Nonlinear Optical Engineering

Optical Solitons (1)
(NFO 5th ed: 5.1 ~ 5.2)

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Modulation Instability (1)

Linear stability analysis:

NLSE: $$i\frac{\partial A}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \gamma |A|^2 A$$

Steady-state solution: $$\bar{A} = \sqrt{P_0}\exp(i\phi_{NL}) \leftarrow \phi_{NL} = \gamma P_0 z.$$

With a perturbation: $$A = (\sqrt{P_0} + a)\exp(i\phi_{NL})$$

$$i\frac{\partial A}{\partial z} = i\frac{\partial}{\partial z}[(\sqrt{P_0} + a)\exp(i\phi_{NL})]$$

$$= i\frac{\partial a}{\partial z}\exp(i\phi_{NL}) + i(\sqrt{P_0} + a)\cdot i\frac{\partial \phi_{NL}}{\partial z}\exp(i\phi_{NL})$$

$$-\gamma |A|^2 A = -\gamma(\sqrt{P_0} + a)\exp(i\phi_{NL})\cdot(\sqrt{P_0} + a^*)\exp(-i\phi_{NL})\cdot(\sqrt{P_0} + a)\exp(i\phi_{NL})$$

$$= -\gamma[P_0(\sqrt{P_0} + a) + P_0(a + a^*) + \sqrt{P_0}(a^2 + 2a^*a) + a^*a^2]\exp(i\phi_{NL})$$

$$\leftarrow (a \to O^2)$$

In result: $$\to i\frac{\partial a}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 a}{\partial T^2} - \gamma P_0(a + a^*)$$

Modulation Instability (2)

Linear stability analysis:

Solution form: $a(z,T) = a_1 \exp[i(Kz - \Omega T)] + a_2 \exp([-i(Kz - \Omega T)]$

$$\rightarrow K = \pm \frac{1}{2} \left| \beta_2 \Omega \right| [\Omega^2 + \mathrm{sgn}(\beta_2)\Omega_c^2]^{1/2}$$

$$\leftarrow \quad \Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|} = \frac{4}{|\beta_2| L_{NL}} \quad \leftarrow \quad L_{NL} = \frac{1}{\gamma P_0}$$

$\rightarrow$ *The sign of β_2 (GVD) determines the stability against the perturbation.*

Modulation Instability (3)

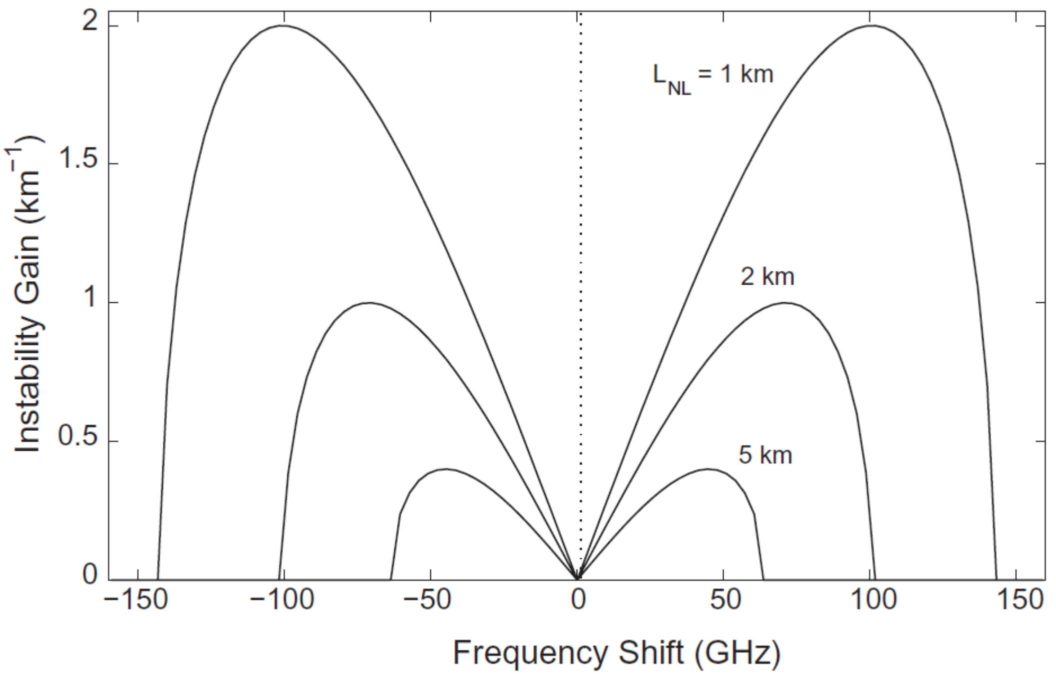
Gain spectrum:

$$K = \pm\frac{1}{2}|\beta_2\Omega|[\Omega^2 + \mathrm{sgn}(\beta_2)\Omega_c^2]^{1/2}$$

$$\rightarrow \beta_2 < 0$$

$$\rightarrow g(\Omega) = 2\,\mathrm{Im}(K) = |\beta_2\Omega|(\Omega_c^2 - \Omega^2)^{1/2}$$

$$\rightarrow \Omega_{\max} = \pm\frac{\Omega_c}{\sqrt{2}} = \pm\left(\frac{2\gamma P_0}{|\beta_2|}\right)^{1/2} \quad \rightarrow g_{\max} = \frac{1}{2}|\beta_2|\Omega_c^2 = 2\gamma P_0$$



$$\leftarrow \beta_2 = -5\,ps^2/km$$

G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Fiber Solitons (1)

Inverse scattering method:

NLSE: $$i\frac{\partial A}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \gamma|A|^2 A$$

Dimensionless variables: $$U = \frac{A}{\sqrt{P_0}},\ \xi = \frac{z}{L_D},\ \tau = \frac{T}{T_0} \quad \leftarrow \ L_D = \frac{T_0^2}{|\beta_2|}.$$

In result: $$\rightarrow i\frac{\partial U}{\partial \xi} = \mathrm{sgn}(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} - N^2|U|^2 U \quad \leftarrow \ N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$$

$$\rightarrow \ u = NU = \sqrt{\gamma L_D}A,\ \ \mathrm{sgn}(\beta_2) = -1$$

Scaling factor

$$\rightarrow \ i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0 \quad \leftarrow u(\xi,\tau) \leftarrow \varepsilon u(\varepsilon^2\xi, \varepsilon\tau)$$

Inverse scattering method: $$\rightarrow \begin{cases} i\dfrac{\partial v_1}{\partial \tau} + u v_2 = \varsigma v_1 \\ i\dfrac{\partial v_2}{\partial \tau} + u^* v_1 = -\varsigma v_2 \end{cases}$$

Complex values (similar to freq. in the Fourier analysis)

Fundamental soliton: (N = 1) $$\rightarrow u(\xi,\tau) = \mathrm{sech}(\tau)\exp(i\xi/2)$$