

# Nonlinear Optical Engineering

Self-Phase Modulation (1)  
(NFO 5<sup>th</sup> ed: 4.1 ~ 4.2)

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: [yunchan@snu.ac.kr](mailto:yunchan@snu.ac.kr)

# SPM-Induced Spectral Changes (1)

Nonlinear phase shift:

$$\rightarrow \frac{\partial U}{\partial z} = \frac{ie^{-\alpha z}}{L_{NL}} |U|^2 U \quad \leftarrow \beta_2 = 0 \quad \leftarrow L_{NL} = (\gamma P_0)^{-1}$$

$$\rightarrow U = V \exp(i\phi_{NL}) \quad \rightarrow \frac{\partial V}{\partial z} = 0, \quad \frac{\partial \phi_{NL}}{\partial z} = \frac{e^{-\alpha z}}{L_{NL}} V^2$$

$$\rightarrow U(L, T) = U(0, T) \exp[i\phi_{NL}(L, T)] \quad \leftarrow L_{eff} = [1 - \exp(-\alpha L)] / \alpha$$

$$\rightarrow \phi_{NL}(L, T) = |U(0, T)|^2 (L_{eff} / L_{NL}) \quad \leftarrow |U(0, 0)| = 1$$

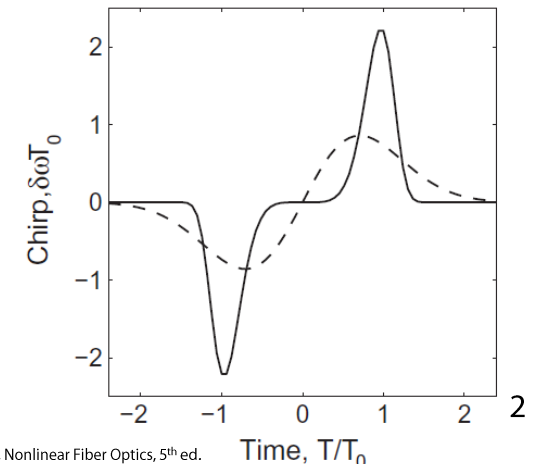
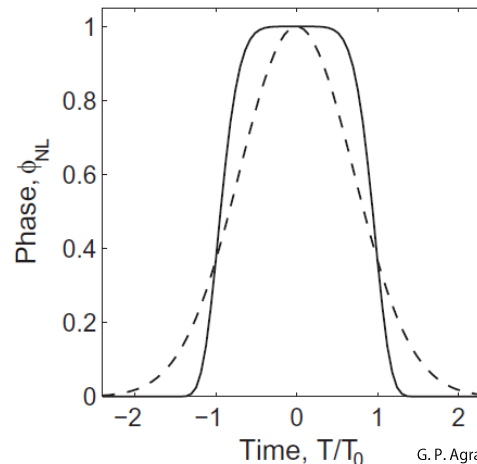
Maximum phase shift:  $\rightarrow \phi_{max} = L_{eff} / L_{NL} = \gamma P_0 L_{eff}$

$$\rightarrow \delta\omega(T) = -\frac{\partial \phi_{NL}}{\partial T} = -\left(\frac{L_{eff}}{L_{NL}}\right) \frac{\partial}{\partial T} |U(0, T)|^2$$

Gaussian/Super-Gaussian pulses:

$$\rightarrow \delta\omega(T) = \frac{2m}{T_0} \left(\frac{L_{eff}}{L_{NL}}\right) \left(\frac{T}{T_0}\right)^{2m-1} \exp\left[-\left(\frac{T}{T_0}\right)^{2m}\right]$$

$$\leftarrow m = 1, 3$$



# SPM-Induced Spectral Changes (2)

Changes in pulse spectra:

$$\rightarrow \delta\omega(T) = \frac{2m}{T_0} \left( \frac{L_{eff}}{L_{NL}} \right) \left( \frac{T}{T_0} \right)^{2m-1} \exp \left[ - \left( \frac{T}{T_0} \right)^{2m} \right]$$

$$\rightarrow \delta\omega_{max} = \frac{mf(m)}{T_0} \phi_{max} \quad \leftarrow f(m) = 2 \left( 1 - \frac{1}{2m} \right)^{1-1/2m} \exp \left[ - \left( 1 - \frac{1}{2m} \right) \right]$$

