Nonlinear Optical Engineering

Cross-Phase Modulation (1)
(NFO 5th ed: 7.1 ~ 7.3)

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XPM-Induced Nonlinear Coupling (1)

Nonlinear refractive index:

Electric field:

$$\rightarrow \mathbf{E}(\mathbf{r},t) = \frac{1}{2}\hat{x}[E_1 \exp(-i\omega_1 t) + E_2 \exp(-i\omega_2 t)] + c.c.$$

$$\rightarrow \mathbf{P}_{NL}(\mathbf{r},t) = \frac{1}{2}\hat{x}[P_{NL}(\omega_1)e^{-i\omega_1 t} + P_{NL}(\omega_2)e^{-i\omega_2 t}$$

$$+ P_{NL}(2\omega_1 - \omega_2)e^{-i(2\omega_1-\omega_2)t} + P_{NL}(2\omega_2 - \omega_1)e^{-i(2\omega_2-\omega_1)t}] + c.c.$$

$$\leftarrow P_{NL}(\omega_1) = \chi_{eff}(|E_1|^2 + 2|E_2|^2)E_1$$

$$\leftarrow P_{NL}(\omega_2) = \chi_{eff}(|E_2|^2 + 2|E_1|^2)E_2$$

$$\leftarrow P_{NL}(2\omega_1 - \omega_2) = \chi_{eff} E_1^2 E_2^*$$

$$\leftarrow P_{NL}(2\omega_2 - \omega_1) = \chi_{eff} E_2^2 E_1^* \qquad \leftarrow \chi_{eff} = \frac{3}{4}\varepsilon_0 \chi_{xxxx}^{(3)}$$

Nonlinear part of the refractive index:

$$\rightarrow \varepsilon_j = \varepsilon_j^L + \varepsilon_j^{NL} = (n_j^L + \Delta n_j)^2 \qquad \rightarrow \Delta n_j \approx \varepsilon_j^{NL} / 2n_j^L \approx n_2(|E_j|^2 + 2|E_{3-j}|^2)$$

Inensity-dependent nonlinear phase shift: $\leftarrow j = 1, 2$

$$\rightarrow \phi_j^{NL}(z) = (\omega_j / c)\Delta n_j z = n_2(\omega_j / c)\Delta z(|E_j|^2 + 2|E_{3-j}|^2)z$$

XPM-Induced Nonlinear Coupling (2)

Coupled NLSE:

Electric field:

$$\rightarrow E_j(\mathbf{r},t) = F_j(x,y)A_j(z,t)\exp(i\beta_{0j}z)$$

Propagation equation:

$$\rightarrow \frac{\partial A_j}{\partial z} + \beta_{1j}\frac{\partial A_j}{\partial t} + \frac{i\beta_{2j}}{2}\frac{\partial^2 A_j}{\partial t^2} + \frac{\alpha_j}{2}A_j = \frac{in_2\omega_j}{c}\left(f_{jj}\left|A_j\right|^2 + 2f_{jk}\left|A_k\right|^2\right)A_j \quad \leftarrow j \neq k$$

$$\leftarrow f_{jk} = \frac{\int\int_{-\infty}^{\infty}\left|F_j(x,y)\right|^2\left|F_k(x,y)\right|^2 dxdy}{\left(\int\int_{-\infty}^{\infty}\left|F_j(x,y)\right|^2 dxdy\right)\left(\int\int_{-\infty}^{\infty}\left|F_k(x,y)\right|^2 dxdy\right)}$$

Coupled NLSE:

$$\rightarrow \frac{\partial A_1}{\partial z} + \frac{1}{v_{g1}}\frac{\partial A_1}{\partial t} + \frac{i\beta_{21}}{2}\frac{\partial^2 A_1}{\partial t^2} + \frac{\alpha_1}{2}A_1 = i\gamma_1\left(\left|A_1\right|^2 + 2\left|A_2\right|^2\right)A_1$$

$$\rightarrow \frac{\partial A_2}{\partial z} + \frac{1}{v_{g2}}\frac{\partial A_2}{\partial t} + \frac{i\beta_{22}}{2}\frac{\partial^2 A_2}{\partial t^2} + \frac{\alpha_2}{2}A_2 = i\gamma_2\left(\left|A_2\right|^2 + 2\left|A_1\right|^2\right)A_2$$

$$\leftarrow \gamma_j = n_2\omega_j/(cA_{eff}) \qquad \leftarrow A_{eff} = 1/f_{11}$$

SMF at 1.55 μm: $\leftarrow \gamma_{1,2} \sim 1\ \mathrm{W}^{-1}/\mathrm{km}$

XPM-Induced Modulation Instability (1)

Linear stability analysis:

Steady-state solution:

$$\rightarrow \bar{A}_j(z) = \sqrt{P_j}\exp[i\phi_j(z)], \quad \phi_j(z) = \gamma_j(P_j + 2P_{3-j})z$$

With a small perturbation:

$$\rightarrow A_j = (\sqrt{P_j} + a_j)\exp(i\phi_j)$$

$$\rightarrow \frac{\partial a_1}{\partial z} + \frac{1}{v_{g1}}\frac{\partial a_1}{\partial t} + \frac{i\beta_{21}}{2}\frac{\partial^2 a_1}{\partial t^2} = i\gamma_1 P_1(a_1 + a_1^*) + 2i\gamma_1\sqrt{P_1P_2}(a_2 + a_2^*)$$

$$\rightarrow \frac{\partial a_2}{\partial z} + \frac{1}{v_{g2}}\frac{\partial a_2}{\partial t} + \frac{i\beta_{22}}{2}\frac{\partial^2 a_2}{\partial t^2} = i\gamma_2 P_2(a_2 + a_2^*) + 2i\gamma_2\sqrt{P_1P_2}(a_1 + a_1^*)$$

General solution:

$$\rightarrow a_j = u_j\exp[i(Kz - \Omega t)] + v_j\exp[-i(Kz - \Omega t)]$$

Dispersion relation:

$$\rightarrow [(K - \Omega / v_{g1})^2 - f_1][(K - \Omega / v_{g2})^2 - f_2] = C_{XPM}$$

$$\leftarrow f_j = \frac{1}{2}\beta_{2j}\Omega^2(\frac{1}{2}\beta_{2j}\Omega^2 + 2\gamma_j P_j)$$

$$\leftarrow C_{XPM} = 4\beta_{21}\beta_{22}\gamma_1\gamma_2 P_1 P_2 \Omega^4$$

XPM-Induced Modulation Instability (2)

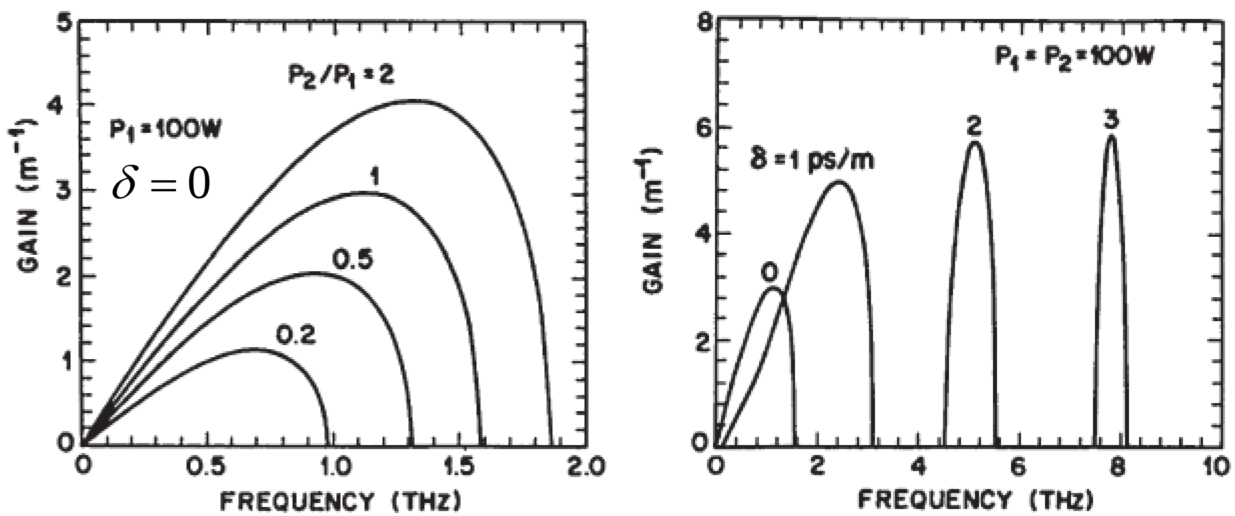
Linear stability analysis:

Dispersion relation:

$$\rightarrow K = \Omega / v_{g1} \pm \left\{ \frac{1}{2}(f_1 + f_2) \pm [(f_1 - f_2)^2 / 4 + C_{XPM}]^{1/2} \right\}^{1/2}$$

$$\rightarrow C_{XPM} > f_1 f_2$$

$$\rightarrow [\Omega^2 / \Omega_{c1}^2 + \mathrm{sgn}(\beta_{21})][\Omega^2 / \Omega_{c2}^2 + \mathrm{sgn}(\beta_{22})] < 4 \qquad \leftarrow \Omega_{cj} = (4\gamma_j P_j / |\beta_{2j}|)^{1/2}$$



$$\leftarrow \delta = |v_{g1}^{-1} - v_{g2}^{-1}|$$

G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

XPM-Paired Solitons (1)

