Nonlinear Optical Engineering

Polarization Effects (4)
(NFO 5th ed: 6.5 ~ 6.6)

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Birefringence and Solitons (1)

Low-birefringence fibers:

Coupled NLSE in terms of circularly polarized components:

$$\rightarrow i\frac{\partial u_+}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_+}{\partial \tau^2} + bu_- + \frac{2}{3}\left(\left|u_+\right|^2 + 2\left|u_-\right|^2\right)u_+ = 0$$

$$\rightarrow i\frac{\partial u_-}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_-}{\partial \tau^2} + bu_- + \frac{2}{3}\left(\left|u_-\right|^2 + 2\left|u_+\right|^2\right)u_- = 0$$

$$\leftarrow \beta_{1x} \approx \beta_{1y}$$

$$\leftarrow b = (\Delta\beta)L_D / 2$$

$$\leftarrow \xi = z / L_D, \quad \tau = (t - \beta_1 z) / T_0, \quad u_\pm = (\gamma L_D)^{1/2} A_\pm$$

Soliton stability:

$$\rightarrow L_{NL} = 1/\gamma P_0, \quad L_B = 2\pi / \Delta\beta$$

$$\rightarrow L_{NL} > L_B \,?$$

$$\rightarrow L_{NL} << L_B \,?$$ ← Onset of polarization instability

Birefringence and Solitons (2)

High-birefringence fibers:

Coupled NLSE in terms of linearly polarized components:

$$\rightarrow i\left(\frac{\partial u}{\partial \xi}+\delta\frac{\partial u}{\partial \tau}\right)+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+\left(|u|^2+B|v|^2\right)u=0$$

$$\rightarrow i\left(\frac{\partial v}{\partial \xi}-\delta\frac{\partial v}{\partial \tau}\right)+\frac{1}{2}\frac{\partial^2 v}{\partial \tau^2}+\left(|v|^2+B|u|^2\right)v=0$$

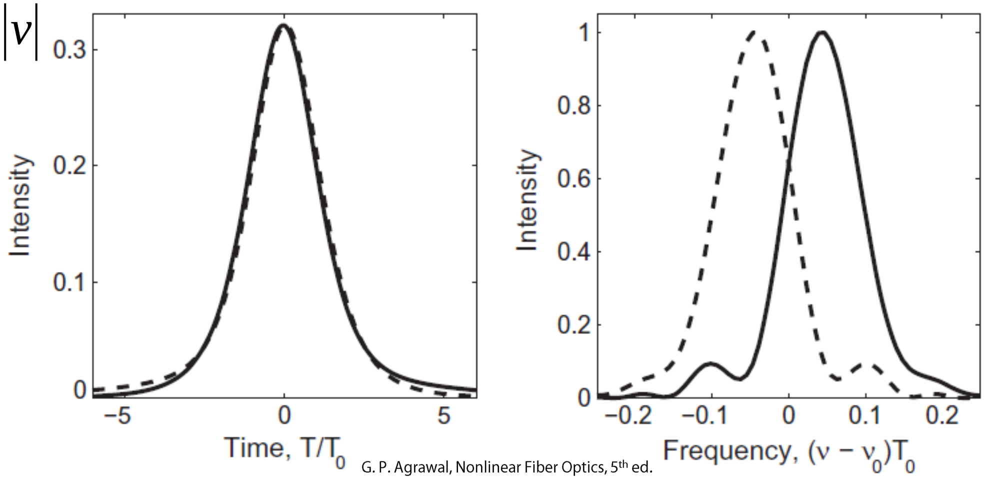
$$\leftarrow \delta=(\beta_{1x}-\beta_{1y})T_0/(2|\beta_2|)$$

$$\leftarrow \tau=(t-\bar{\beta}_1 z)/T_0,\quad \bar{\beta}_1=\tfrac{1}{2}(\beta_{1x}+\beta_{1y})$$

$$\leftarrow B=\tfrac{2}{3}$$

Numerical results: $\rightarrow u(0,\tau)=N\cos\theta\,\mathrm{sech}(\tau),\quad v(0,\tau)=N\sin\theta\,\mathrm{sech}(\tau)$ ← Input

