Nonlinear Optical Engineering

Polarization Effects (2)
(NFO 5th ed: 6.3)

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Polarization of Monochromatic Plane Waves (1)

Electric field vector:

$$\mathbf{E}(z,t) = \mathrm{Re}[\mathbf{A}e^{i(kz-\omega t)}] \quad \leftarrow \quad \mathbf{A} = \hat{e}_x A_x e^{i\delta_x} + \hat{e}_y A_y e^{i\delta_y}$$

$$\rightarrow \; E_x = A_x \cos(kz - \omega t + \delta_x)$$

$$\rightarrow \; E_y = A_y \cos(kz - \omega t + \delta_y)$$

Derivation of the time-evolution locus:

$$\left\{ \begin{array}{l} \rightarrow \; E_x = A_x[\cos(kz-\omega t)\cos\delta_x - \sin(kz-\omega t)\sin\delta_x] \\ \rightarrow \; E_y = A_y[\cos(kz-\omega t)\cos\delta_y - \sin(kz-\omega t)\sin\delta_y] \end{array} \right.$$

$$\left\{ \begin{array}{l} \rightarrow \; \dfrac{E_x}{A_x}\cos\delta_y = \cos(kz-\omega t)\cos\delta_x\cos\delta_y - \sin(kz-\omega t)\sin\delta_x\cos\delta_y \\ \rightarrow \; \dfrac{E_y}{A_y}\cos\delta_x = \cos(kz-\omega t)\cos\delta_x\cos\delta_y - \sin(kz-\omega t)\cos\delta_x\sin\delta_y \end{array} \right.$$

$$\left\{ \begin{array}{l} \rightarrow \; \dfrac{E_x}{A_x}\sin\delta_y = \cos(kz-\omega t)\cos\delta_x\sin\delta_y - \sin(kz-\omega t)\sin\delta_x\sin\delta_y \\ \rightarrow \; \dfrac{E_y}{A_y}\sin\delta_x = \cos(kz-\omega t)\sin\delta_x\cos\delta_y - \sin(kz-\omega t)\sin\delta_x\sin\delta_y \end{array} \right.$$

Polarization of Monochromatic Plane Waves (2)

Continued:

$$\rightarrow \frac{E_x}{A_x}\cos\delta_y - \frac{E_y}{A_y}\cos\delta_x = \sin(kz-\omega t)[\cos\delta_x \sin\delta_y - \sin\delta_x \cos\delta_y]$$

$$\rightarrow \frac{E_x}{A_x}\sin\delta_y - \frac{E_y}{A_y}\sin\delta_x = \cos(kz-\omega t)[\cos\delta_x \sin\delta_y - \sin\delta_x \cos\delta_y]$$

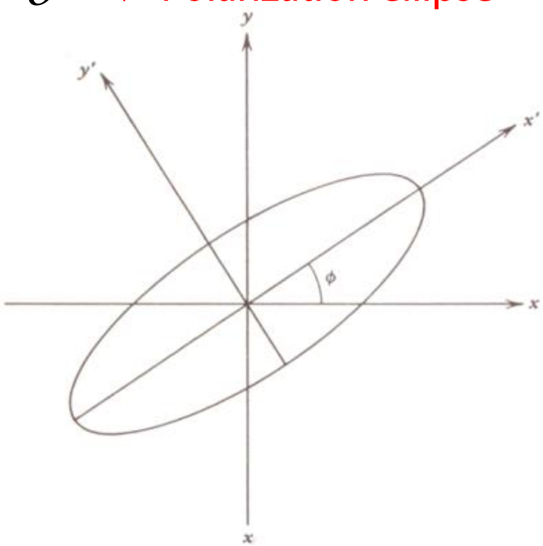
$$\sin\delta \quad \leftarrow \quad \delta = \delta_y - \delta_x$$

Time-evolution locus:

$$\rightarrow \left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - 2\frac{\cos\delta}{A_x A_y}E_x E_y = \sin^2\delta$$ ← Polarization ellipse

In the principal coordinate system:

$$\rightarrow \begin{pmatrix} E_{x'} \\ E_{y'} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

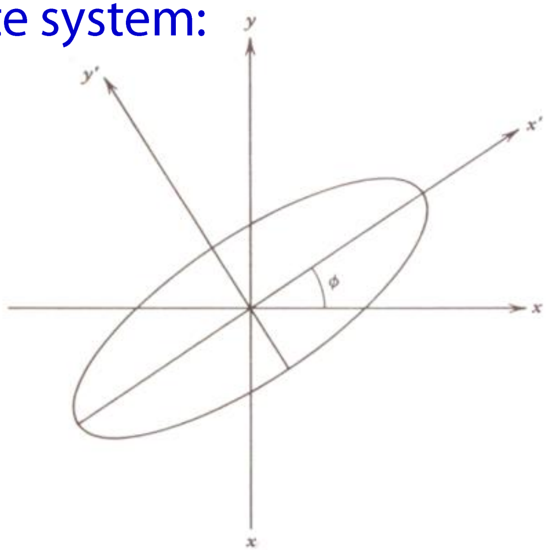


A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

Polarization of Monochromatic Plane Waves (3)

Time-evolution locus in the principal coordinate system:

$$\rightarrow \left(\frac{E_{x'}}{a}\right)^2 + \left(\frac{E_{y'}}{b}\right)^2 = 1$$



A. Yariv and P. Yeh, *Optical Waves in Crystals, 1984*

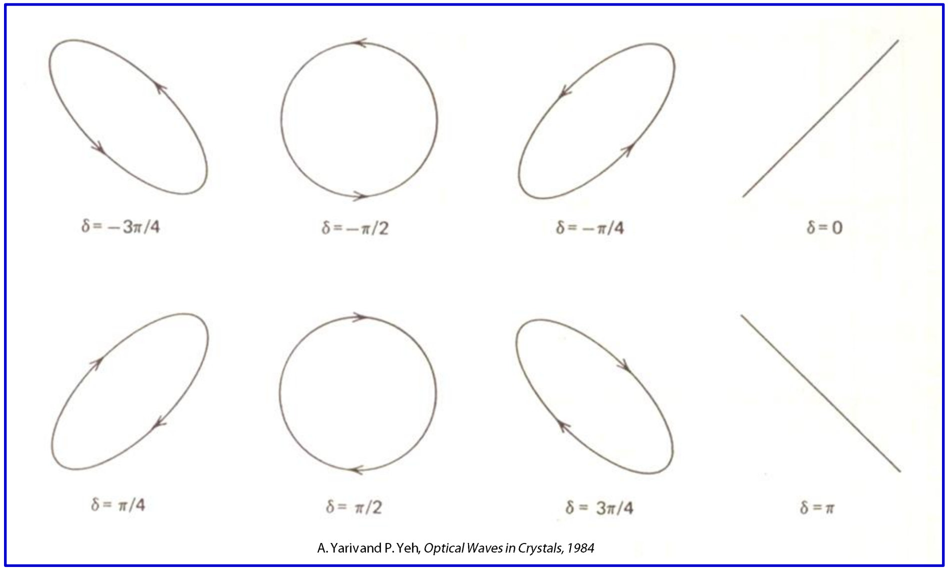
Azimuth of the polarization ellipse:

$$\rightarrow \tan 2\phi = \frac{2A_x A_y}{A_x^2 - A_y^2}\cos\delta$$

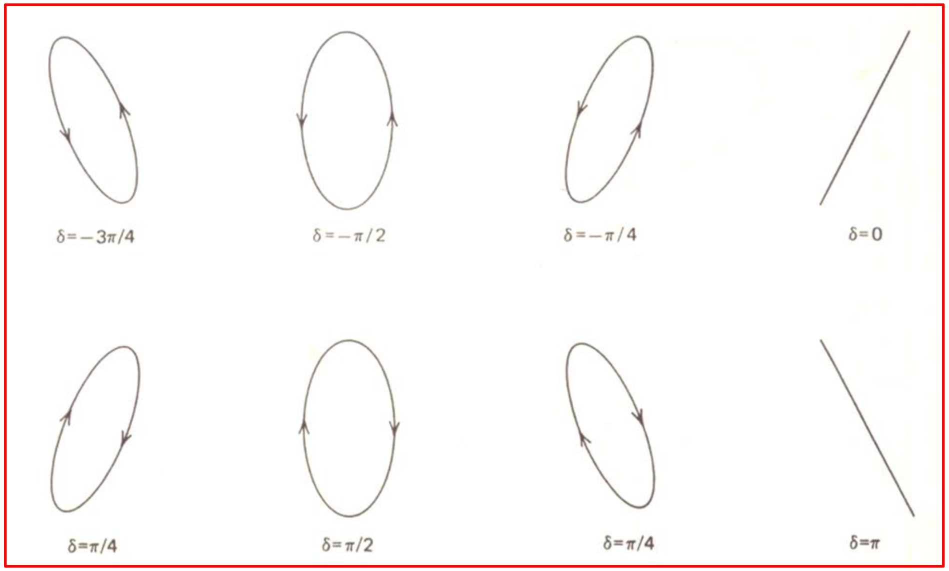
$$\leftarrow \; a^2 = A_x^2\cos^2\phi + A_y^2\sin^2\phi + 2A_x A_y\cos\delta\cos\phi\sin\phi$$

$$\leftarrow \; b^2 = A_x^2\sin^2\phi + A_y^2\cos^2\phi - 2A_x A_y\cos\delta\cos\phi\sin\phi$$

Polarization Ellipses



$$\leftarrow \quad \begin{aligned} E_x &= \cos(kz - \omega t) \\ E_y &= \cos(kz - \omega t + \delta) \end{aligned}$$



$$\leftarrow \quad \begin{aligned} E_x &= \frac{1}{2}\cos(kz - \omega t) \\ E_y &= \cos(kz - \omega t + \delta) \end{aligned}$$

Linear and Circular Polarizations

Linear polarization:

$$\delta = \delta_y - \delta_x = m\pi \quad (m = 0, 1)$$

$$\rightarrow \frac{E_y}{E_x} = (-1)^m \frac{A_y}{A_x}$$

Recall:

$$\rightarrow E_x = A_x \cos(kz - \omega t + \delta_x)$$

$$\rightarrow E_y = A_y \cos(kz - \omega t + \delta_y)$$

Circular polarization:

$$\delta = \delta_y - \delta_x = \pm \frac{1}{2}\pi \quad \& \quad A_y = A_x$$

$$\rightarrow \delta = -\frac{1}{2}\pi$$ ← Right-hand circularly polarized

$$\rightarrow \delta = +\frac{1}{2}\pi$$ ← Left-hand circularly polarized

Elliptical polarization:

$$e = \pm \frac{b}{a}$$ ← Ellipticity of a polarization ellipse

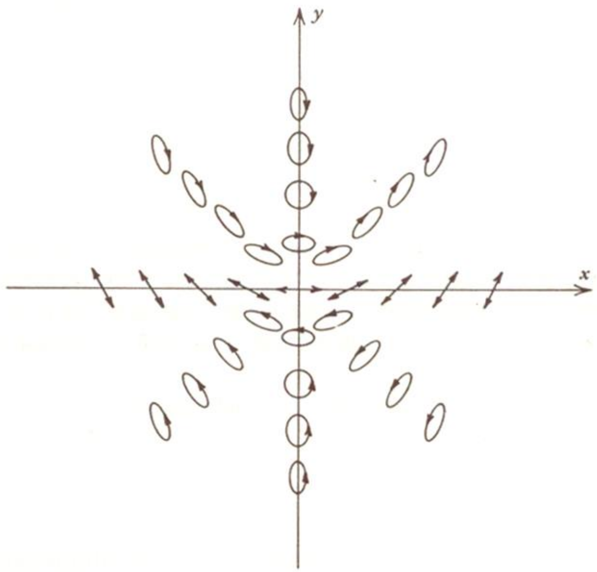
(±: Right- & left-handed rotation)

