Nonlinear Optical Engineering

Polarization Effects (3)
(NFO 5th ed: 6.3 ~ 6.4)

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Evolution of Polarization State (1)

Analytic solution:

Coupled NLSE in terms of circularly polarized components:

$$\rightarrow \frac{\partial A_+}{\partial z} = \frac{i}{2}(\Delta\beta)A_- + \frac{2i\gamma}{3}\left(|A_+|^2 + 2|A_-|^2\right)A_+$$

$$\rightarrow \frac{\partial A_-}{\partial z} = \frac{i}{2}(\Delta\beta)A_+ + \frac{2i\gamma}{3}\left(|A_-|^2 + 2|A_+|^2\right)A_-$$

← Quasi-CW case

$$\rightarrow \gamma = 0$$ (Low-power case)

$$\rightarrow A_+(z) = \sqrt{P_0}\cos(\pi z / L_B), \quad A_-(z) = i\sqrt{P_0}\sin(\pi z / L_B)$$

$$\leftarrow L_B = 2\pi / \Delta\beta$$

Ellipticity and azimuth of polarization ellipse:

$$\rightarrow e_p = \frac{|A_+| - |A_-|}{|A_+| + |A_-|}, \quad \theta = \frac{1}{2}\arg\left(\frac{A_+}{A_-}\right)$$

Incl. nonlinear effects:

$$\rightarrow A_\pm = \left(\frac{3\Delta\beta}{2\gamma}\right)^{1/2}\sqrt{p_\pm}\exp(i\phi_\pm) \;\leftarrow Z = (\Delta\beta)z/2, \quad \psi = \phi_+ - \phi_-$$

$$\rightarrow \frac{dp_\pm}{dZ} = \pm 2\sqrt{p_+ p_-}\sin\psi$$

$$\rightarrow \frac{d\psi}{dZ} = \frac{p_- - p_+}{\sqrt{p_+ p_-}}\cos\psi + 2(p_- - p_+)$$

Evolution of Polarization State (2)

Analytic solution:

Two invariant quantities:

$$\rightarrow p = p_+ + p_- \qquad \leftarrow p = P_0 / P_{cr}, \quad P_{cr} = 3\left|\Delta\beta / (2\gamma)\right|$$

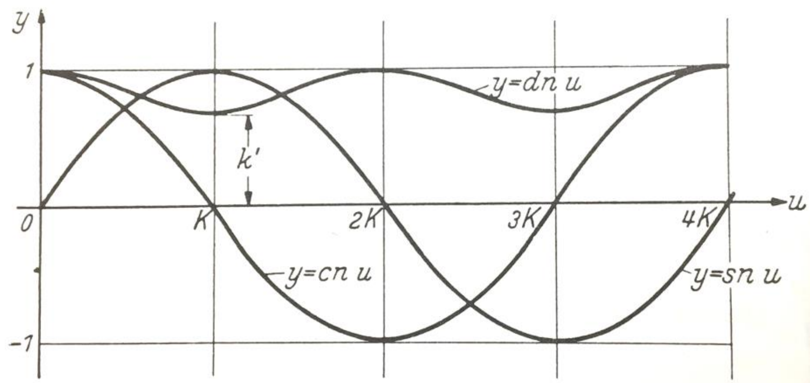
$$\rightarrow \Gamma = \sqrt{p_+ p_-}\cos\psi + p_+ p_-$$

$$\rightarrow \left(\frac{dp_+}{dZ}\right)^2 = 4p_+p_-\sin^2\psi = 4p_+p_-(1-\cos^2\psi) = 4\{p_+p_- - (\Gamma - p_+p_-)^2\}$$

$$= 4(-p_+^4 + a_3 p_+^3 + a_2 p_+^2 + a_1 p_+^2 + a_0)$$

Recall: $\left[\frac{d}{du}(\mathrm{cn}\,u)\right]^2 = (1-\mathrm{cn}^2 u)(k'^2 + k^2\mathrm{cn}^2 u) \qquad \leftarrow k' = \sqrt{1-k^2}$

"Jacobian elliptic function"



P. F. Byrd and M. D. Friedman, Handbook of Elliptic Integrals for Engineers and Scientists, 2nd ed.

Solution:

$$\leftarrow p_+(Z) = b_0 + b_1\tilde{p}_+(c_1 Z + c_0)$$

$$\rightarrow p_+(z) = \frac{1}{2}p - \sqrt{m|q|}\,cn(x,m)$$

$$\leftarrow x = \sqrt{|q|}(\Delta\beta)z + K(m) \;\leftarrow m \equiv k$$ ← K: "Quarter-period"

$$\leftarrow m = \frac{1}{2}[1 - \mathrm{Re}(q)/|q|], \quad q = 1 + p\exp(i\psi_0)$$

Evolution of Polarization State (3)

Poincaré-Sphere Representation:

Stokes parameters:

$$S_0 = |\bar{A}_x|^2 + |\bar{A}_y|^2, \quad S_1 = |\bar{A}_x|^2 - |\bar{A}_y|^2, \quad S_2 = 2\,\mathrm{Re}\left(\bar{A}_x^* \bar{A}_y\right), \quad S_3 = 2\,\mathrm{Im}\left(\bar{A}_x^* \bar{A}_y\right)$$

$$\rightarrow S_0^2 = S_1^2 + S_2^2 + S_3^2$$

Coupled NLSE in terms of Stokes parameters:

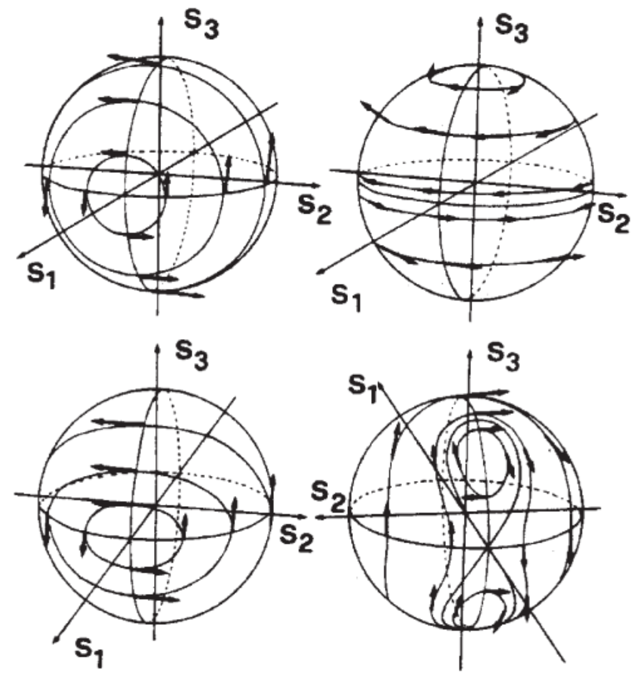
$$\rightarrow \frac{dS_0}{dz} = 0, \quad \frac{dS_1}{dz} = \frac{2\gamma}{3} S_2 S_3$$

$$\rightarrow \frac{dS_2}{dz} = -(\Delta\beta) S_3 - \frac{2\gamma}{3} S_1 S_3, \quad \frac{dS_3}{dz} = (\Delta\beta) S_2$$

$$\rightarrow \frac{d\mathbf{S}}{dz} = \mathbf{W} \times \mathbf{S} \quad \leftarrow \mathbf{W} = \mathbf{W}_L + \mathbf{W}_{NL}$$

$$\leftarrow \mathbf{W}_L = (\Delta\beta, 0, 0)$$

$$\leftarrow \mathbf{W}_{NL} = (0, 0, -2\gamma S_3 / 3)$$



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

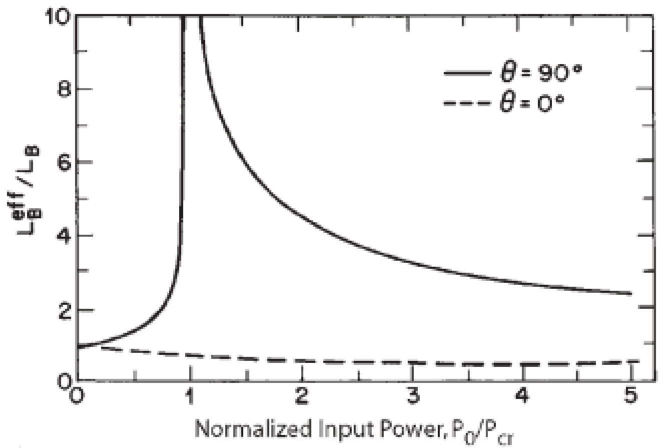
Consider:

$$(\pm S_0, 0, 0) \;\&\; (-P_{cr}, 0, \pm\sqrt{P_0^2 - P_{cr}^2})$$

Evolution of Polarization State (4)

Polarization instability:

Effective beat length: $\rightarrow L_B^{eff} = \dfrac{4K(m)}{\sqrt{|q|}(\Delta\beta)} = \dfrac{2K(m)}{\pi\sqrt{|q|}} L_B \quad \leftarrow \Delta\beta = \dfrac{2\pi}{L_B}$



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Polarization chaos:

Coupled NLSE with twisting of birefringent fibers:

$$\rightarrow \frac{\partial A_+}{\partial z} = ib_c A_+ + \frac{i\Delta\beta}{2} e^{2ir_t z} A_- + \frac{2i\gamma}{3}\left(|A_+|^2 + 2|A_-|^2\right) A_+ \qquad \leftarrow b_c = \frac{hr_t}{2\bar{n}}$$

$$\rightarrow \frac{\partial A_-}{\partial z} = -ib_c A_- + \frac{i\Delta\beta}{2} e^{-2ir_t z} A_+ + \frac{2i\gamma}{3}\left(|A_-|^2 + 2|A_+|^2\right) A_-$$

"Circular birefringence"

Vector Modulation Instability (1)

Low-birefringence fibers:

In case the input pol. aligned along the fast axis: $\rightarrow A_x = 0$

Steady-state solution:

$$\rightarrow \bar{A}_{\pm}(z) = \pm i\sqrt{P_0/2}\exp(i\gamma P_0 z)$$

With a small perturbation:

$$\rightarrow A_{\pm}(z,t) = \pm[i\sqrt{P_0/2} + a_{\pm}(z,t)]\exp(i\gamma P_0 z)$$

$$\leftarrow a_{\pm} = u_{\pm}\exp[i(Kz-\Omega t)] + v_{\pm}\exp[-i(Kz-\Omega t)]$$

Dispersion relation:

$$\rightarrow [(K-\beta_1\Omega)^2 - C_1][(K-\beta_1\Omega)^2 - C_2] = 0$$

$$\leftarrow C_1 = \tfrac{1}{2}\beta_2\Omega^2(\tfrac{1}{2}\beta_2\Omega^2 + 2\gamma P_0)$$

$$\leftarrow C_2 = (\tfrac{1}{2}\beta_2\Omega^2 + \Delta\beta - 2\gamma P_0/3)(\tfrac{1}{2}\beta_2\Omega^2 + \Delta\beta)$$

Power gain: $\rightarrow g = 2\,\mathrm{Im}(K)$

$P_0 < P_{cr}$: $\rightarrow$ MI only for anomalous dispersion

$P_0 > P_{cr}$: $\rightarrow$ MI also for normal dispersion $\leftarrow$ XPM effects

In case the input pol. aligned along the slow axis: $\rightarrow A_y = 0$

$P_0 > P_{cr}$: $\rightarrow$ MI also for normal dispersion $\leftarrow$ XPM effects

Vector Modulation Instability (2)

High-birefringence fibers:

Steady-state solution:

$$\to \bar{A}_x(z) = \sqrt{P_x}\exp[i\phi_x(z)], \quad \bar{A}_y(z) = \sqrt{P_y}\exp[i\phi_y(z)]$$

$$\leftarrow \phi_x(z) = \gamma(P_x + BP_y)z, \quad \phi_y(z) = \gamma(P_y + BP_x)z \quad \leftarrow B = \tfrac{2}{3}$$

With a small perturbation:

$$\to A_j(z,t) = (\sqrt{P_j} + a_j)\exp(i\phi_j)$$

$$\leftarrow a_j = u_j \exp[i(Kz - \Omega t)] + v_j \exp[-i(Kz - \Omega t)]$$

Dispersion relation for linear pol. of $\theta = \pi/4$: $\to P_x = P_y = P$

$$\to [(K-b)^2 - H][(K+b)^2 - H] = C_X^2 \quad \leftarrow b = (\beta_{1x} - \beta_{1y})\Omega / 2$$

$$\leftarrow H = \beta_2\Omega^2(\beta_2\Omega^2 / 4 + \gamma P)$$

$$\leftarrow C_X = B\beta_2\gamma P\Omega^2$$

$$\to K^2 = H + b^2 \pm [(H+b^2)^2 + C_X^2 - (H-b^2)^2]^{1/2} \quad \to g = 2\,\mathrm{Im}(K)$$

$\to$ "MI can occur irrespective of the sign of the GVD parameter."

$\to$ MI for normal GVD: $\to C_X > |H - b^2|$

$\to$ No MI: $\leftarrow P > P_c \quad \leftarrow P_c \approx 3(\beta_{1x} - \beta_{1y})^2 / (4\beta_2\gamma)$

$$\leftarrow \theta \approx 0 \text{ or } \pi/2$$

Vector Modulation Instability (3)

Isotropic fibers: $\rightarrow \Delta\beta = 0$

Steady-state solution:

$$\rightarrow \bar{A}_{\pm}(z) = \sqrt{P_{\pm}}\exp[i\phi_{\pm}(z)] \quad \leftarrow \phi_{\pm}(z) = \gamma'(P_{\pm} + 2P_{\mp})z, \quad \gamma' = \tfrac{2}{3}\gamma$$

With a small perturbation:

$$\rightarrow A_{\pm}(z,t) = [\sqrt{P_{\pm}} + a_{\pm}(z,t)]\exp(i\phi_{\pm})$$

$$\leftarrow a_{\pm} = u_j \exp[i(Kz - \Omega t)] + v_{\pm}\exp[-i(Kz - \Omega t)]$$

Dispersion relation:

$$\rightarrow (K^2 - H_+)(K^2 - H_-) = C_X^2 \quad \leftarrow H_{\pm} = \frac{1}{2}\beta_2\Omega^2(\frac{1}{2}\beta_2\Omega^2 + 2\gamma' P_{\pm})$$

$$\leftarrow C_X = 2\beta_2\gamma'\Omega^2\sqrt{P_+P_-}$$

$$\rightarrow K^2 = \frac{1}{2}\left[H_+ + H_- \pm \sqrt{(H_+ + H_-)^2 - 4(H_+H_- - C_X^2)}\right]$$

$\rightarrow$ MI for normal GVD: $\rightarrow C_X^2 > H_+H_-$

$\rightarrow$ No MI for a circular polarization beam