$\rightarrow |u|=|v|$



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

← Solid: slow (u)
← Dashed: fast (v)
← $\theta=\pi/4$

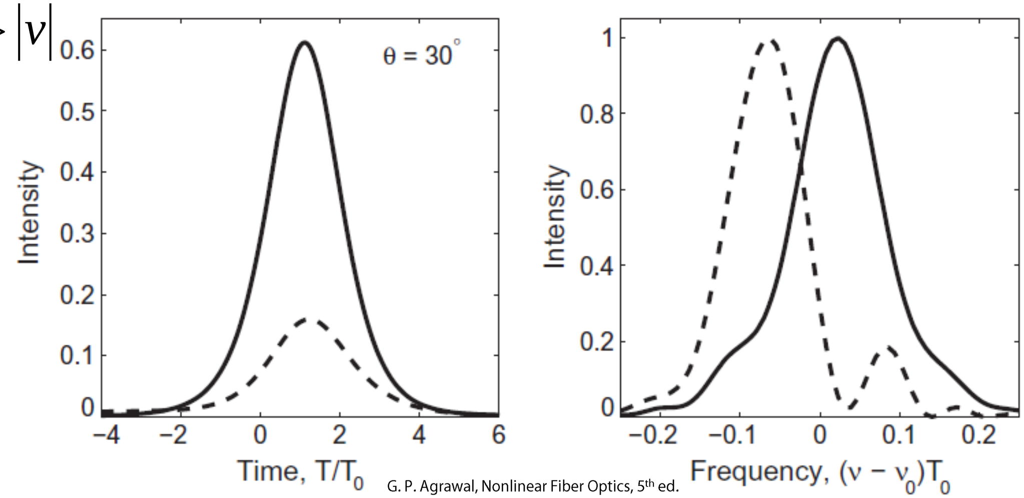
→ Temporal trapping
→ Spectral shift

Birefringence and Solitons (3)

High-birefringence fibers:

Numerical results:

$\rightarrow |u| > |v|$



← Solid: slow (u)
← Dashed: fast (v)
← $\theta = \pi/6$

→ Temporal trapping & dragging
→ Spectral shift

→ "Soliton trapping" → All-optical switching

→ "Soliton-dragging logic gates"

Birefringence and Solitons (4)

Vector solitons:

Consider high-birefringence fibers:

$$\rightarrow u = \tilde{u}\exp(i\delta^2\xi/2 - i\delta\tau), \qquad v = \tilde{v}\exp(i\delta^2\xi/2 + i\delta\tau)$$

$$\rightarrow i\frac{\partial\tilde{u}}{\partial\xi} + \frac{1}{2}\frac{\partial^2\tilde{u}}{\partial\tau^2} + \left(|\tilde{u}|^2 + B|\tilde{v}|^2\right)\tilde{u} = 0$$

$$\rightarrow i\frac{\partial\tilde{v}}{\partial\xi} + \frac{1}{2}\frac{\partial^2\tilde{v}}{\partial\tau^2} + \left(|\tilde{v}|^2 + B|\tilde{u}|^2\right)\tilde{v} = 0$$

Analytic solutions:

$$\rightarrow B = 1$$

$$\rightarrow \tilde{u}(\xi,\tau) = \cos\theta\,\mathrm{sech}(\tau)\exp(i\xi/2)$$

$$\rightarrow \tilde{v}(\xi,\tau) = \sin\theta\,\mathrm{sech}(\tau)\exp(i\xi/2)$$

$$\rightarrow \tilde{u} = \tilde{v}$$

$$\rightarrow \tilde{u} = \tilde{v} = \eta\,\mathrm{sech}[(1+B)^{1/2}\eta\tau]\exp[i(1+B)\eta^2\xi/2]$$

$$\rightarrow \eta = 1$$

Spectral shift

$$\rightarrow u(\xi,\tau) = \mathrm{sech}[(1+B)^{1/2}\tau]\exp[i(1+B+\delta^2)\xi/2 - i\delta\tau]$$

$$\rightarrow v(\xi,\tau) = \mathrm{sech}[(1+B)^{1/2}\tau]\exp[i(1+B+\delta^2)\xi/2 + i\delta\tau]$$

Random Birefringence (1)

Polarization-mode dispersion (PMD):

RMS value of the differential group delay ΔT:

$$\rightarrow \sigma_T^2(z) = 2(\Delta\beta_1)^2 l_c^2 [\exp(-z/l_c) + z/l_c - 1]$$

Correlation length

$$\rightarrow z << l_c$$

$$\rightarrow \sigma_T \approx (\Delta\beta_1) z$$

$$\rightarrow z >> l_c$$

$$\rightarrow \sigma_T \approx \Delta\beta_1 \sqrt{2 l_c z} \equiv D_p \sqrt{z}$$

Typical PMD parameter:

$$\rightarrow D_p = 0.1 \mathrm{ps}\sqrt{\mathrm{km}}$$

$\rightarrow$ Relatively small in comparison with the GVD effects $(D =\sim 10\,\mathrm{ps/km \cdot nm})$