For an unchirped Gaussian pulse:  $\rightarrow \delta\omega_{max} = 0.86 \Delta\omega_0 \phi_{max}$

Pulse spectrum:

$$\rightarrow S(\omega) = |\tilde{U}(L, \omega)|^2$$

$$\rightarrow \phi_{max} \approx \left( M - \frac{1}{2} \right) \pi$$

$\leftarrow M$  : # of peaks

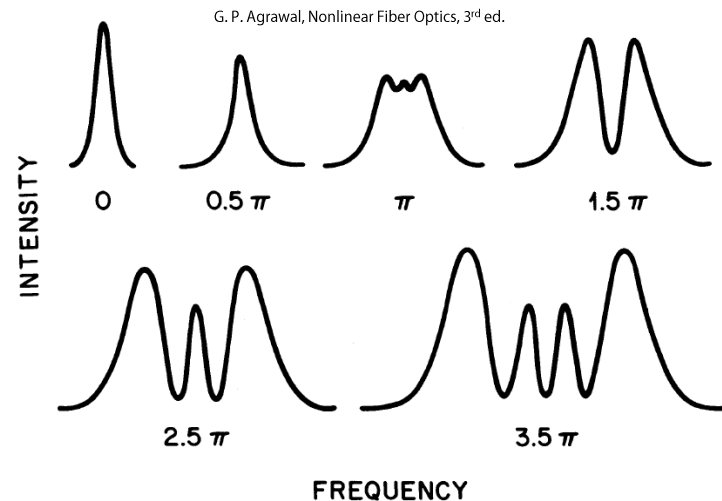
( $\phi_{max} \gg 1$ )

RMS spectral width:

$$\rightarrow \Delta\omega_{rms}^2 = \langle (\omega - \omega_0)^2 \rangle - \langle (\omega - \omega_0) \rangle^2$$

For an unchirped Gaussian pulse:

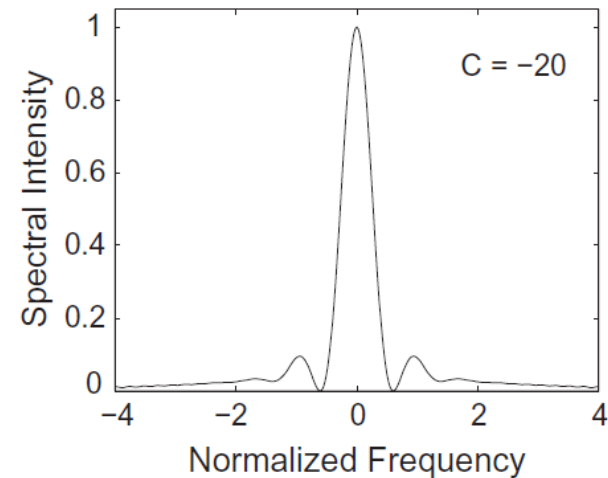
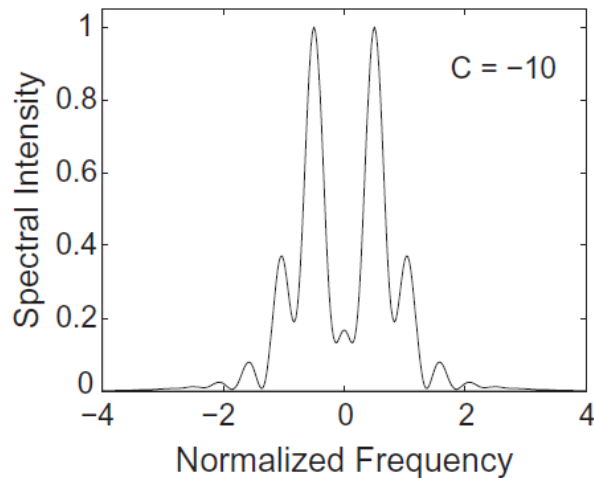
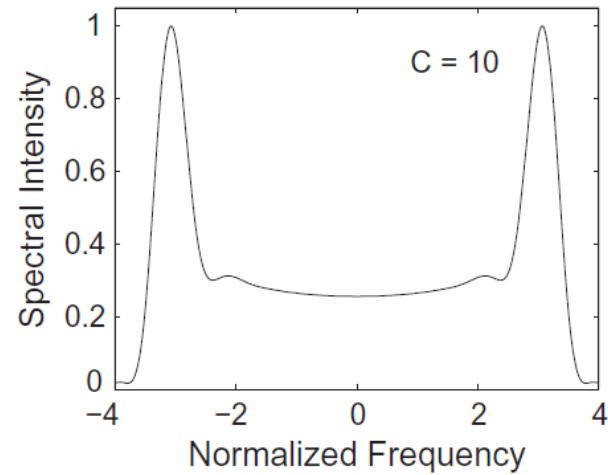
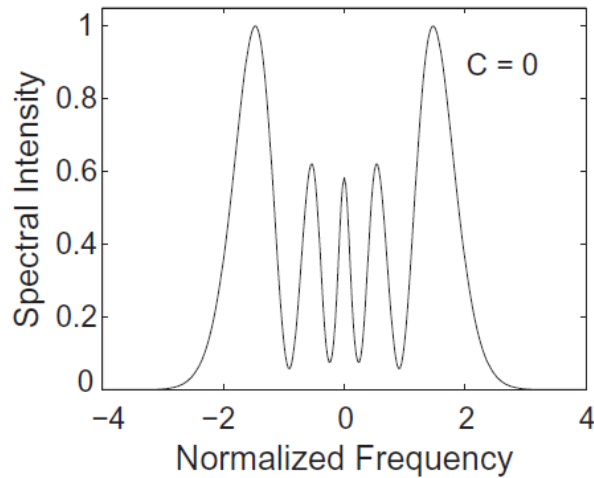
$$\rightarrow \frac{\Delta\omega_{rms}}{\Delta\omega_0} = \left( 1 + \frac{4}{3\sqrt{3}} \phi_{max}^2 \right)^{1/2}$$



$$\leftarrow \langle (\omega - \omega_0)^n \rangle = \frac{\int_{-\infty}^{\infty} (\omega - \omega_0)^n S(\omega) d\omega}{\int_{-\infty}^{\infty} S(\omega) d\omega}$$

# SPM-Induced Spectral Changes (3)

Effect of pulse shape and initial chirp (Gaussian pulses):



# SPM-Induced Spectral Changes (4)

Effect of partial coherence:

→ H.W.