Complex-Number Representation (1)

Polarization state in the complex plane:

$$\chi = e^{i\delta}\tan\psi = \frac{A_y}{A_x}e^{i(\delta_y-\delta_x)}$$

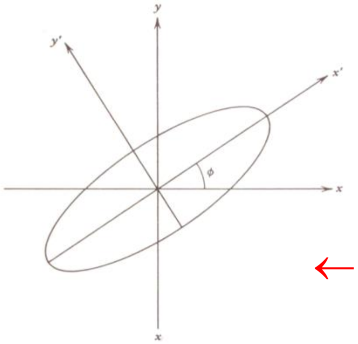
Complex-number representation →



A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

Inclination angle:

$$\tan 2\phi = \frac{2A_xA_y}{A_x^2-A_y^2}\cos\delta = \frac{2\,\mathrm{Re}[\chi]}{1-|\chi|^2}$$



← Polarization ellipse

A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

$$\leftarrow \mathrm{Re}[\chi] = \frac{A_y}{A_x}\cos\delta$$

$$\leftarrow 1-|\chi|^2 = 1-\frac{A_y^2}{A_x^2}$$

Complex-Number Representation (2)

Ellipticity angle:

$$\theta = \tan^{-1} e = \tan^{-1} \pm \frac{b}{a}$$ ← ±: Right- & left-handed rotation

$$\rightarrow \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{\pm 2\frac{b}{a}}{1+\frac{b^2}{a^2}} = \frac{\pm 2ab}{a^2+b^2}$$

$$\leftarrow a^2 = A_x^2 \cos^2\phi + A_y^2 \sin^2\phi + 2A_x A_y \cos\delta \cos\phi \sin\phi$$

$$\leftarrow b^2 = A_x^2 \sin^2\phi + A_y^2 \cos^2\phi - 2A_x A_y \cos\delta \cos\phi \sin\phi$$

$$\rightarrow a^2 + b^2 = A_x^2 + A_y^2$$

$$\rightarrow a^2 b^2 = A_x^2 A_y^2 \sin^2\delta \quad \rightarrow ab = \mp A_x A_y \sin\delta$$

$$\rightarrow \sin 2\theta = \frac{\pm 2(\mp A_x A_y \sin\delta)}{A_x^2 + A_y^2} = \frac{-2\frac{A_y}{A_x}\sin\delta}{1+\left(\frac{A_y}{A_x}\right)^2} = -\frac{2\,\mathrm{Im}[\chi]}{1+|\chi|^2}$$

Jones-Vector Representation (1)

Column vector of complex amplitudes:

$$\mathbf{J} = \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix}$$

Normalized Jones vector:

$$\rightarrow \ \mathbf{J}^* \cdot \mathbf{J} = 1$$

Linearly polarized light:

$$\begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} \perp \begin{pmatrix} -\sin\psi \\ \cos\psi \end{pmatrix}$$

$$\rightarrow \ \hat{e}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \hat{e}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Circularly polarized light:

$$\hat{e}_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \hat{e}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\rightarrow \ \hat{e}_+^* \cdot \hat{e}_- = 0$$

Jones-Vector Representation (2)

Superposition of polarizations:

$$\hat{e}_+ = \frac{1}{\sqrt{2}}\left(\hat{e}_x - i\hat{e}_y\right)$$

$$\hat{e}_- = \frac{1}{\sqrt{2}}\left(\hat{e}_x + i\hat{e}_y\right)$$

$$\hat{e}_x = \frac{1}{\sqrt{2}}\left(\hat{e}_+ + \hat{e}_-\right)$$

$$\hat{e}_y = \frac{i}{\sqrt{2}}\left(\hat{e}_+ - \hat{e}_-\right)$$

General elliptical polarization:

$$\chi = e^{i\delta}\tan\psi = \frac{A_y}{A_x}e^{i(\delta_y - \delta_x)}$$

$$\rightarrow \ \mathbf{J} = \begin{pmatrix} \cos\psi \\ e^{i\delta}\sin\psi \end{pmatrix}$$