$\rightarrow$ Still a limiting factor to long-distance/high-bit-rate light wave systems

Random Birefringence (2)

Vector form of the NLSE:

Coupled NLSE:

$$\rightarrow u = A_x\sqrt{\gamma L_D}e^{i\Delta\beta z/2}, \quad v = A_y\sqrt{\gamma L_D}e^{-i\Delta\beta z/2} \quad \rightarrow \xi = z/L_D, \quad \tau = (t-\bar{\beta}_1 z)/T_0$$

$$\rightarrow i\left(\frac{\partial u}{\partial \xi} + \delta\frac{\partial u}{\partial \tau}\right) + bu + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + \left(|u|^2 + \frac{2}{3}|v|^2\right)u + \frac{1}{3}v^2u^* = 0$$

$$\rightarrow i\left(\frac{\partial v}{\partial \xi} - \delta\frac{\partial v}{\partial \tau}\right) - bv + \frac{1}{2}\frac{\partial^2 v}{\partial \tau^2} + \left(|v|^2 + \frac{2}{3}|u|^2\right)v + \frac{1}{3}u^2v^* = 0$$

$$\leftarrow b = \frac{T_0^2\Delta\beta}{2|\beta_2|}, \quad \delta = \frac{T_0\Delta\beta_1}{2|\beta_2|}$$

Vector form:

$$\rightarrow |U\rangle = \begin{pmatrix} u \\ v \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\rightarrow s_0 = \langle U|U\rangle = |u|^2 + |v|^2, \quad s_1 = \langle U|\sigma_1|U\rangle = |u|^2 - |v|^2$$

$$\rightarrow s_2 = \langle U|\sigma_2|U\rangle = 2\,\mathrm{Re}(u^*v), \quad s_3 = \langle U|\sigma_3|U\rangle = 2\,\mathrm{Im}(u^*v)$$

$$\rightarrow i\frac{\partial|U\rangle}{\partial\xi} + \sigma_1\left(b|U\rangle + i\delta\frac{\partial|U\rangle}{\partial\tau}\right) + \frac{1}{2}\frac{\partial^2|U\rangle}{\partial\tau^2} + s_0|U\rangle - \frac{1}{3}s_3\sigma_3|U\rangle = 0$$

Random Birefringence (3)

Vector form of the NLSE:

Random rotation matrix:

$$\rightarrow R = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ -\sin\theta e^{-i\phi} & \cos\theta \end{pmatrix} \qquad \begin{aligned} &\rightarrow \overline{\Delta\beta(z)} = 0 \\ &\rightarrow \overline{\Delta\beta(z)\Delta\beta(z')} = \sigma_\beta^2 \exp(-|z-z'|/l_c) \end{aligned}$$

Vector form reduced with averaging:

$$\rightarrow i\frac{\partial|U\rangle}{\partial\xi} + \sigma_1\left(b|U\rangle + i\delta\frac{\partial|U\rangle}{\partial\tau}\right) + \frac{1}{2}\frac{\partial^2|U\rangle}{\partial\tau^2} + s_0|U\rangle - \frac{1}{3}s_3\sigma_3|U\rangle = 0$$

$$\rightarrow i\frac{\partial|U\rangle}{\partial\xi} + \frac{1}{2}\frac{\partial^2|U\rangle}{\partial\tau^2} + \frac{8}{9}s_0|U\rangle = 0$$

$$\leftarrow \overline{s_3\sigma_3}|U\rangle = \frac{1}{3}s_0|U\rangle$$

$$\because s_0|U\rangle = \sum_{j=1}^{3} s_j\sigma_j|U\rangle$$

$$\rightarrow u = \sqrt{\tfrac{9}{8}}\tilde{u}, \quad v = \sqrt{\tfrac{9}{8}}\tilde{v}$$

$$\rightarrow i\frac{\partial\tilde{u}}{\partial\xi} + \frac{1}{2}\frac{\partial^2\tilde{u}}{\partial\tau^2} + \left(|\tilde{u}|^2 + |\tilde{v}|^2\right)\tilde{u} = 0$$

$$\rightarrow i\frac{\partial\tilde{v}}{\partial\xi} + \frac{1}{2}\frac{\partial^2\tilde{v}}{\partial\tau^2} + \left(|\tilde{v}|^2 + |\tilde{u}|^2\right)\tilde{v} = 0$$

$\rightarrow$ The same form as already discussed!

Otherwise, one can solve the exact equation via numerical or perturbation methods.