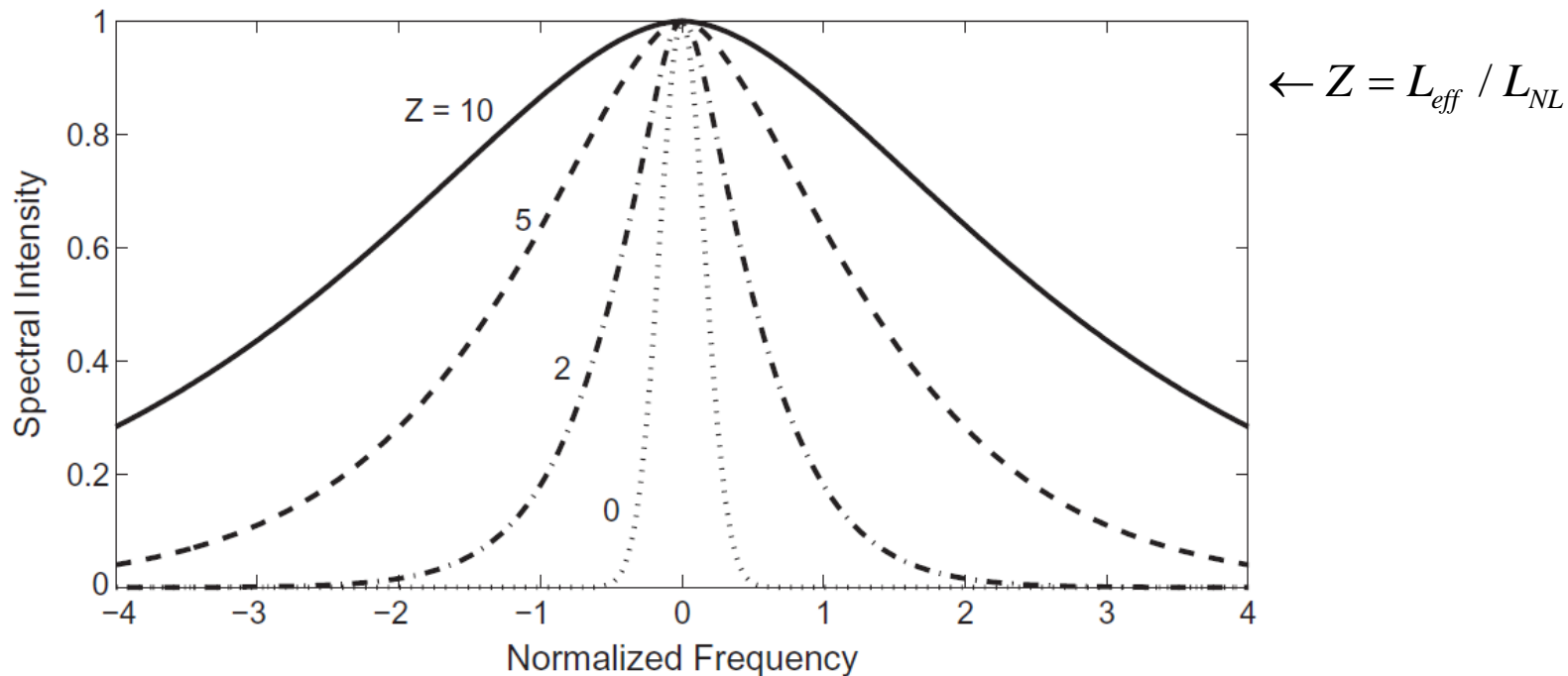
Optical spectrum:  $\rightarrow S(\omega) = \int_{-\infty}^{\infty} \Gamma(z, \tau) \exp(i\omega\tau) d\tau$  ← Wiener-Khintchine theorem

$$\leftarrow \Gamma(z, \tau) = \langle U^*(z, T) U(z, T + \tau) \rangle$$

← Coherent function

Gaussian form for the input coherent function:

$$\rightarrow \Gamma(0, \tau) = \exp[-\tau^2 / (2T_c^2)]$$

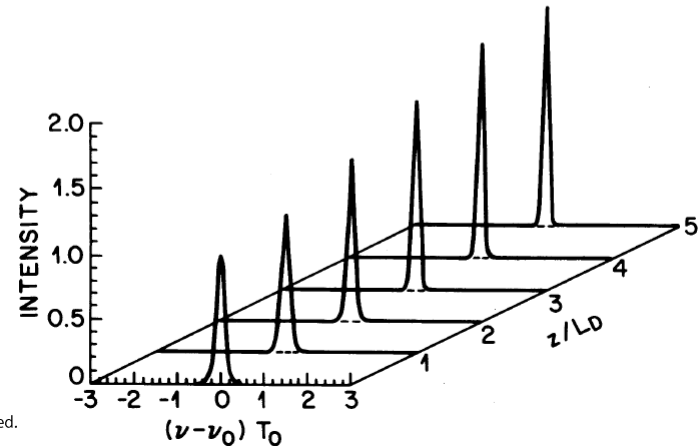
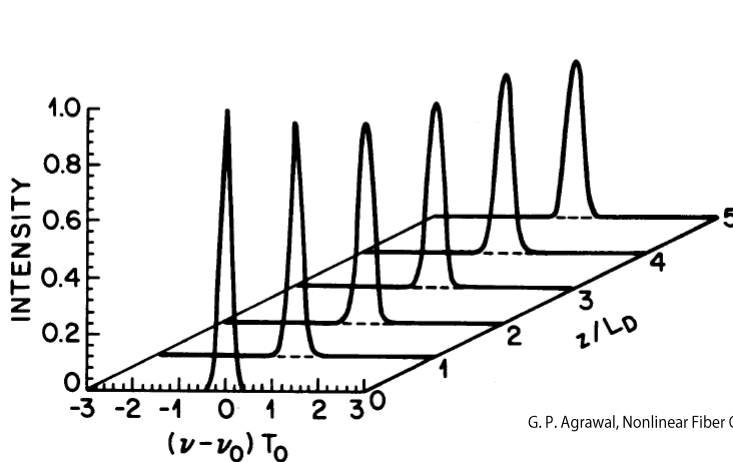
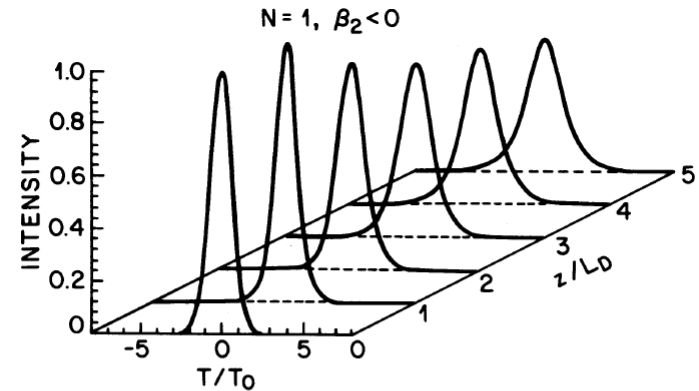
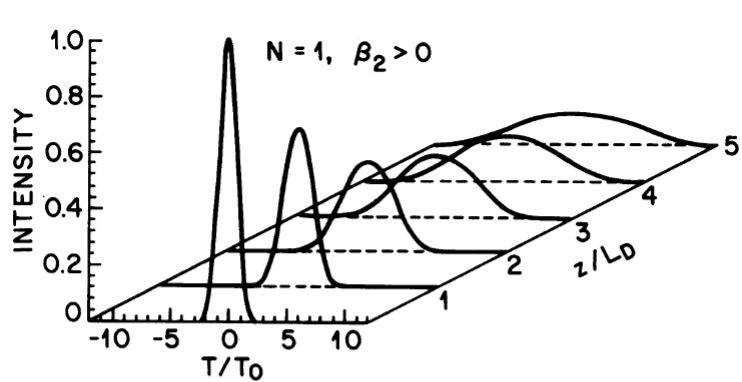


# Effect of Group-Velocity Dispersion (1)

Pulse evolution:

$$\rightarrow i \frac{\partial U}{\partial \xi} = \text{sgn}(\beta_2) \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} - N^2 e^{-\alpha z} |U|^2 U$$

