Nonlinear Optical Engineering

Self-Phase Modulation (2)
(NFO 5^{th} ed: 4.3 ~ 4.4)

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Semianalytic Techniques (1)

Nonlinear Schrödinger equation: $\rightarrow i\frac{\partial U}{\partial z}-\frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2}+\gamma P_0 e^{-\alpha z}|U|^2 U=0$

Moment method: $\leftarrow$ Approximation: pulse shape maintained ("like a particle")

Energy: $\rightarrow E_p=\int_{-\infty}^{\infty}|U|^2\,dT$

RMS width: $\rightarrow \sigma_p^2=\frac{1}{E_p}\int_{-\infty}^{\infty}T^2|U|^2\,dT$

Chirp: $\rightarrow C_p=\frac{i}{E_p}\int_{-\infty}^{\infty}T\left(U^*\frac{\partial U}{\partial T}-U\frac{\partial U^*}{\partial T}\right)dT$

Derivatives: $\rightarrow \frac{dE_p}{dz}=0 \quad \rightarrow \frac{d\sigma_p^2}{dz}=\frac{\beta_2}{E_p}\int_{-\infty}^{\infty}T^2\,\mathrm{Im}\left(U^*\frac{\partial^2 U}{\partial T^2}\right)dT$

$$\rightarrow \frac{dC_p}{dz}=\frac{2\beta_2}{E_p}\int_{-\infty}^{\infty}\left|\frac{\partial U}{\partial T}\right|^2 dT+e^{-\alpha z}\frac{\gamma P_0}{E_p}\int_{-\infty}^{\infty}|U|^4\,dT$$

For a chirped Gaussian pulse:

$$\rightarrow U(z,T)=a_p\exp\left[-\frac{1}{2}(1+iC_p)(T/T_p)^2+i\phi_p\right]$$

$$\rightarrow E_p=\sqrt{\pi}a_p^2T_p=\sqrt{\pi}T_0 \rightarrow T_p=\sqrt{2}\sigma_p$$

$$\rightarrow T_p^2(z)=T_0^2+2\int_0^z\beta_2(z)C_p(z)dz$$

$$\rightarrow \frac{dT_p}{dz}=\frac{\beta_2 C_p}{T_p}$$

$$\rightarrow \frac{dC_p}{dz}=\left(1+C_p^2\right)\frac{\beta_2}{T_p^2}+\frac{\gamma P_0 T_0}{\sqrt{2}T_p}e^{-\alpha z}$$

Semianalytic Techniques (2)

Nonlinear Schrödinger equation: $\rightarrow i\frac{\partial U}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} + \gamma P_0 e^{-\alpha z}|U|^2 U = 0$

Variational method: $\leftarrow$ Approximation by the use of Lagrangian

$$\rightarrow \mathcal{L} = \int_{-\infty}^{\infty} \mathcal{L}_d(q, q^*)dT$$

$$\rightarrow \mathcal{S} = \int_{-\infty}^{\infty} \mathcal{L}dz$$ $\leftarrow$ Minimization of the "action" functional

$$\rightarrow \frac{\partial}{\partial T}\left(\frac{\partial \mathcal{L}_d}{\partial q_T}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \mathcal{L}_d}{\partial q_z}\right) - \frac{\partial \mathcal{L}_d}{\partial q} = 0$$ $\leftarrow$ Euler-Lagrange eq.

NLS equation: $\leftarrow \mathcal{L}_d = \frac{i}{2}\left(U^*\frac{\partial U}{\partial z} - U\frac{\partial U^*}{\partial z}\right) + \frac{\beta_2}{2}\left|\frac{\partial U}{\partial T}\right|^2 + \frac{1}{2}\gamma P_0 e^{-\alpha z}|U|^4$ $(U^* \rightarrow q)$

For a chirped Gaussian pulse:

$$\rightarrow \mathcal{L} = \frac{\beta_2 E_p}{4T_p^2}(1 + C_p^2) + \frac{\gamma e^{-\alpha z}E_p^2}{\sqrt{8\pi}T_p} + \frac{E_p}{4}\left(\frac{dC_p}{dz} - \frac{2C_p}{T_p}\frac{dT_p}{dz}\right) - E_p\frac{d\phi_p}{dz}$$

