses

Incident field:

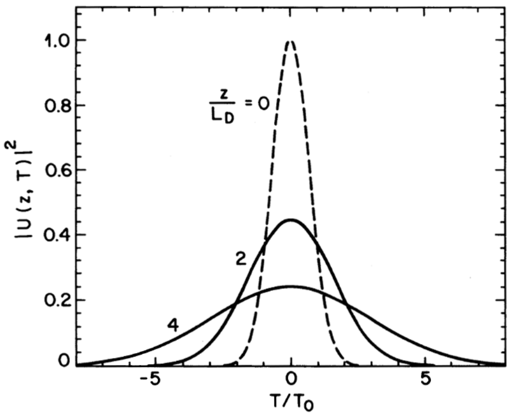
$$\rightarrow U(0,T)=\exp\left(-\frac{T^2}{2T_0^2}\right)$$

← Half-width (at 1/e-intensity point)

$$\rightarrow T_{FWHM}=2(\ln 2)^{1/2}T_0\approx 1.665T_0$$

Field amplitude in the time domain:

$$\rightarrow U(z,T)=\frac{T_0}{(T_0^2-i\beta_2 z)^{1/2}}\exp\left(-\frac{T^2}{2(T_0^2-i\beta_2 z)}\right)$$



G. P. Agrawal, Nonlinear Fiber Optics, 3rd ed.

Pulse width evolution:

$$\rightarrow T_1(z)=T_0[1+(z/L_D)^2]^{1/2}$$

Phase evolution:

$$\rightarrow U(z,T)=|U(z,T)|\exp[i\phi(z,T)]$$

$$\rightarrow \phi(z,T)=-\frac{\mathrm{sgn}(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{T^2}{2T_0^2}+\frac{1}{2}\tan^{-1}\left(\mathrm{sgn}(\beta_2)\frac{z}{L_D}\right)$$

Frequency chirp:

$$\rightarrow \delta\omega=-\frac{\partial\phi(z,T)}{\partial t}=\frac{\mathrm{sgn}(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{T}{T_0^2}$$

Chirped Gaussian Pulses (1)

"Chirp parameter"

Incident field:

$$\to U(0,T) = \exp\left(-\frac{(1+iC)}{2}\frac{T^2}{T_0^2}\right)$$

$\to$ "Up-chirp" for $C > 0$

$\to$ "Down-chirp" for $C < 0$

In the Fourier domain:

$$\to \tilde{U}(0,\omega) = \left(\frac{2\pi T_0^2}{1+iC}\right)^{1/2} \exp\left(-\frac{\omega^2 T_0^2}{2(1+iC)}\right)$$

$$\to \Delta\omega = (1+C^2)^{1/2} / T_0$$ $\leftarrow$ Half-width (at 1/e-intensity point)

$$\to \Delta\omega T_0 = 1$$ $\leftarrow$ "Transform-limited" if $C = 0$

In the time domain:

$$\to U(z,T) = \frac{T_0}{[T_0^2 - i\beta_2 z(1+iC)]^{1/2}} \exp\left(-\frac{(1+iC)T^2}{2[T_0^2 - i\beta_2 z(1+iC)]}\right)$$

Pulse width evolution:

$$\to \frac{T_1}{T_0} = \left[\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2\right]^{1/2}$$

Chirp parameter evolution:

$$\to C_1(z) = C + (1+C^2)(\beta_2 z / T_0^2)$$

Chirped Gaussian Pulses (2)

In case: $\rightarrow \beta_2 C < 0$

$$\rightarrow \frac{T_1}{T_0} = \left[\left(1 + \frac{C\beta_2 z}{T_0^2} \right)^2 + \left(\frac{\beta_2 z}{T_0^2} \right)^2 \right]^{1/2}$$

$$\rightarrow z_{\min} = \frac{|C|}{1+C^2} L_D, \quad T_1^{\min} = \frac{T_0}{(1+C^2)^{1/2}}$$

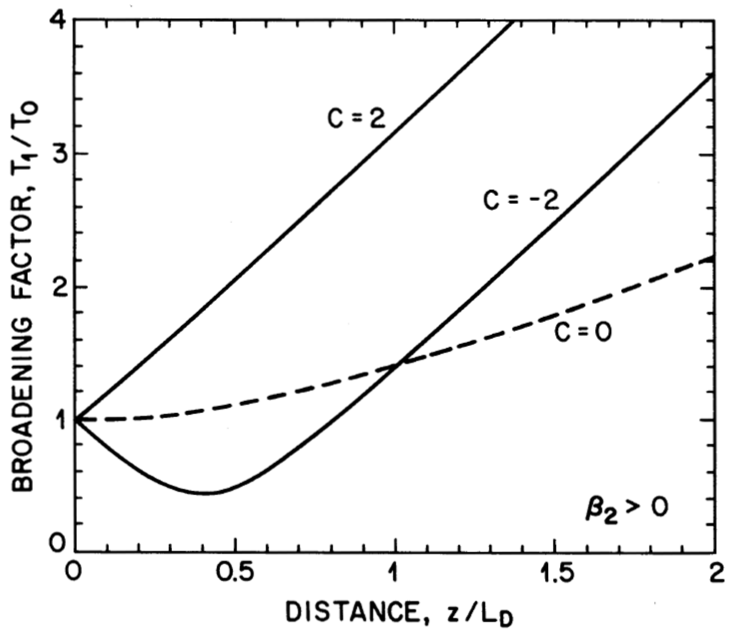
Recall:

$$\rightarrow \Delta\omega = (1+C^2)^{1/2} / T_0$$

$$\rightarrow \Delta\omega T_1^{\min} = 1$$

$\rightarrow$ "Transform-limited " at z_{min}

$$\rightarrow C_1^{\min} = ?$$



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Hyperbolic-Secant Pulses

Incident field:

$$\rightarrow U(0,T) = \operatorname{sech}\left(\frac{T}{T_0}\right)\exp\left(-\frac{iCT^2}{2T_0^2}\right) \quad \leftarrow \text{"Optical solitons"}$$

$$\rightarrow T_{FWHM} = 2\ln(1+\sqrt{2})T_0 \approx 1.763T_0$$

Super-Gaussian Pulses

Incident field:

$$\rightarrow U(0,T) = \exp\left[-\frac{1+iC}{2}\left(\frac{T}{T_0}\right)^{2m}\right] \quad \leftarrow \text{"Modulated semiconductor lasers"}$$

$$\rightarrow T_r \approx (\ln 9)\frac{T_0}{2m} \approx \frac{T_0}{m} \quad \leftarrow \text{Rise time from 10 to 90\%}$$

Root-mean-square (RMS) width (σ):

$$\rightarrow \sigma = \left[\left\langle (T - \langle T \rangle)^2 \right\rangle\right]^{1/2} = \left[\left\langle T^2 \right\rangle - \langle T \rangle^2\right]^{1/2} \qquad \leftarrow \left\langle T^p \right\rangle = \frac{\int_{-\infty}^{\infty} T^p \left|U(z,T)\right|^2 dT}{\int_{-\infty}^{\infty} \left|U(z,T)\right|^2 dT}$$

$$\rightarrow \frac{\sigma}{\sigma_0} = \left[1 + \frac{\Gamma(1/2m)}{\Gamma(3/2m)}\frac{C\beta_2 z}{T_0^2} + m^2(1+C^2)\frac{\Gamma(2-1/2m)}{\Gamma(3/2m)}\left(\frac{\beta_2 z}{T_0^2}\right)^2\right]^{1/2} \quad \rightarrow \text{H.W.}$$