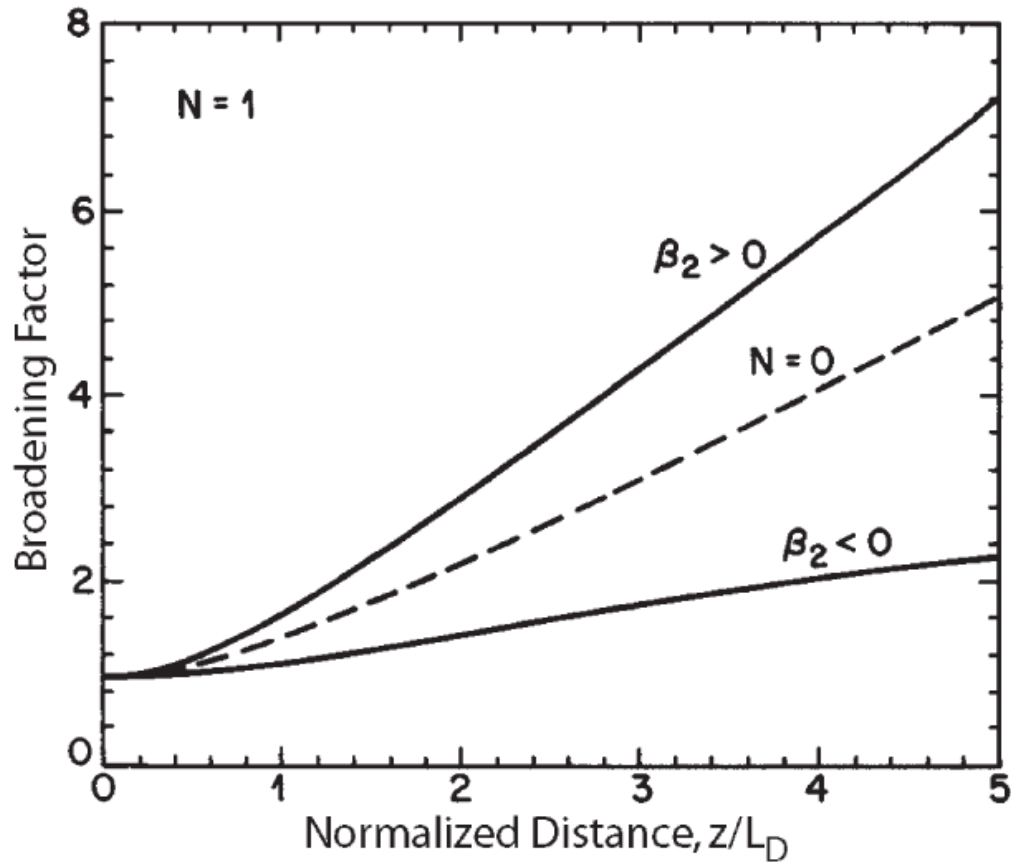
$$\leftarrow \xi = z / L_D, \quad \tau = T / T_0, \quad N^2 = \frac{L_D}{L_{NL}} \equiv \frac{\gamma P_0 T_0^2}{|\beta_2|}$$



# Effect of Group-Velocity Dispersion (2)

Broadening factor:

Gaussian pulses:

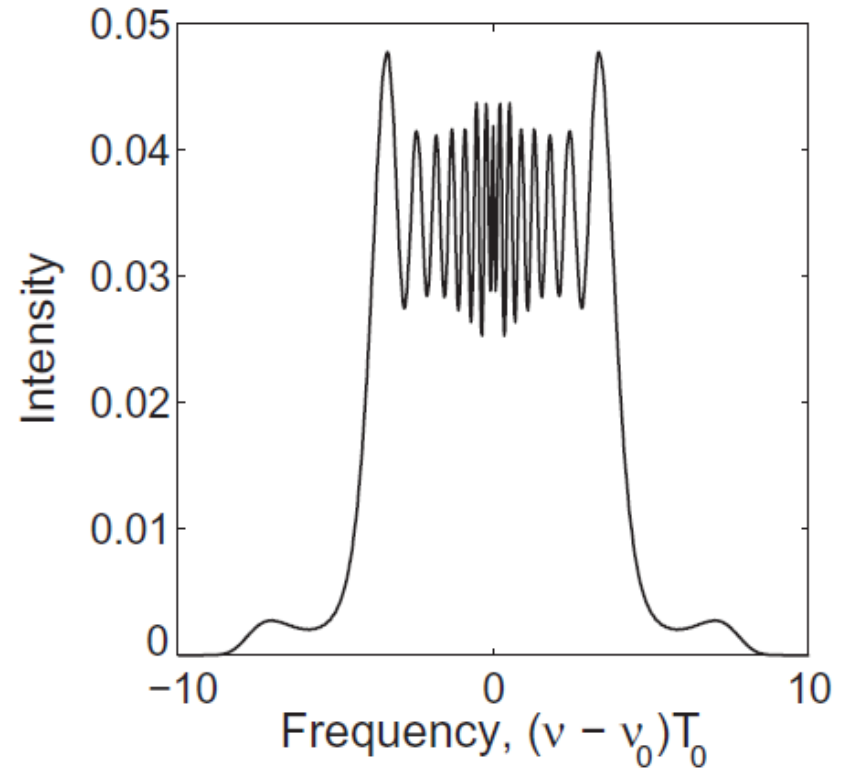
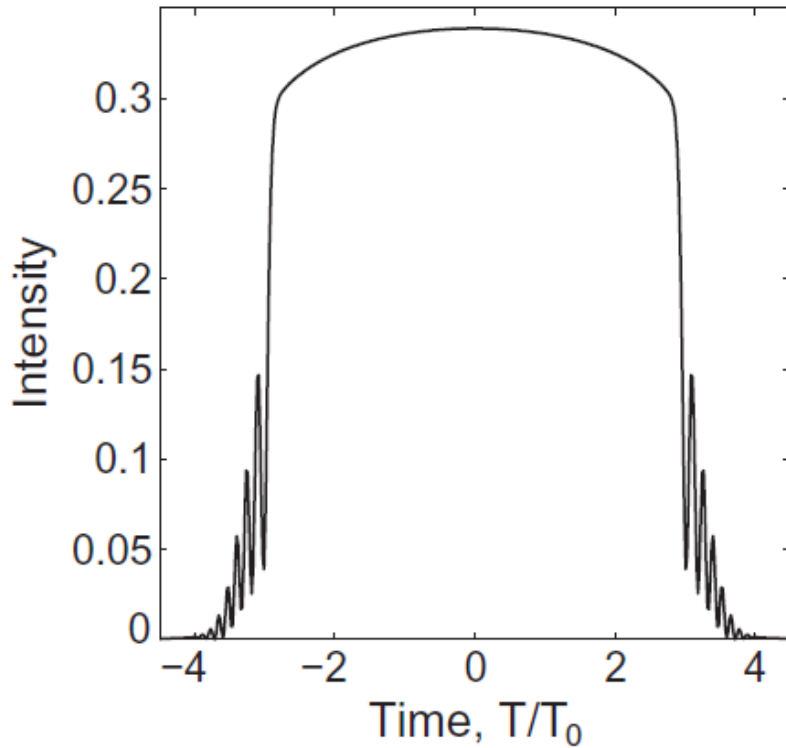


G. P. Agrawal, Nonlinear Fiber Optics, 5<sup>th</sup> ed.

# Effect of Group-Velocity Dispersion (3)

Optical wave breaking :

Sech pulse:



G. P. Agrawal, Nonlinear Fiber Optics, 5<sup>th</sup> ed.



# Effect of Group-Velocity Dispersion (4)

Effect of third-order dispersion:

$$\rightarrow i \frac{\partial U}{\partial \xi'} = \text{sgn}(\beta_3) \frac{i}{6} \frac{\partial^3 U}{\partial \tau^3} - \bar{N}^2 e^{-\alpha z} |U|^2 U$$

$$\leftarrow \xi = z / L'_D, \quad \tau = T / T_0, \quad \bar{N}^2 = \frac{L'_D}{L_{NL}} \equiv \frac{\gamma P_0 T_0^3}{|\beta_3|}$$

Unchirped Gaussian pulse:  $\rightarrow \beta_3 > 0$