Poincaré-Sphere Representation

Stokes parameters:

$$S_0 = |\bar{A}_x|^2 + |\bar{A}_y|^2$$

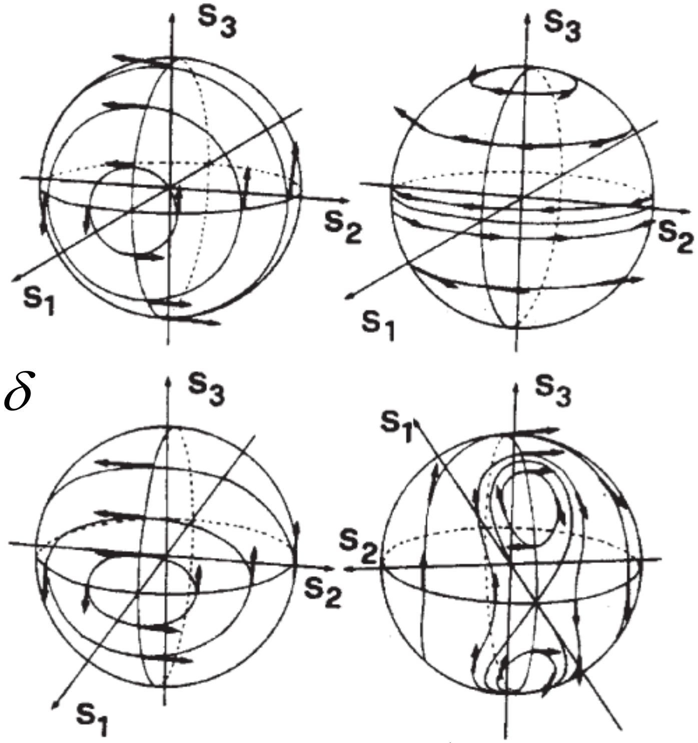
$$S_1 = |\bar{A}_x|^2 - |\bar{A}_y|^2 \quad \rightarrow S_0^2 = S_1^2 + S_2^2 + S_3^2 \quad \leftarrow \text{Poincaré-Sphere}$$

$$S_2 = 2\,\mathrm{Re}\left(\bar{A}_x^* \bar{A}_y\right)$$

$$S_3 = 2\,\mathrm{Im}\left(\bar{A}_x^* \bar{A}_y\right)$$

Recall:

$$\rightarrow \tan 2\phi = \frac{2A_x A_y}{A_x^2 - A_y^2} \cos\delta$$



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Polarization Ellipses

Recall:

$$\rightarrow \tan 2\phi = \frac{2A_x A_y}{A_x^2 - A_y^2}\cos\delta \qquad \leftarrow \bar{A}_x = \left|\bar{A}_x\right| e^{i\delta_x}, \quad \bar{A}_y = \left|\bar{A}_y\right| e^{i\delta_y}$$

$$= \frac{2\,\mathrm{Re}(\bar{A}_x^* \bar{A}_y)}{\left|\bar{A}_x\right|^2 - \left|\bar{A}_y\right|^2} \qquad \leftarrow \bar{A}_x = \frac{A_+ + A_-}{\sqrt{2}}, \quad \bar{A}_y = \frac{-i(A_+ + A_-)}{\sqrt{2}}$$

$$= -\frac{\mathrm{Im}\left(A_+^* A_-\right)}{\mathrm{Re}\left(A_+^* A_-\right)} \qquad \leftarrow A_+ = \left|A_+\right| e^{i\delta_+}, \quad A_- = \left|A_-\right| e^{i\delta_-}$$

$$= \tan\delta' \qquad \leftarrow \delta' = \delta_+ - \delta_-$$

Azimuth of the polarization ellipse:

$$\rightarrow \phi = \frac{\delta'}{2} \quad = \frac{1}{2}\arg\left(\frac{A_+}{A_-}\right)$$