Nonlinear Optical Engineering

Group-Velocity Dispersion (2)
(NFO 5th ed: 3.3 ~ 3.4)

Yoonchan Jeong
School of Electrical Engineering, Seoul National University
Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Third-Order Dispersion (TOD) (1)

Nonlinear Schrödinger equation ($\beta_2 \approx 0$ or $T_0 < 1$ ps):

$$\rightarrow i\frac{\partial U}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 U}{\partial T^3} \qquad \leftarrow \gamma = 0$$

$$\rightarrow U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{U}(0,\omega)\exp\left(\frac{i}{2}\beta_2\omega^2 z + \frac{i}{6}\beta_3\omega^3 z - i\omega T\right)d\omega$$

Evolution of chirped Gaussian pulses:

$$\rightarrow \tilde{U}(0,\omega) = \left(\frac{2\pi T_0^2}{1+iC}\right)^{1/2}\exp\left(-\frac{\omega^2 T_0^2}{2(1+iC)}\right)$$

$$\rightarrow U(z,T) = \frac{A_0}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp\left(-x^2 + \frac{ib}{3}x^3 - \frac{iT}{p}x\right)dx$$

$$\leftarrow x = \omega p, \quad p^2 = \frac{T_0^2}{2}\left(\frac{1}{1+iC} - \frac{i\beta_2}{T_0^2}\right), \quad b = \beta_3 z/(2p^3)$$

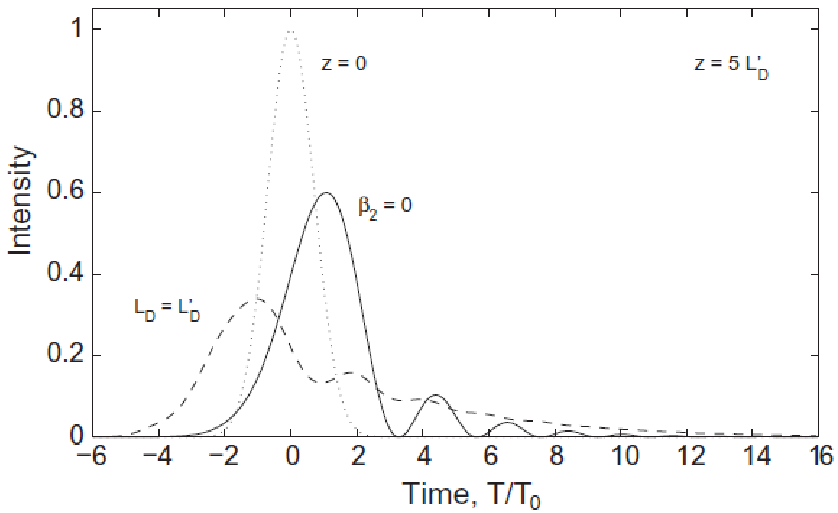
$$\leftarrow x = b^{-1/3}u - i/b$$

$$\rightarrow U(z,T) = \frac{2A_0\sqrt{\pi}}{|b|^{1/3}}\exp\left(\frac{2p-3bT}{3pb^2}\right)\mathrm{Ai}\left(\frac{p-bT}{p|b|^{4/3}}\right)$$

Third-Order Dispersion (2)

Changes in pulse shape:

$$\rightarrow L_D' = T_0^3 / |\beta_3| \quad \leftarrow \text{Dispersion length with TOD}$$



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Third-Order Dispersion (3)

FT of the pulse intensity:

$$\tilde{I}(z,\omega)=\int_{-\infty}^{\infty}\left|U(z,T)\right|^{2}\exp\left(i\omega T\right)dT$$

$$\rightarrow \lim_{\omega\to 0}\frac{\partial^n}{\partial\omega^n}\tilde{I}(z,\omega)=(i)^n\int_{-\infty}^{\infty}T^n\left|U(z,T)\right|^{2}dT$$

$$\rightarrow \left\langle T^n\right\rangle=\frac{(-i)^n}{N_c}\lim_{\omega\to 0}\frac{\partial^n}{\partial\omega^n}\tilde{I}(z,\omega)$$

$$\leftarrow N_c=\int_{-\infty}^{\infty}\left|U(z,T)\right|^{2}dT\equiv\int_{-\infty}^{\infty}\left|U(0,T)\right|^{2}dT$$

$$\rightarrow \tilde{I}(z,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{U}(z,\omega+\omega')\tilde{U}^*(z,\omega')d\omega'$$ ← Convolution theorem

$$\rightarrow \left\langle T^n\right\rangle=\frac{(i)^n}{2\pi N_c}\int_{-\infty}^{\infty}\tilde{U}^*(z,\omega)\frac{\partial^n}{\partial\omega^n}\tilde{U}(z,\omega)d\omega$$

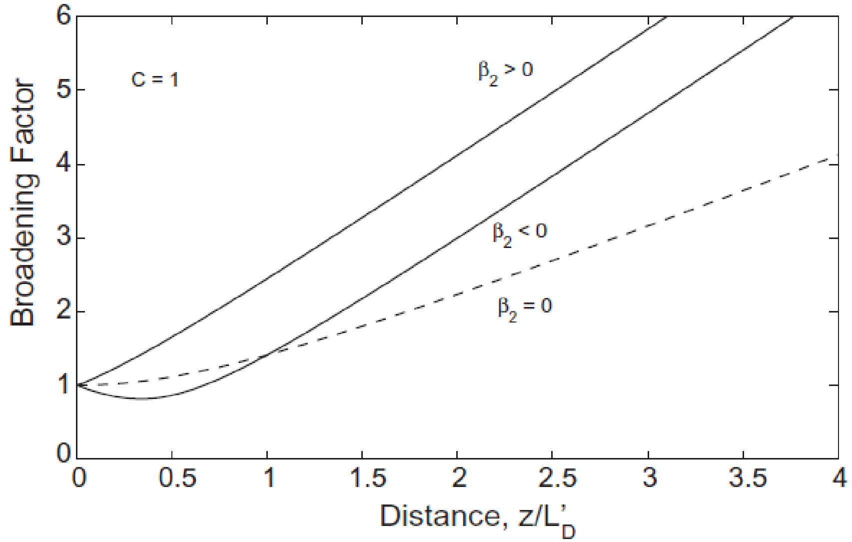
Chirped Gaussian pulse:

$$\rightarrow \tilde{U}(z,\omega)=\left(\frac{2\pi T_0^2}{1+iC}\right)^{1/2}\exp\left[\frac{i\omega^2}{2}\left(\beta_2 z+\frac{iT_0^2}{1+iC}\right)+\frac{i}{6}\beta_3\omega^3 z\right]$$

$$\rightarrow \sigma_0=T_0/\sqrt{2}\qquad \rightarrow \frac{\sigma}{\sigma_0}=\left[\left(1+\frac{C\beta_2 z}{2\sigma_0^2}\right)^2+\left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2+(1+C^2)^2\frac{1}{2}\left(\frac{\beta_3 z}{4\sigma_0^3}\right)^2\right]^{1/2}$$

Third-Order Dispersion (4)

Broadening factor for a chirped Gaussian pulse:



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

In case: $\rightarrow z >> L_D / |C|$

$$\rightarrow \frac{\sigma}{\sigma_0} = (1+C^2)^{1/2}[1+(1+C^2)(L_D/2L'_D)^2]^{1/2}(z/L_D)$$

Incl. a Gaussian source spectrum: $\rightarrow G(\omega) = \frac{1}{\sigma_\omega\sqrt{2\pi}}\exp\left(-\frac{\omega^2}{2\sigma_\omega^2}\right)$

$$\rightarrow \frac{\sigma}{\sigma_0} = \left[\left(1+\frac{C\beta_2 z}{2\sigma_0^2}\right)^2 + \left(1+V_\omega^2\right)\left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + (1+C^2+V_\omega^2)^2\frac{1}{2}\left(\frac{\beta_3 z}{4\sigma_0^3}\right)^2\right]^{1/2} \rightarrow \text{H.W.}$$

$$\leftarrow V_\omega = 2\sigma_\omega\sigma_0$$

Third-Order Dispersion (5)

