Nonlinear Optical Engineering

Optical Solitons (3)
(NFO 5th ed: 5.4)

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Perturbation of Solitons (1)

Perturbation methods:

NLSE: $\rightarrow i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+|u|^2 u = i\varepsilon(u)$

$$\rightarrow u(\xi,\tau)=\eta(\xi)\,\mathrm{sech}[\eta(\xi)(\tau-q(\xi))]\exp[i\phi(\xi)-i\delta(\xi)\tau]$$

Variational method: $\rightarrow \mathcal{L}_d=\frac{i}{2}\left(u^*\frac{du}{d\xi}-u\frac{\partial u^*}{\partial \xi}\right)+\frac{1}{2}\left(|u|^4-\left|\frac{du}{d\tau}\right|^2\right)+i(\varepsilon^* u-\varepsilon u^*)$

Reduced E-L eq.: $\rightarrow \mathcal{L}=\int_{-\infty}^{\infty}\mathcal{L}_d\,d\tau \quad \rightarrow \frac{d}{d\xi}\left(\frac{\partial \mathcal{L}}{\partial q_\xi}\right)-\frac{\partial \mathcal{L}}{\partial q}=0$

Four differential eqs.: $\rightarrow \frac{d\eta}{d\xi}=\mathrm{Re}\int_{-\infty}^{\infty}\varepsilon(u)u^*\,d\tau$

$$\rightarrow \frac{d\delta}{d\xi}=-\mathrm{Im}\int_{-\infty}^{\infty}\varepsilon(u)\tanh[\eta(\tau-q)]u^*\,d\tau$$

$$\rightarrow \frac{dq}{d\xi}=-\delta+\frac{1}{\eta}\mathrm{Re}\int_{-\infty}^{\infty}\varepsilon(u)(\tau-q)u^*\,d\tau$$

$$\rightarrow \frac{d\phi}{d\xi}=\mathrm{Im}\int_{-\infty}^{\infty}\varepsilon(u)\{1/\eta-(\tau-q)\tanh[\eta(\tau-q)]\}u^*\,d\tau$$

$$+\frac{1}{2}(\eta^2-\delta^2)+q\frac{d\delta}{d\xi}$$

Perturbation of Solitons (2)

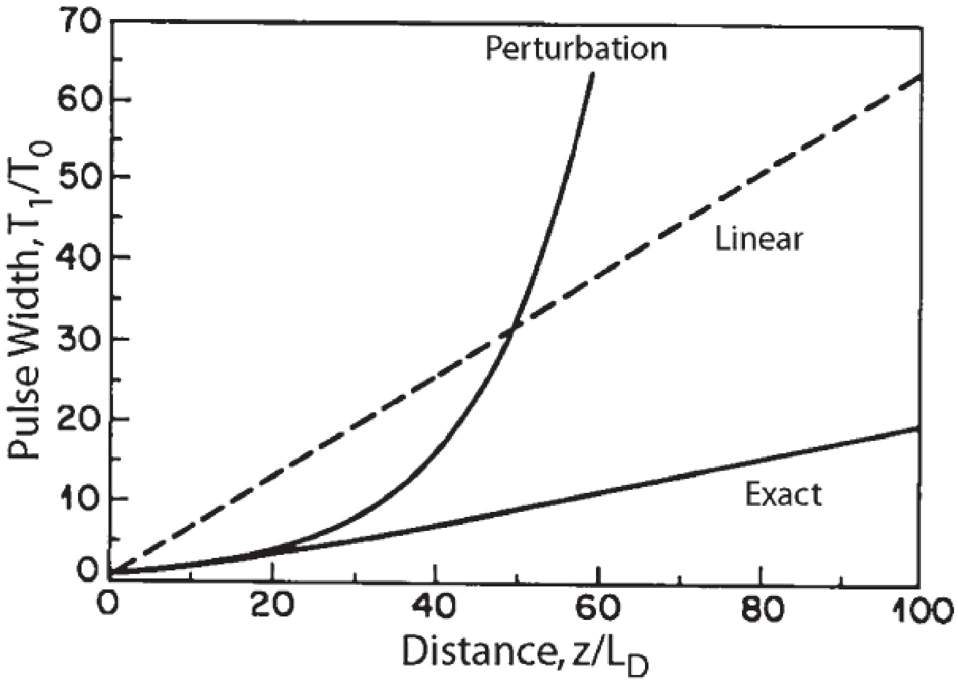
Fiber Losses:

NLSE: $\rightarrow i\dfrac{\partial u}{\partial \xi} + \dfrac{1}{2}\dfrac{\partial^2 u}{\partial \tau^2} + |u|^2 u = -\dfrac{i}{2}\Gamma u \quad \leftarrow \Gamma = \alpha L_D = \alpha T_0^2 / |\beta_2|$

$$\rightarrow \eta = \exp(-\Gamma\xi) \qquad \leftarrow \eta(0) = 1, \delta(0) = 0, \& \ q(0) = 0$$

$$\rightarrow \phi(\xi) = \phi(0) + [1 - \exp(-2\Gamma\xi)] / (4\Gamma)$$

Soliton broadening: $\rightarrow T_1(z) = T_0 \exp(\Gamma\xi) = T_0 \exp(\alpha z) \quad \leftarrow \tau = T / T_0$



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Perturbation of Solitons (3)

How can a soliton survive inside lossy optical fibers?

NLSE: $$\rightarrow\ i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+|u|^2 u=-\frac{i}{2}\Gamma u$$

$$\leftarrow u(\xi,\tau)=v(\xi,\tau)\exp(-\Gamma\xi/2)$$

$$\rightarrow\ i\frac{\partial v}{\partial \xi}+\frac{d(\xi)}{2}\frac{\partial^2 v}{\partial \tau^2}+e^{-\Gamma\xi}|v|^2 v=0 \quad \leftarrow d(\xi)=|\beta_2(\xi)/\beta_2(0)|$$

$$\leftarrow \xi'=\int_0^{\xi} d(\xi)d\xi$$

$$\rightarrow\ i\frac{\partial v}{\partial \xi'}+\frac{1}{2}\frac{\partial^2 v}{\partial \tau^2}+\frac{e^{-\Gamma\xi}}{d(\xi)}|v|^2 v=0$$

Dispersion-decreasing fibers: $$\rightarrow d(\xi)=\exp(-\Gamma\xi)$$

$$\rightarrow |\beta_2(z)|=|\beta_2(0)|\exp(-\alpha z)$$

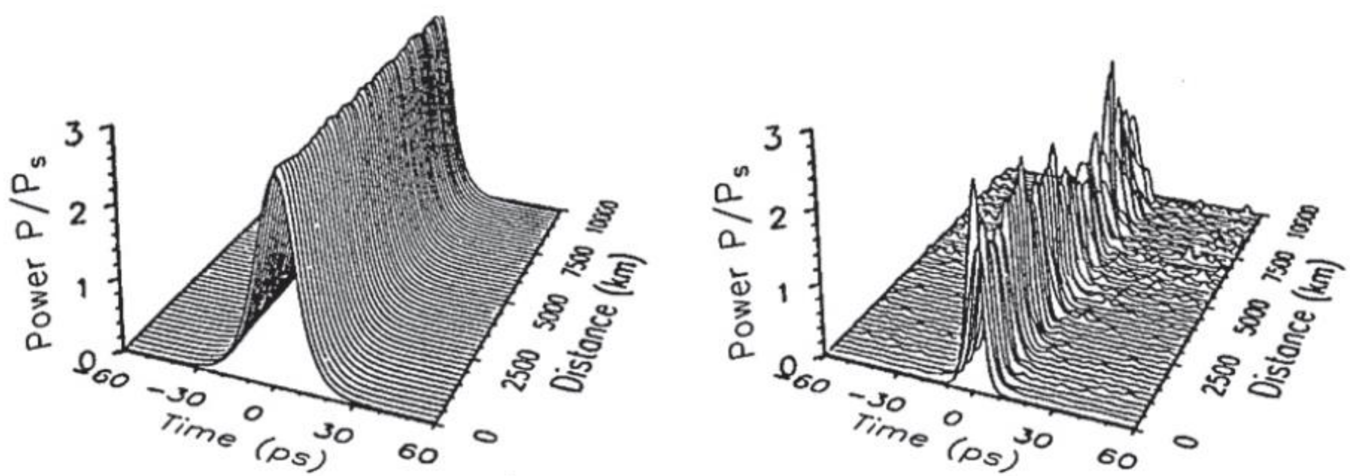
Perturbation of Solitons (4)

Soliton amplification:

NLSE: $$\rightarrow\; i\frac{\partial v}{\partial \xi}+\frac{1}{2}\frac{\partial^2 v}{\partial \tau^2}+a^2(\xi)\left|v\right|^2 v=0$$

$$\leftarrow u(\xi,\tau)=\exp\left(\int_0^{\xi}-\frac{1}{2}\Gamma(\xi)d\xi\right)v(\xi,\tau)=a(\xi)v(\xi,\tau)$$

Distributed amp. vs. lumped amp.:



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Perturbation of Solitons (5)

Soliton interaction:

Two solitons: $\rightarrow u_j(\xi,\tau) = \eta_j \text{sech}[\eta_j(\tau - q_j)]\exp[i\phi_j - i\delta_j \tau]$

$\rightarrow u = u_1 + u_2$ $\rightarrow$ NLSE

Perturbed NLSE: $\rightarrow \; i\frac{\partial u_1}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_1}{\partial \tau^2} + |u_1|^2 u_1 = -2|u_1|^2 u_2 - u_1^2 u_2^*$

$$\rightarrow \eta_\pm = \eta_1 \pm \eta_2, \; q_\pm = q_1 \pm q_2, \; \delta_\pm = \delta_1 \pm \delta_2, \; \phi_\pm = \phi_1 \pm \phi_2$$

$$\rightarrow \frac{d\eta_+}{d\xi} = 0, \; \frac{d\eta_-}{d\xi} = \eta_+^3 \exp(-q_-)\sin\phi_-$$

$$\rightarrow \frac{d\delta_+}{d\xi} = 0, \; \frac{d\delta_-}{d\xi} = \eta_+^3 \exp(-q_-)\cos\phi_-$$

$$\rightarrow \frac{dq_-}{d\xi} = -\delta_-, \; \frac{d\phi_-}{d\xi} = \frac{1}{2}\eta_+\eta_- \quad \leftarrow q = q_-/2, \; \psi = \phi_-/2$$

In result: $\rightarrow \frac{d^2 q}{d\xi^2} = -4e^{-2q}\cos(2\psi), \; \frac{d^2\psi}{d\xi^2} = 4e^{-2q}\sin(2\psi) \;\; \leftarrow \eta_-(0) = 0, \; \delta_-(0) = 0$

Solution: $\rightarrow q(\xi) = q_0 + \tfrac{1}{2}\ln[\cosh^2(2\xi e^{-q_0}\sin\psi_0) + \cos^2(2\xi e^{-q_0}\cos\psi_0) - 1]$

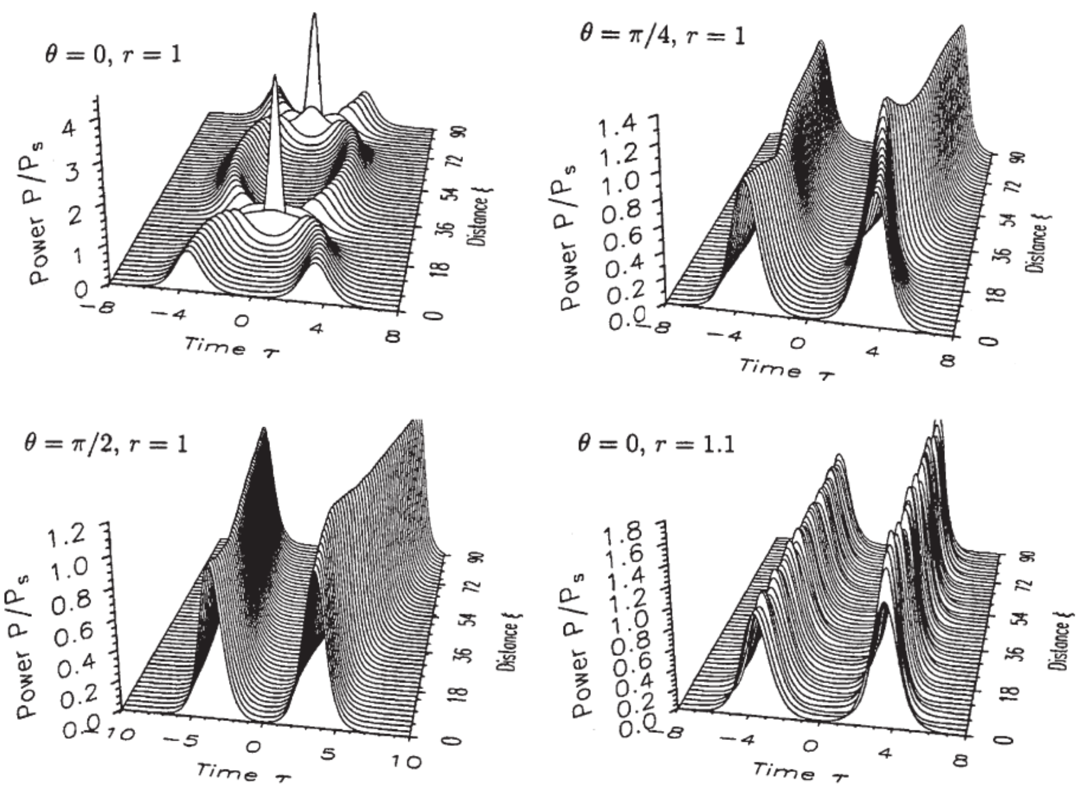
Perturbation of Solitons (6)

Soliton interaction:

Solution: $\rightarrow\ q(\xi) = q_0 + \frac{1}{2}\ln[\cosh^2(2\xi e^{-q_0}\sin\psi_0) + \cos^2(2\xi e^{-q_0}\cos\psi_0) - 1]$

In phase ($\psi = 0$): $\rightarrow\ q(\xi) = q_0 + \ln\left|\cos(2\xi e^{-q_0})\right|$ ← Periodically attractive

Out of phase ($\psi = \pi/2$): $\rightarrow\ q(\xi) = q_0 + \ln[\cosh(2\xi e^{-q_0})]$ ← Monotonically repulsive



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