Reduced E-L eq.

$$\rightarrow \frac{d}{dz}\left(\frac{\partial \mathcal{L}}{\partial q_z}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0$$

$\rightarrow q = \phi_p \quad \rightarrow dE_p / dz = 0$

$\rightarrow q = C_p$ or $T_p \quad \rightarrow$ Eq. (4.3.7) or (4.3.8)

$\rightarrow q = E_p \quad \rightarrow \frac{d\phi_p}{dz} = \frac{\beta_2}{2T_p^2} + \frac{5\gamma e^{-\alpha z}E_p}{4\sqrt{2\pi}T_p}$ $\rightarrow$ H.W.

Higher-Order Nonlinear Effects (1)

Nonlinear Schrödinger equation for ultrashort pulses ($T_0 < 1$ ps):

$$\rightarrow \frac{\partial U}{\partial z} + i\frac{\mathrm{sgn}(\beta_2)}{2L_D}\frac{\partial^2 U}{\partial \tau^2} - \frac{\mathrm{sgn}(\beta_3)}{6L_D'}\frac{\partial^3 U}{\partial \tau^3} = i\frac{e^{-\alpha z}}{L_{NL}}\left(|U|^2 U + is\frac{\partial}{\partial \tau}(|U|^2 U) - \tau_R U\frac{\partial |U|^2}{\partial \tau}\right)$$

$$\leftarrow L_D = \frac{T_0^2}{|\beta_2|}, \quad L_D' = \frac{T_0^3}{|\beta_3|}, \quad L_{NL} = \frac{1}{\gamma P_0}$$

$$\leftarrow s = \frac{1}{\omega_0 T_0}, \quad \tau_R = \frac{T_R}{T_0}$$

"Self-steepening" "Intrapulse Raman scattering"

Self-steepening: ← Intensity dependence of the group velocity

→ Asymmetry in the SPM-broadened spectra

$$\rightarrow \frac{\partial U}{\partial Z} + s\frac{\partial}{\partial \tau}(|U|^2 U) = i|U|^2 U \qquad \leftarrow \beta_2 = \beta_3 = 0, \quad \tau_R = 0, \quad \alpha = 0, \quad Z = z/L_{NL}$$

$$\leftarrow U = \sqrt{I}\exp(i\phi)$$

$$\rightarrow \frac{\partial I}{\partial Z} + 3sI\frac{\partial I}{\partial \tau} = 0$$

$$\rightarrow \frac{\partial \phi}{\partial Z} + sI\frac{\partial \phi}{\partial \tau} = I$$

General solution form:

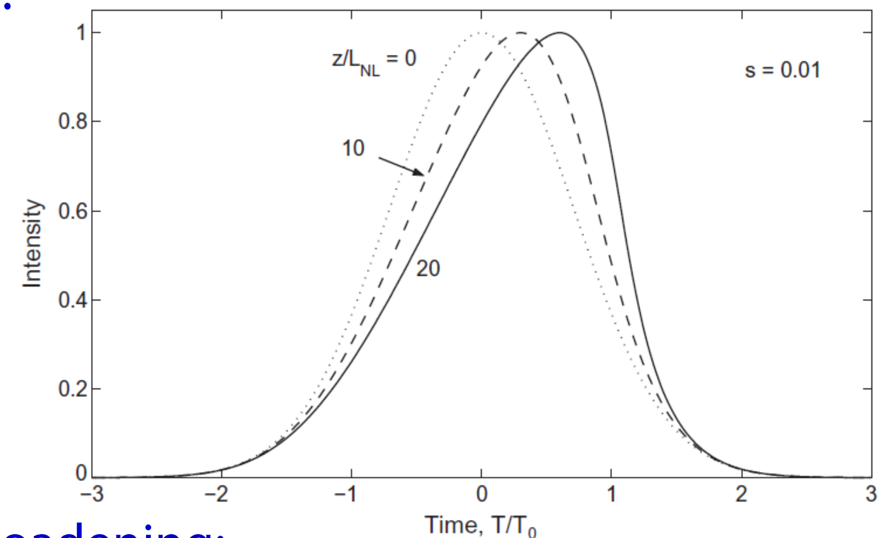
$$\leftarrow I(Z,\tau) = f(\tau - 3sIZ)$$