Arbitrary-shape pulses:

$$\rightarrow i\frac{\partial U}{\partial z} = \hat{H}U \qquad \leftarrow \hat{H} = -\sum_{n=2}^{\infty}\frac{i^n}{n!}\left(\frac{\partial}{\partial T}\right)^n = \frac{\beta_2}{2}\frac{\partial^2}{\partial T^2} + \frac{i\beta_3}{6}\frac{\partial^3}{\partial T^3} + \cdots$$

$$\leftarrow \int_{-\infty}^{\infty}|U|^2\,dT = 1$$

Evolution of the first and second moments of *T*:

$$\rightarrow \frac{d\langle T\rangle}{dz} = i\int_{-\infty}^{\infty}U^*(z,T)[\hat{H},T]U(z,T)dT \qquad \leftarrow \text{Recall: } \langle T^p\rangle = \frac{\int_{-\infty}^{\infty}T^p|U(z,T)|^2\,dT}{\int_{-\infty}^{\infty}|U(z,T)|^2\,dT}$$

$$\rightarrow \frac{d\langle T^2\rangle}{dz^2} = i^2\int_{-\infty}^{\infty}U^*(z,T)[\hat{H},[\hat{H},T^2]]U(z,T)dT$$

$$\rightarrow \langle T\rangle = a_0 + a_1 z$$

$$\rightarrow \langle T^2\rangle = b_0 + b_1 z + b_2 z^2$$

$$\begin{cases} a_0 = \int_{-\infty}^{\infty}U_0^*(T)TU_0(T)dT & \leftarrow U_0(T) \equiv U(0,T) \\ a_1 = i\int_{-\infty}^{\infty}U_0^*(T)[\hat{H},T]U_0(T)dT & \\ b_0 = \int_{-\infty}^{\infty}U_0^*(T)T^2U_0(T)dT & \rightarrow \text{H.W.} \\ b_1 = i\int_{-\infty}^{\infty}U_0^*(T)[\hat{H},T^2]U_0(T)dT & \\ b_2 = -\frac{1}{2}\int_{-\infty}^{\infty}U_0^*(T)[\hat{H},[\hat{H},T^2]]U_0(T)dT & \end{cases}$$

Dispersion Management

GVD-induced limitations:

$$\rightarrow \frac{\sigma}{\sigma_0} = \left[\left(1+\frac{C\beta_2 z}{2\sigma_0^2}\right)^2 + \left(1+V_\omega^2\right)\left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + (1+C^2+V_\omega^2)^2 \frac{1}{2}\left(\frac{\beta_3 z}{4\sigma_0^3}\right)^2\right]^{1/2}$$

$$\rightarrow \sigma = [\sigma_0^2 + (\beta_2 L\sigma_\omega)^2]^{1/2} = [\sigma_0^2 + (DL\sigma_\lambda)^2]^{1/2} \quad \leftarrow \beta_3 \approx 0, C=0, \& V_\omega >> 1$$

$$\rightarrow \sigma = \left[\sigma_0^2 + \frac{1}{2}\left(\beta_3 L/4\sigma_0^2\right)^2\right]^{1/2} \quad \leftarrow \beta_2 = 0, C=0, \& V_\omega << 1$$

Dispersion Compensation

For dispersion map consisting of two fiber segments:

$$\rightarrow U(L_m,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{U}(0,\omega)\exp\left[\frac{i}{2}\omega^2(\beta_{21}L_1+\beta_{22}L_2) + \frac{i}{6}\omega^3(\beta_{31}L_1+\beta_{32}L_2) - i\omega t\right]d\omega$$

$$\rightarrow \beta_{21}L_1 + \beta_{22}L_2 = 0 \quad \leftarrow \text{GVD}$$

$$\rightarrow \beta_{31}L_1 + \beta_{32}L_2 = 0 \quad \leftarrow \text{TOD}$$