

Nonlinear Optical Engineering

Optical Solitons (2)
(NFO 5th ed: 5.2 ~ 5.3)

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Fiber Solitons (2)

Fundamental soliton:

Shape-preserving solution: $\rightarrow u(\xi, \tau) = V(\tau) \exp[i\phi(\xi, \tau)]$

$$\text{NLSE: } \rightarrow i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$$

$$\rightarrow \phi(\xi, \tau) = K\xi - \delta\tau$$

$$\rightarrow \frac{d^2 V}{d\tau^2} = 2V(K - V^2) \quad \leftarrow \times 2 \frac{dV}{d\tau}$$

$$\rightarrow \left(\frac{dV}{d\tau} \right)^2 = 2KV^2 - V^4 + C$$

$\leftarrow V = 1 \text{ \& } dV / d\tau = 0$ \leftarrow At the soliton peak

$$\rightarrow K = \frac{1}{2}$$

$$\rightarrow V = \text{sech}(\tau)$$

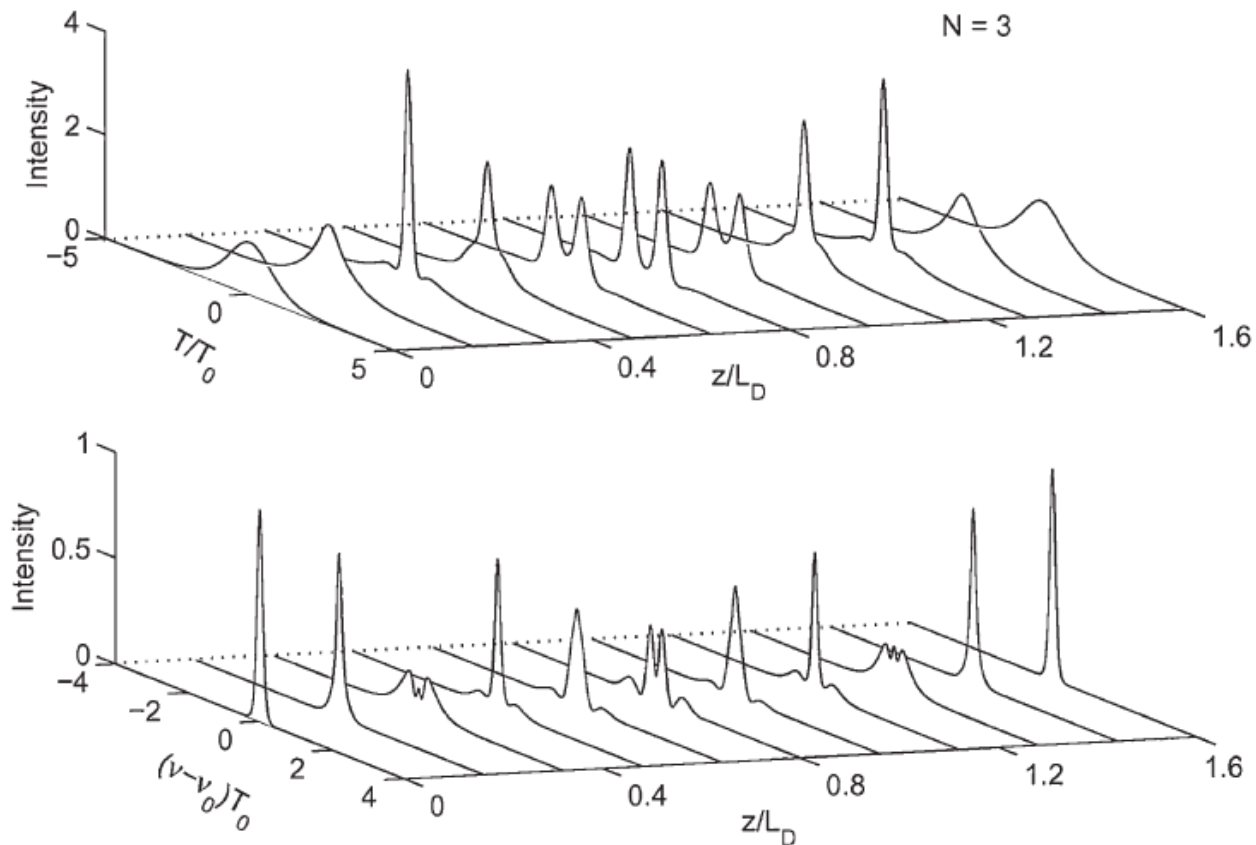
In result: $\rightarrow u(\xi, \tau) = \text{sech}(\tau) \exp(i\xi / 2)$