Bright-dark soliton pair:

In case: $\rightarrow \beta_{21} < 0 \;\&\; \beta_{22} > 0, \quad \alpha_1 = \alpha_2 = 0, \quad v_{g1} = v_{g2}$

$$\rightarrow A_1(z,t) = B_1 \tanh[W(t - z/V)]\exp[i(K_1 z - \Omega_1 t)]$$

$$\rightarrow A_2(z,t) = B_2 \,\text{sech}[W(t - z/V)]\exp[i(K_2 z - \Omega_2 t)]$$

$$\leftarrow B_1^2 = (2\gamma_1\beta_{22} + \gamma_2|\beta_{21}|)W^2/(3\gamma_1\gamma_2)$$

$$\leftarrow B_2^2 = (2\gamma_2\beta_{21} + \gamma_1|\beta_{22}|)W^2/(3\gamma_1\gamma_2)$$

$$\leftarrow K_1 = \gamma_1 B_1^2 - |\beta_{21}|\Omega_1^2/2, \quad K_2 = \beta_{22}(\Omega_2^2 - W^2)/2$$

$$\leftarrow V^{-1} = v_g^{-1} - |\beta_{21}|\Omega_1 = v_g^{-1} + |\beta_{22}|\Omega_2 \qquad \leftarrow \text{sgn}(\Omega_1/\Omega_2) = -1$$

Bright-gray soliton pair:

$$\rightarrow A_j(z,t) = Q_j(t - z/V)\exp[i(K_j z - \Omega_j t + \phi_j)]$$

$$\leftarrow Q_1(\tau) = B_1[1 - b^2\text{sech}^2(W\tau)], \quad Q_2(\tau) = B_2\text{sech}(W\tau) \qquad \leftarrow \tau = t - z/V$$

$$\leftarrow W = \left(\frac{3\gamma_1\gamma_2}{2\gamma_1\beta_{22} - 4\gamma_2\beta_{21}}\right)^{1/2} B_2, \quad b = \left(\frac{2\gamma_1\beta_{22} - \gamma_2\beta_{21}}{\gamma_1\beta_{22} - 2\gamma_2\beta_{21}}\right)^{1/2} \frac{B_2}{B_1}$$

Conditions:

$\rightarrow \beta_{21} < 0 \;\&\; \beta_{22} > 0$ (o) $\qquad \rightarrow \beta_{21} > 0 \;\&\; \beta_{22} < 0$ (×)

$\rightarrow \gamma_1\beta_{22} > 2\,\gamma_2\beta_{21}$ (Normal GVD) $\qquad \rightarrow 2\gamma_1|\beta_{22}| < \gamma_2|\beta_{21}|$ (Anomalous GVD)):

XPM-Paired Solitons (2)

Periodic solitons:

Coupled NLSE: $\rightarrow \xi = z / L_D, \quad \tau = (t - z / v_{g1}) / T_0, \quad A_j = \gamma_1 L_D u_j$

$$\rightarrow i\frac{\partial u_1}{\partial \xi} - \frac{d_1}{2}\frac{\partial^2 u_1}{\partial \tau^2} + (|u_1|^2 + \sigma |u_2|^2)u_1 = 0$$

$$\rightarrow i\frac{\partial u_2}{\partial \xi} + \frac{d_2}{2}\frac{\partial^2 u_2}{\partial \tau^2} + (|u_2|^2 + \sigma |u_1|^2)u_2 = 0 \qquad \leftarrow d_j = |\beta_{2j} / \beta_{20}|$$

$\rightarrow$ Periodic solitons in terms of the elliptic functions

Multiple coupled NLSEs:

Coupled NLSE:

$$\rightarrow \frac{\partial A_j}{\partial z} + \frac{1}{v_{gj}}\frac{\partial A_j}{\partial t} + \frac{i\beta_{2j}}{2}\frac{\partial^2 A_j}{\partial t^2} = i(\gamma_j |A_j|^2 + \sigma \sum_{k \neq j} \gamma_k |A_k|^2) A_j$$

$$\rightarrow i\left(\frac{\partial u_j}{\partial \xi} + \delta_j \frac{\partial u_j}{\partial \tau}\right) + \frac{d_j}{2}\frac{\partial^2 u_j}{\partial \tau^2} + (\bar{\gamma}_j |u_j|^2 + \sigma \sum_{k \neq j} \bar{\gamma}_k |u_k|^2) u_j = 0$$

$$\leftarrow \delta_j = v_{gj}^{-1} - v_{g0}^{-1}, \quad d_j = \beta_{2j} / \beta_{20}, \quad \bar{\gamma}_j = \gamma_j / \gamma_0$$

$\rightarrow$ Multicomponent vector solitons