

# Nonlinear Optical Engineering

Optical Solitons (2)  
(NFO 5<sup>th</sup> ed: 5.2 ~ 5.3)

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: [yunchan@snu.ac.kr](mailto:yunchan@snu.ac.kr)

# Fiber Solitons (2)

## Fundamental soliton:

Shape-preserving solution:  $\rightarrow u(\xi, \tau) = V(\tau) \exp[i\phi(\xi, \tau)]$

$$\text{NLSE: } \rightarrow i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$$

$$\rightarrow \phi(\xi, \tau) = K\xi - \delta\tau$$

$$\rightarrow \frac{d^2 V}{d\tau^2} = 2V(K - V^2) \quad \leftarrow \times 2 \frac{dV}{d\tau}$$

$$\rightarrow \left( \frac{dV}{d\tau} \right)^2 = 2KV^2 - V^4 + C$$

$\leftarrow V = 1 \text{ \& } dV / d\tau = 0$   $\leftarrow$  At the soliton peak

$$\rightarrow K = \frac{1}{2}$$

$$\rightarrow V = \text{sech}(\tau)$$

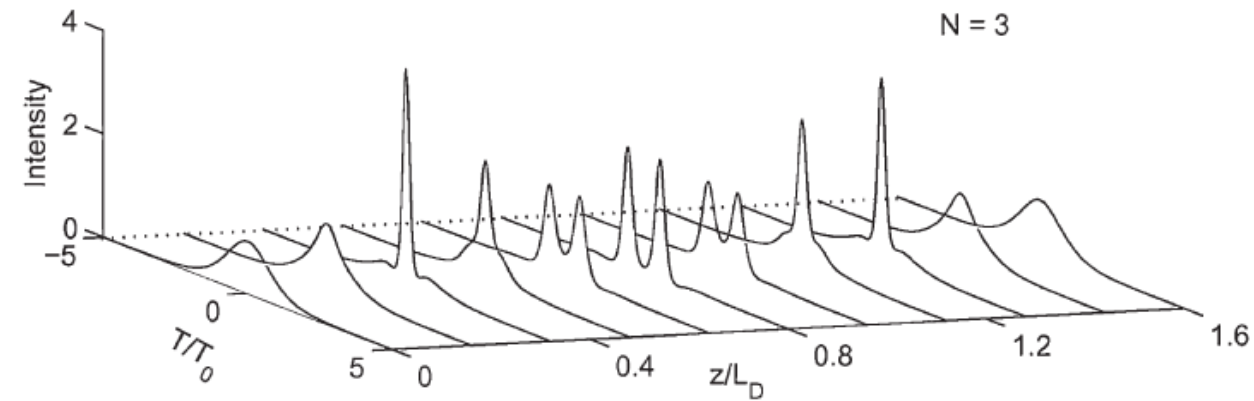
In result:  $\rightarrow u(\xi, \tau) = \text{sech}(\tau) \exp(i\xi / 2)$

# Fiber Solitons (3)

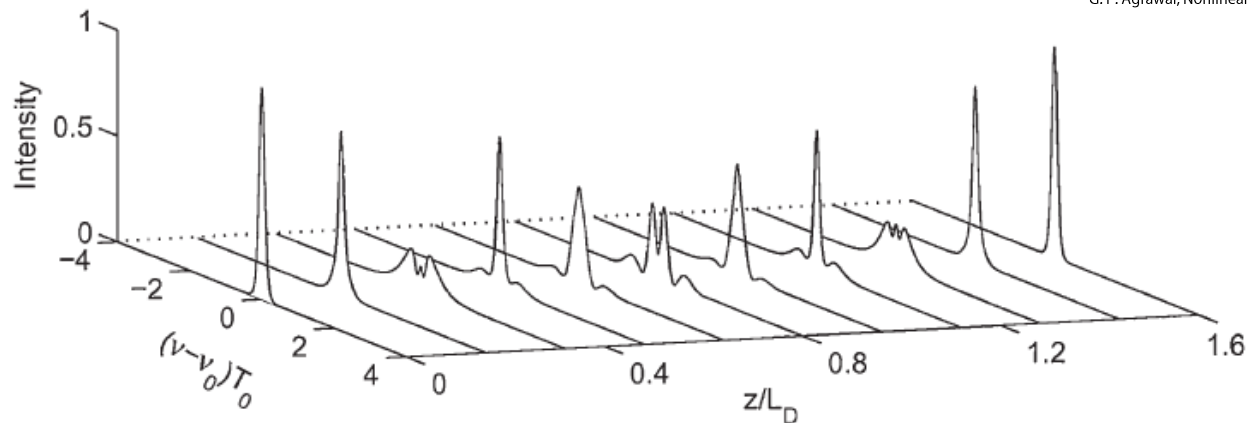
Higher-order solitons:

Periodically evolving solution:  $\rightarrow N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$

Third-order soliton:



G. P. Agrawal, Nonlinear Fiber Optics, 5<sup>th</sup> ed.



# Fiber Solitons (4)

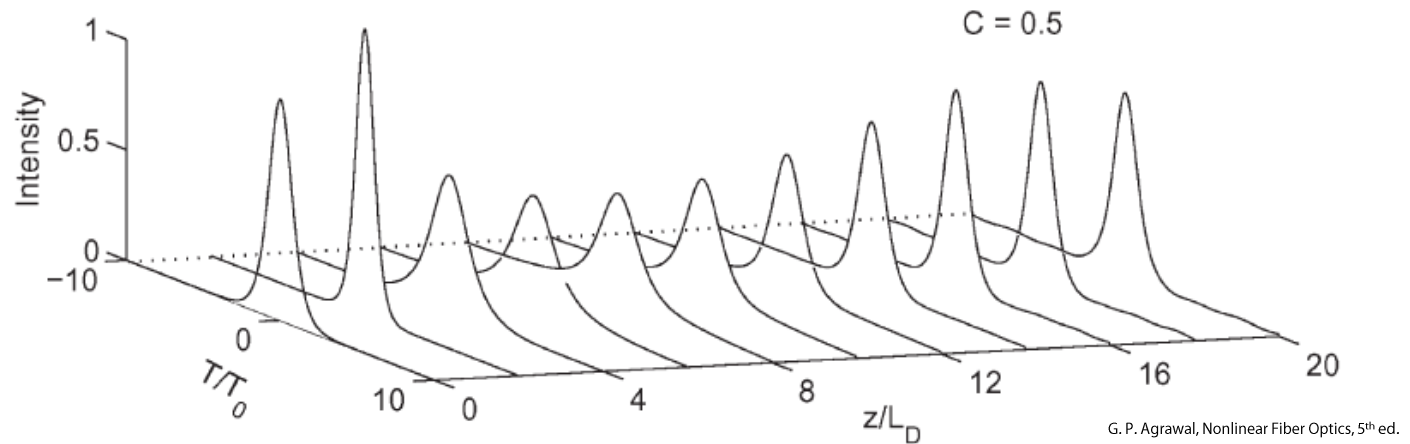
## Soliton stability:

Perturbations:  $\rightarrow u(0, \tau) = (\tilde{N} + 2\varepsilon)\text{sech}[(1 + 2\varepsilon / \tilde{N})\tau]$

$$\rightarrow u(0, \tau) = N\text{sech}(\tau) \exp(-iC\tau^2 / 2)$$

$\rightarrow$  Numerical analysis

Soliton formation in the presence of an initial linear chirp:



$\leftarrow$  Pulse energy loss via forming dispersive waves

# Other Types of Solitons (1)

Dark solitons:

NLSE:  $\rightarrow i \frac{\partial u}{\partial \xi} - \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$   $\leftarrow$  In the normal GVD condition

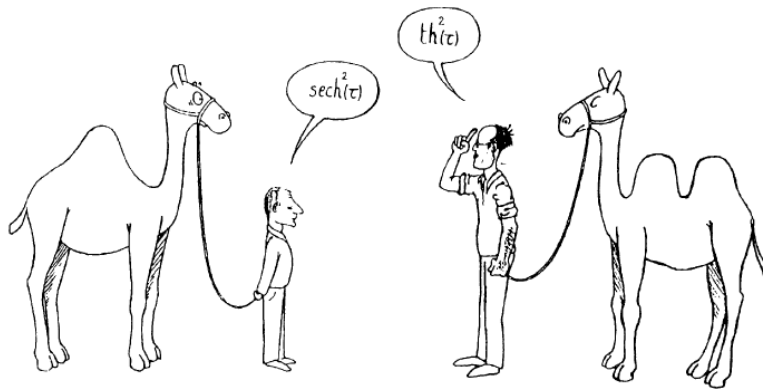
General solution:  $\rightarrow u(\xi, \tau) = \eta [B \tanh(\zeta) - i\sqrt{1-B^2}] \exp(i\eta^2 \xi)$

$\leftarrow \zeta = \eta B(\tau - \tau_s - \eta B \sqrt{1-B^2})$

$\leftarrow |B| < 1$   $\leftarrow$  Grey solitons

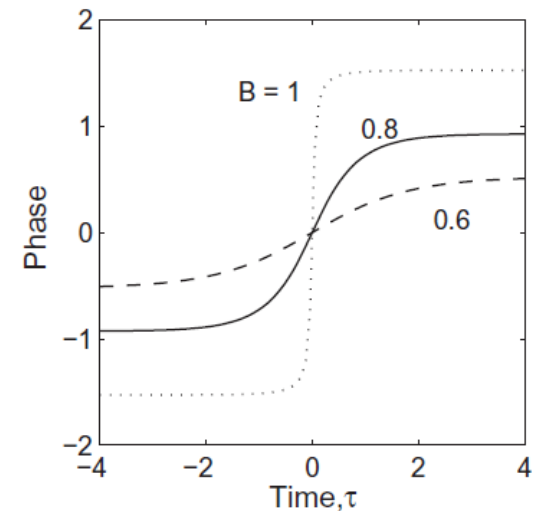
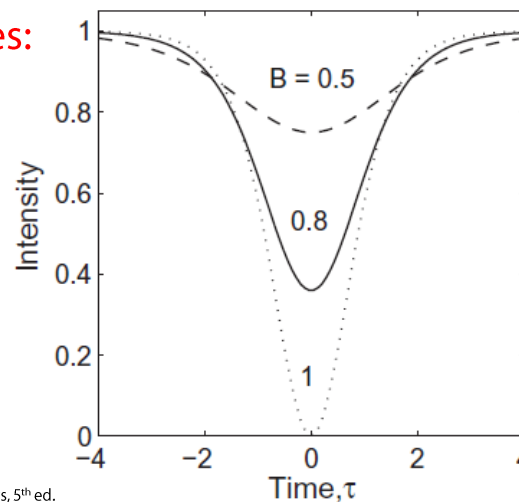
$\leftarrow |B| = 1$   $\leftarrow$  Black soliton

$\rightarrow u(\xi, \tau) = \tanh(\tau) \exp(i\xi)$

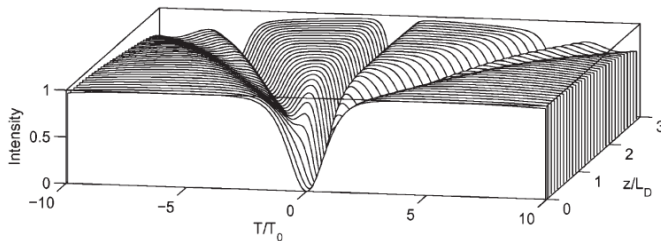


Physics Reports 298 (1998) 81-197

Intensity and phase profiles:



Third-order dark soliton:



G. P. Agrawal, Nonlinear Fiber Optics, 5<sup>th</sup> ed.

# Other Types of Solitons (2)

## Bistable solitons:

Intensity-dependent refractive index:  $\rightarrow \tilde{n}(I) = n + n_2 I \rightarrow \tilde{n}(I) = n + n_2 f(I)$

← "Saturation" considered

NLSE:  $\rightarrow i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + f(|u|^2)u = 0$

$\rightarrow u(\xi, \tau) = V(\tau) \exp(iK\xi)$

$\rightarrow \frac{d^2 V}{d\tau^2} = 2V[K - f(V^2)] \rightarrow \left( \frac{dV}{d\tau} \right)^2 = 4 \int_0^V [K - f(V^2)] V dV$

$\rightarrow 2\tau = \int_0^V \left( \int_0^{V^2} [K - f(P)] dP \right)^{-1/2} dV$

Soliton energy:  $\rightarrow E_s(K) = \frac{1}{2} \int_0^{P_m} [K - F(P)]^{-1/2} dP \leftarrow F(P) = \frac{1}{P} \int_0^P f(P) dP$   
 $\leftarrow F(0) = 0$

←  $P_m$ : the smallest positive root of  $F(P) = K$

Bistable soliton solutions: ←  $K$  &  $P_m$

# Other Types of Solitons (3)

Dispersion-managed solitons:

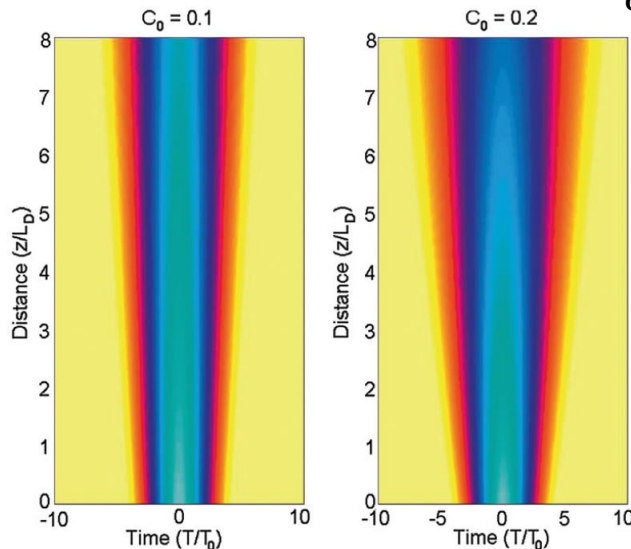
$$\text{NLSE: } \rightarrow i \frac{\partial u}{\partial \xi} + \frac{d(\xi)}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$$

Optical similaritons:

$$\text{NLSE: } \rightarrow i \frac{\partial A}{\partial z} - \frac{\beta_2(z)}{2} \frac{\partial^2 A}{\partial T^2} + \gamma(z) |A|^2 A = i \frac{g(z)}{2} A$$

$$\text{Compatibility condition: } \rightarrow g(z) = C(z) \frac{\beta_2(z)}{T_0^2} + \frac{d}{dz} \ln \left[ \frac{\beta_2(z)}{\gamma(z)} \right]$$

$$\rightarrow i \frac{\partial U}{\partial \zeta} - \text{sgn}(\beta_2) \frac{1}{2} \frac{\partial^2 U}{\partial \chi^2} + |U|^2 U = 0$$



→ Self-similar transformation into a chirped parabolic-shape pulse

→ Good for forming a high-energy ultra-short pulse