Higher-Order Nonlinear Effects (2)

Self-steepening for a Gaussian pulse:

$$\rightarrow I(0,\tau) \equiv f(\tau) = \exp(-\tau^2)$$

$$\rightarrow I(Z,\tau) = \exp[(\tau - 3sIZ)^2]$$



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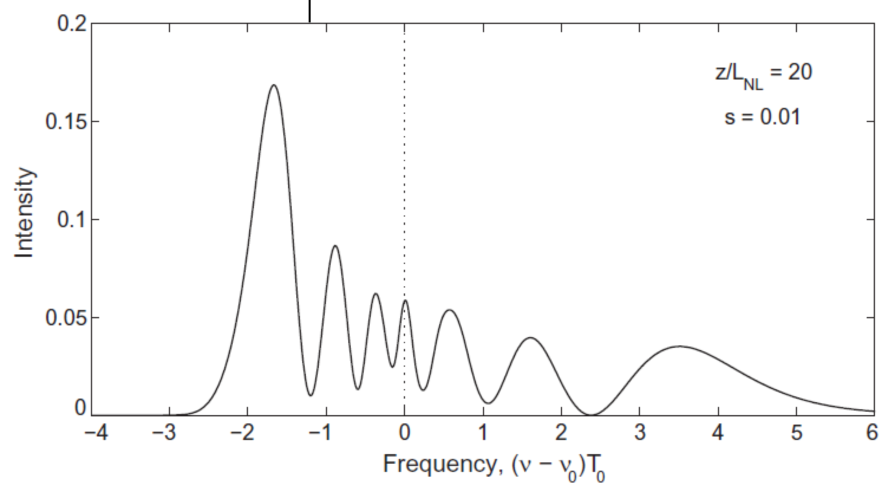
Shock location: $\rightarrow \partial I / \partial \tau = \infty$

$$\rightarrow z_s = \left(\frac{e}{2}\right)^{1/2} \frac{L_{NL}}{3s} \approx 0.39(L_{NL}/s)$$

Impacts on SPM-induced spectral broadening:

$$\rightarrow S(\omega) = \left| \int_{-\infty}^{\infty} [I(z,\tau)]^{1/2} \exp[i\phi(z,\tau) + i(\omega - \omega_0)\tau] d\tau \right|^2$$



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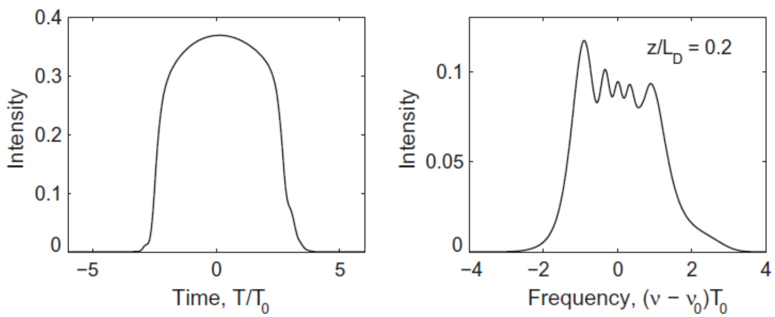
Effect of GVD on optical shocks:



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Higher-Order Nonlinear Effects (3)

Intrapulse Raman scattering:

$$\rightarrow q_p(z) = \frac{1}{E_p}\int_{-\infty}^{\infty} T\left|U(z,T)\right|^2 dT$$

$$\rightarrow \Omega_p(z) = \frac{i}{2E_p}\int_{-\infty}^{\infty}\left(U^*\frac{\partial U}{\partial T} - U\frac{\partial U^*}{\partial T}\right) dT$$

$\rightarrow$ For a Gaussian pulse:

$$\rightarrow \frac{dq_p}{dz} = \beta_2\Omega_p \quad \rightarrow \frac{d\Omega_p}{dz} = -T_R e^{-\alpha z}\frac{\gamma P_0 T_0}{\sqrt{2}T_p^3}$$

Raman-induced frequency shift: