Nonlinear Optical Engineering

Polarization Effects (1)
(NFO 5th ed: 6.1~6.2)

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Nonlinear Birefringence (1)

Origin of nonlinear birefringence:

Electric field: $\mathbf{E}(\mathbf{r},t) = \frac{1}{2}(\hat{x}E_x + \hat{y}E_y)\exp(-i\omega_0 t) + c.c.$

Third-order susceptibility: How many elements for $\chi^{(3)}_{ijkl}$? $\rightarrow$ *81 elements*

For isotropic media: $\rightarrow$ *21 nonzero elements*

$$\chi^{(3)}_{xxxx} = \chi^{(3)}_{yyyy} = \chi^{(3)}_{zzzz},$$

$$\chi^{(3)}_{xxyy} = \chi^{(3)}_{xxzz} = \chi^{(3)}_{yyxx} = \chi^{(3)}_{yyzz} = \chi^{(3)}_{zzxx} = \chi^{(3)}_{zzyy},$$

$$\chi^{(3)}_{xyxy} = \chi^{(3)}_{xzxz} = \chi^{(3)}_{yzyz} = \chi^{(3)}_{yxyx} = \chi^{(3)}_{zxzx} = \chi^{(3)}_{zyzy},$$

$\leftarrow$ *Centrosymmetry*

$$\chi^{(3)}_{xyyx} = \chi^{(3)}_{xzzx} = \chi^{(3)}_{yxxy} = \chi^{(3)}_{yzzy} = \chi^{(3)}_{zxxz} = \chi^{(3)}_{zyyz}.$$

$$\rightarrow \chi^{(3)}_{xxxx} = \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx}$$

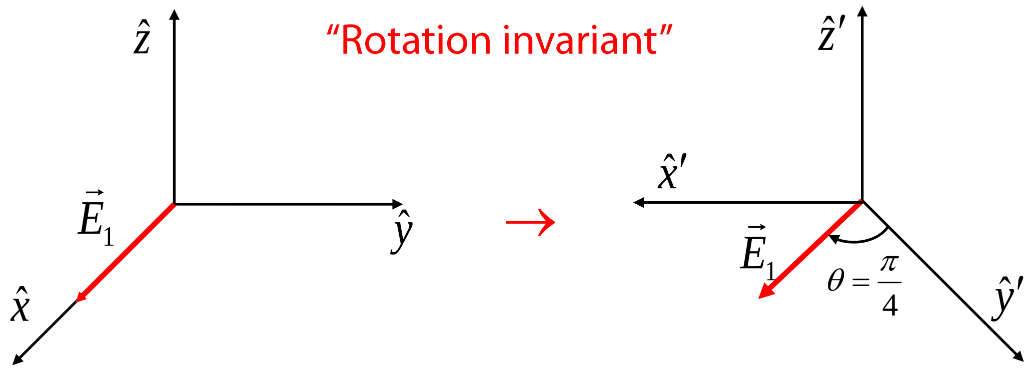
In the compact form:

$$\rightarrow \chi^{(3)}_{ijkl} = \chi^{(3)}_{xxyy}\delta_{ij}\delta_{kl} + \chi^{(3)}_{xyxy}\delta_{ik}\delta_{jl} + \chi^{(3)}_{xyyx}\delta_{il}\delta_{jk}$$

Nonlinear Birefringence (2)

Third-order susceptibility:

$$\vec{P}_{NL} = \hat{x}\frac{3}{4}\varepsilon_0\chi^{(3)}_{xxxx}\left|E_1^2\right|E_1 = \vec{P}'_{NL}$$



$$\vec{E}'_1 = \hat{x}'\frac{1}{\sqrt{2}}E_1 + \hat{y}'\frac{1}{\sqrt{2}}E_1 = \hat{x}'E'_{1,x} + \hat{y}'E'_{1,y}$$

$$P'_{NL,x'} = \frac{3}{4}\varepsilon_0(\chi^{(3)}_{xxxx}\left|E'^2_{1,x}\right|E_{1,x} + \chi^{(3)}_{xxyy}\left|E'^2_{1,y}\right|E_{1,x} + \chi^{(3)}_{xyxy}\left|E'^2_{1,y}\right|E_{1,x} + \chi^{(3)}_{xyyx}\left|E'^2_{1,y}\right|E_{1,x})$$

$$= \frac{3}{4}\varepsilon_0\left|E_1\right|^2 E_1\frac{1}{2\sqrt{2}}(\chi^{(3)}_{xxxx} + \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx})$$

$$P'_{NL,y'} = \frac{3}{4}\varepsilon_0(\chi^{(3)}_{yyyy}\left|E'^2_{1,y}\right|E_{1,y} + \chi^{(3)}_{yyxx}\left|E'^2_{1,x}\right|E_{1,y} + \chi^{(3)}_{yxyx}\left|E'^2_{1,x}\right|E_{1,y} + \chi^{(3)}_{yxxy}\left|E'^2_{1,x}\right|E_{1,y})$$

$$= \frac{3}{4}\varepsilon_0\left|E_1\right|^2 E_1\frac{1}{2\sqrt{2}}(\chi^{(3)}_{xxxx} + \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx})$$

$$\vec{P}'_{NL} = \hat{x}'P'_{NL,x'} + \hat{y}'P'_{NL,y'}$$

$$\rightarrow \quad P'_{NL} = \sqrt{2}P'_{NL,x'} = \frac{3}{4}\varepsilon_0\left|E_1\right|^2 E_1\frac{1}{2}(\chi^{(3)}_{xxxx} + \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx})$$

$$\rightarrow \boxed{\chi^{(3)}_{xxxx} = \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx}}$$

Nonlinear Birefringence (3)

Nonlinear polarization:

$$\rightarrow \mathbf{P}'_{NL}(\mathbf{r},t) = \varepsilon_0 \chi^{(3)} \vdots \mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)$$

$$\rightarrow \mathbf{P}_{NL}(\mathbf{r},t) = \frac{1}{2}(\hat{x}P_x + \hat{y}P_y)\exp(-i\omega_0 t) + c.c. \leftarrow \text{Oscillating at} \sim \omega_0$$

$$\rightarrow P_i = \frac{3\varepsilon_0}{4}\sum_j \left(\chi^{(3)}_{xxyy}E_iE_jE_j^* + \chi^{(3)}_{xyxy}E_jE_iE_j^* + \chi^{(3)}_{xyyx}E_jE_jE_i^*\right)$$

$$\leftarrow \chi^{(3)}_{ijkl} = \chi^{(3)}_{xxyy}\delta_{ij}\delta_{kl} + \chi^{(3)}_{xyxy}\delta_{ik}\delta_{jl} + \chi^{(3)}_{xyyx}\delta_{il}\delta_{jk}$$

$$\rightarrow P_x = \frac{3\varepsilon_0}{4}\chi^{(3)}_{xxxx}\left[\left(|E_x|^2 + \frac{2}{3}|E_y|^2\right)E_x + \frac{1}{3}(E_x^*E_y)E_y\right] \quad \leftarrow \chi^{(3)}_{xxyy} \approx \chi^{(3)}_{xyxy} \approx \chi^{(3)}_{xyyx}$$

"Mostly electronic"

$$\rightarrow P_y = \frac{3\varepsilon_0}{4}\chi^{(3)}_{xxxx}\left[\left(|E_y|^2 + \frac{2}{3}|E_x|^2\right)E_y + \frac{1}{3}(E_y^*E_x)E_x\right]$$

Nonlinear refractive indices:

$$\rightarrow P_j = \varepsilon_0 \varepsilon_j^{NL} E_j \quad \rightarrow \varepsilon_j = \varepsilon_j^L + \varepsilon_j^{NL} = (n_j^L + \Delta n_j)^2$$

$$\rightarrow \Delta n_x = n_2\left(|E_x|^2 + \frac{2}{3}|E_y|^2\right)$$

$$\rightarrow \Delta n_y = n_2\left(|E_y|^2 + \frac{2}{3}|E_x|^2\right)$$

Nonlinear Birefringence (4)

Coupled-mode equations:

Electric fields: $\rightarrow E_j(\mathbf{r},t) = F(x,y)A_j(z,t)\exp(i\beta_{0j}z)$

Coupled NLSE in terms of *x* and *y* pols.:

$$\rightarrow \frac{\partial A_x}{\partial z} + \beta_{1x}\frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2}A_x$$

$$= i\gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x + \frac{i\gamma}{3}A_x^* A_y^2 \exp(-2i\Delta\beta z)$$

$$\rightarrow \frac{\partial A_y}{\partial z} + \beta_{1y}\frac{\partial A_y}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2}A_y$$

$$= i\gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y + \frac{i\gamma}{3}A_y^* A_x^2 \exp(2i\Delta\beta z)$$

$$\leftarrow \Delta\beta = \beta_{0x} - \beta_{0y} = \frac{2\pi}{\lambda}B_m = \frac{2\pi}{L_B}$$

Nonlinear Birefringence (5)

Coupled-mode equations:

Coupled NLSE in terms of circularly polarized components:

$$\rightarrow A_+ = (\bar{A}_x + i\bar{A}_y)/\sqrt{2}, \quad A_- = (\bar{A}_x - i\bar{A}_y)/\sqrt{2}$$

$$\rightarrow \frac{\partial A_+}{\partial z} + \beta_1 \frac{\partial A_+}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_+}{\partial t^2} + \frac{\alpha}{2} A_+$$

$$= \frac{i}{2}(\Delta\beta)A_- + \frac{2i\gamma}{3}\left(|A_+|^2 + 2|A_-|^2\right)A_+$$

$$\rightarrow \frac{\partial A_-}{\partial z} + \beta_1 \frac{\partial A_-}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_-}{\partial t^2} + \frac{\alpha}{2} A_-$$

$$= \frac{i}{2}(\Delta\beta)A_+ + \frac{2i\gamma}{3}\left(|A_-|^2 + 2|A_+|^2\right)A_-$$

$\leftarrow \beta_{1x} \approx \beta_{1y} = \beta_1$ ← Low birefringence

Nonlinear Birefringence (6)

Elliptically birefringent fibers:

Electric field: $\rightarrow \mathbf{E}(\mathbf{r},t) = \frac{1}{2}(\hat{e}_x E_x + \hat{e}_y E_y)\exp(-i\omega_0 t) + c.c.$

$$\leftarrow \hat{e}_x = \frac{\hat{x} + ir\hat{y}}{\sqrt{1+r^2}}, \quad \hat{e}_y = \frac{r\hat{x} - i\hat{y}}{\sqrt{1+r^2}}$$ ← Orthogonal pol. eigenvectors

$$\leftarrow r = \tan(\theta/2)$$ ← θ: Ellipticity angle

Coupled NLSE:

$$\rightarrow \frac{\partial A_x}{\partial z} + \beta_{1x}\frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2}A_x = i\gamma\left[\left(|A_x|^2 + B|A_y|^2\right)A_x + CA_x^* A_y^2 e^{-2i\Delta\beta z}\right] + i\gamma D\left[A_y^* A_x^2 e^{i\Delta\beta z} + \left(|A_y|^2 + 2|A_x|^2\right)A_y e^{-i\Delta\beta z}\right]$$

$$\rightarrow \frac{\partial A_y}{\partial z} + \beta_{1y}\frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2}A_y = i\gamma\left[\left(|A_y|^2 + B|A_x|^2\right)A_y + CA_y^* A_x^2 e^{2i\Delta\beta z}\right] + i\gamma D\left[A_x^* A_y^2 e^{-i\Delta\beta z} + \left(|A_x|^2 + 2|A_y|^2\right)A_x e^{i\Delta\beta z}\right]$$

$$\leftarrow B = \frac{2 + 2\sin^2\theta}{2 + \cos^2\theta}, \quad C = \frac{\cos^2\theta}{2 + \cos^2\theta}, \quad D = \frac{\sin\theta\cos\theta}{2 + \cos^2\theta}$$

$\rightarrow L >> L_B$

$$\rightarrow \frac{\partial A_x}{\partial z} + \beta_{1x}\frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2}A_x = i\gamma\left(|A_x|^2 + B|A_y|^2\right)A_x$$

$$\rightarrow \frac{\partial A_y}{\partial z} + \beta_{1y}\frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2}A_y = i\gamma\left(|A_y|^2 + B|A_x|^2\right)A_y$$

Nonlinear Phase Shift (1)

Nondispersive XPM:

Coupled NLSE: $\rightarrow \frac{\partial A_x}{\partial z} + \frac{\alpha}{2} A_x = i\gamma\left(|A_x|^2 + B|A_y|^2\right)A_x$

$\rightarrow \frac{\partial A_y}{\partial z} + \frac{\alpha}{2} A_y = i\gamma\left(|A_y|^2 + B|A_x|^2\right)A_y$ $\leftarrow$ CW radiation

$$\leftarrow A_x = \sqrt{P_x} e^{-\alpha z/2} e^{i\phi_x}, \quad A_y = \sqrt{P_y} e^{-\alpha z/2} e^{i\phi_y}$$

$$\rightarrow \frac{\partial \phi_x}{\partial z} = \gamma e^{-\alpha z}\left(P_x + BP_y\right), \quad \frac{\partial \phi_y}{\partial z} = \gamma e^{-\alpha z}\left(P_y + BP_x\right)$$

$$\rightarrow \phi_x = \gamma(P_x + BP_y)L_{eff}, \quad \phi_y = \gamma(P_y + BP_x)L_{eff} \quad \leftarrow L_{eff} = [1 - \exp(-\alpha L)]/\alpha$$

Relative phase difference:

$$\rightarrow \Delta\phi_{NL} = \phi_x - \phi_y = \gamma L_{eff}(1-B)(P_x - P_y)$$

$$\leftarrow B = \tfrac{2}{3}, \quad P_x = P_0 \cos^2\theta, \quad P_y = P_0 \sin^2\theta \quad (\theta : \text{Pol. angle to } \hat{x})$$

$$\rightarrow \Delta\phi_{NL} = (\gamma P_0 L_{eff}/3)\cos(2\theta)$$

Nonlinear Phase Shift (2)

Optical Kerr effect:

Relative phase difference:

$$\rightarrow \Delta\phi = (2\pi/\lambda)(\tilde{n}_x - \tilde{n}_y)L$$

$$\leftarrow \tilde{n}_x = n_x + \Delta n_x, \quad \tilde{n}_y = n_y + \Delta n_y$$

By pump: $\rightarrow \Delta n_x = 2n_2 |E_p|^2 \leftarrow \mathbf{E}_p = E_p \hat{x}$

$$\rightarrow \Delta n_y = 2n_2 b |E_p|^2 \leftarrow b = \chi^{(3)}_{xxyy} / \chi^{(3)}_{xxxx} \quad \leftarrow b = \tfrac{1}{3}$$ ← Purely electronic

$$\rightarrow \Delta\phi \equiv \Delta\phi_L + \Delta\phi_{NL} = (2\pi L/\lambda)(\Delta n_L + n_{2B}|E_p|^2) \leftarrow n_{2B} = 2n_2(1-b)$$

→ Kerr shutter

→ Pulse shaping