Fiber Solitons (3)

Higher-order solitons:

Periodically evolving solution: $\rightarrow N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$

Third-order soliton:



Fiber Solitons (4)

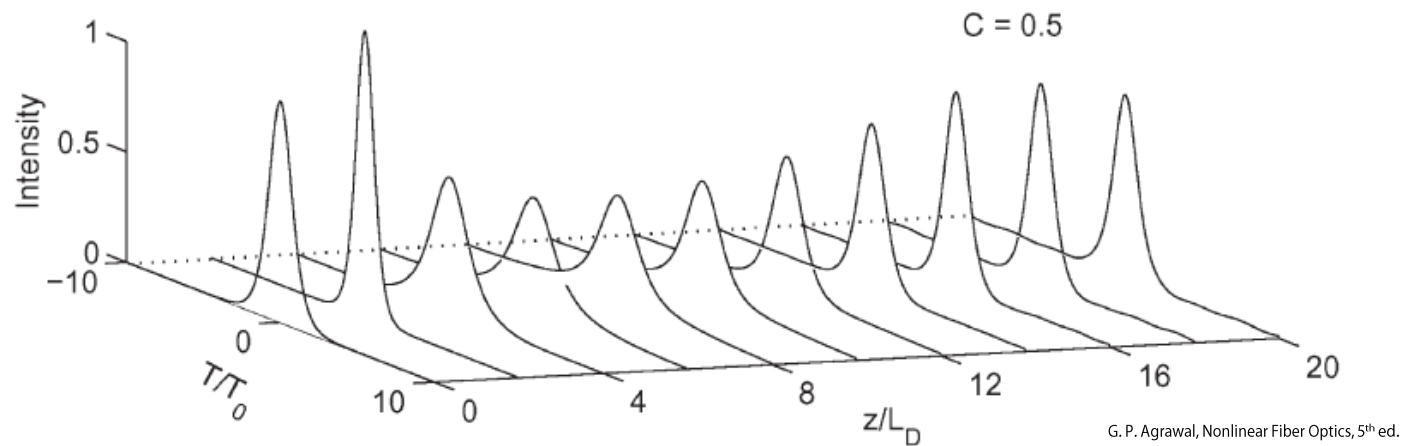
Soliton stability:

Perturbations: $\rightarrow u(0, \tau) = (\tilde{N} + 2\varepsilon)\text{sech}[(1 + 2\varepsilon / \tilde{N})\tau]$

$$\rightarrow u(0, \tau) = N\text{sech}(\tau) \exp(-iC\tau^2 / 2)$$

\rightarrow Numerical analysis

Soliton formation in the presence of an initial linear chirp:



\leftarrow Pulse energy loss via forming dispersive waves

Other Types of Solitons (1)

Dark solitons:

NLSE: $\rightarrow i \frac{\partial u}{\partial \xi} - \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$ \leftarrow In the normal GVD condition

General solution: $\rightarrow u(\xi, \tau) = \eta [B \tanh(\zeta) - i\sqrt{1-B^2}] \exp(i\eta^2 \xi)$

$\leftarrow \zeta = \eta B(\tau - \tau_s - \eta B\sqrt{1-B^2})$

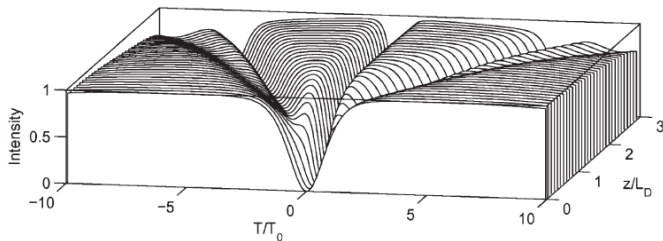
$\leftarrow |B| < 1$ \leftarrow Grey solitons

$\leftarrow |B| = 1$ \leftarrow Black soliton

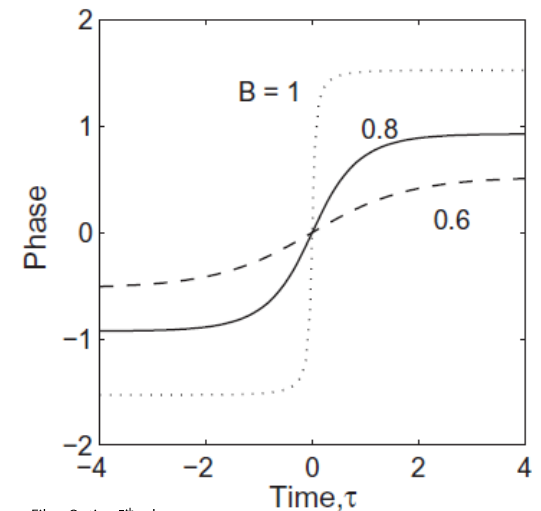
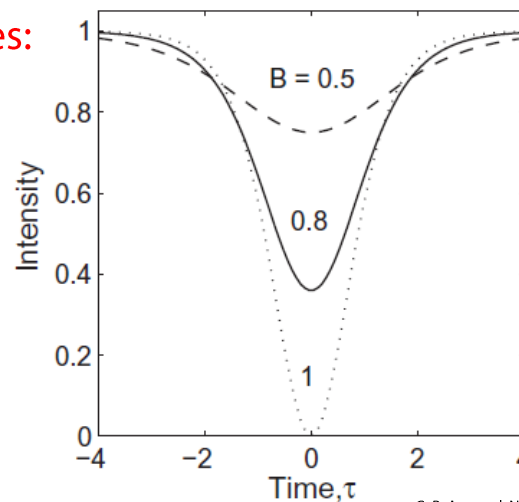
$\rightarrow u(\xi, \tau) = \tanh(\tau) \exp(i\xi)$

Intensity and phase profiles:

Third-order dark soliton:



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Other Types of Solitons (2)

Bistable solitons:

Intensity-dependent refractive index: $\rightarrow \tilde{n}(I) = n + n_2 I \rightarrow \tilde{n}(I) = n + n_2 f(I)$

← “Saturation” considered

NLSE: $\rightarrow i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + f(|u|^2)u = 0$

$\rightarrow u(\xi, \tau) = V(\tau) \exp(iK\xi)$

$\rightarrow \frac{d^2 V}{d\tau^2} = 2V[K - f(V^2)] \rightarrow \left(\frac{dV}{d\tau} \right)^2 = 4 \int_0^V [K - f(V^2)] V dV$

$\rightarrow 2\tau = \int_0^V \left(\int_0^{V^2} [K - f(P)] dP \right)^{-1/2} dV$

Soliton energy: $\rightarrow E_s(K) = \frac{1}{2} \int_0^{P_m} [K - F(P)]^{-1/2} dP \leftarrow F(P) = \frac{1}{P} \int_0^P f(P) dP$
 $\leftarrow F(0) = 0$

← P_m : the smallest positive root of $F(P) = K$

Bistable soliton solutions: ← K & P_m

Other Types of Solitons (3)

Dispersion-managed solitons:

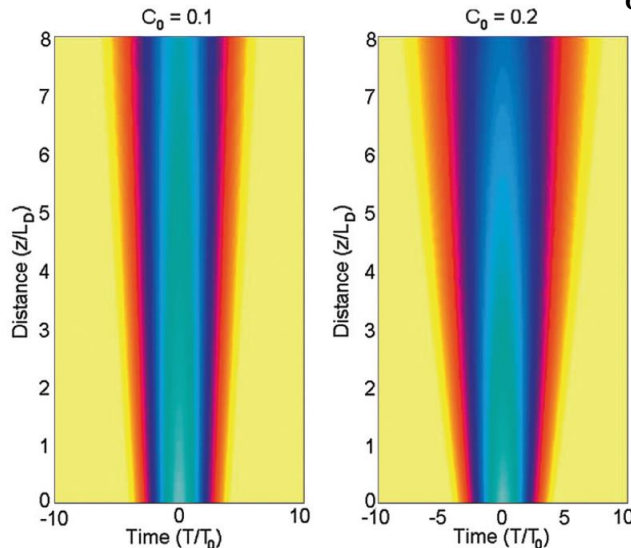
$$\text{NLSE: } \rightarrow i \frac{\partial u}{\partial \xi} + \frac{d(\xi)}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$$

Optical similaritons:

$$\text{NLSE: } \rightarrow i \frac{\partial A}{\partial z} - \frac{\beta_2(z)}{2} \frac{\partial^2 A}{\partial T^2} + \gamma(z) |A|^2 A = i \frac{g(z)}{2} A$$

$$\text{Compatibility condition: } \rightarrow g(z) = C(z) \frac{\beta_2(z)}{T_0^2} + \frac{d}{dz} \ln \left[\frac{\beta_2(z)}{\gamma(z)} \right]$$

$$\rightarrow i \frac{\partial U}{\partial \zeta} - \text{sgn}(\beta_2) \frac{1}{2} \frac{\partial^2 U}{\partial \chi^2} + |U|^2 U = 0$$



→ Self-similar transformation into a chirped parabolic-shape pulse

→ Good for forming a high-energy ultra-short pulse