

Nonlinear Optical Engineering

Cross-Phase Modulation (2) (NFO 5th ed: 7.4 ~ 7.5)

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Spectral and Temporal Effects (1)

Asymmetric spectral broadening:

Coupled NLSE:

$$\begin{aligned} \rightarrow \frac{\partial A_1}{\partial z} + \frac{i\beta_{21}}{2} \frac{\partial^2 A_1}{\partial T^2} &= i\gamma_1(|A_1|^2 + 2|A_2|^2)A_1 & \leftarrow T = t - \frac{z}{v_{g1}}, \quad d = \frac{v_{g1} - v_{g2}}{v_{g1}v_{g2}} \\ \rightarrow \frac{\partial A_2}{\partial z} + d \frac{\partial A_2}{\partial T} + \frac{i\beta_{22}}{2} \frac{\partial^2 A_2}{\partial T^2} &= i\gamma_2(|A_2|^2 + 2|A_1|^2)A_2 \end{aligned}$$

Walk-off and dispersion lengths:

$$\rightarrow L_W = T_0 / |d|, \quad L_D = T_0^2 / |\beta_{21}|$$

Only ignoring the GVD effects: $\rightarrow L_W < L \ll L_D$

General solution at $z = L$:

$$\begin{aligned} \rightarrow A_1(L, T) &= A_1(0, T)e^{i\phi_1}, \quad A_2(L, T) = A_2(0, T - dL)e^{i\phi_2} \\ \leftarrow \phi_1(T) &= \gamma_1 \left(L|A_1(0, T)|^2 + 2 \int_0^L |A_2(0, T - zd)|^2 dz \right) \\ \leftarrow \phi_2(T) &= \gamma_2 \left(L|A_2(0, T)|^2 + 2 \int_0^L |A_1(0, T + zd)|^2 dz \right) \end{aligned}$$

Spectral and Temporal Effects (2)

Asymmetric spectral broadening:

Unchirped Gaussian pulses:

$$\rightarrow A_1(0, T) = \sqrt{P_1} \exp\left(-\frac{T^2}{2T_0^2}\right), \quad A_2(0, T) = \sqrt{P_2} \exp\left(-\frac{(T - T_d)^2}{2T_0^2}\right)$$

$$\rightarrow \phi_1(\tau) = \gamma_1 L \left(P_1 e^{-\tau^2} + P_2 \frac{\sqrt{\pi}}{\delta} [\operatorname{erf}(\tau - \tau_d) - \operatorname{erf}(\tau - \tau_d - \delta)] \right)$$

$$\leftarrow \tau = \frac{T}{T_0}, \quad \tau_d = \frac{T_d}{T_0}, \quad \delta = \frac{dT}{T_0}$$

XPM-induced frequency chirp:

$$\rightarrow \Delta \nu_1(\tau) = \frac{-1}{2\pi} \frac{\partial \phi_1}{\partial T} = \frac{\gamma_1 L}{\pi T_0} \left[P_1 \tau e^{-\tau^2} - \frac{P_2}{\delta} \left(e^{-(\tau - \tau_d)^2} - e^{-(\tau - \tau_d - \delta)^2} \right) \right]$$

$$\leftarrow \tau_d = 0, \quad |\delta| \ll 1 \quad (L \ll L_w)$$

$$\rightarrow \Delta \nu_1(\tau) \approx \frac{\gamma_1 L}{\pi T_0} e^{-\tau^2} [P_1 \tau + P_2 (2\tau - \delta)], \quad \Delta \nu_2(\tau) \approx \frac{\gamma_2 L}{\pi T_0} e^{-\tau^2} [P_2 \tau + P_1 (2\tau + \delta)]$$

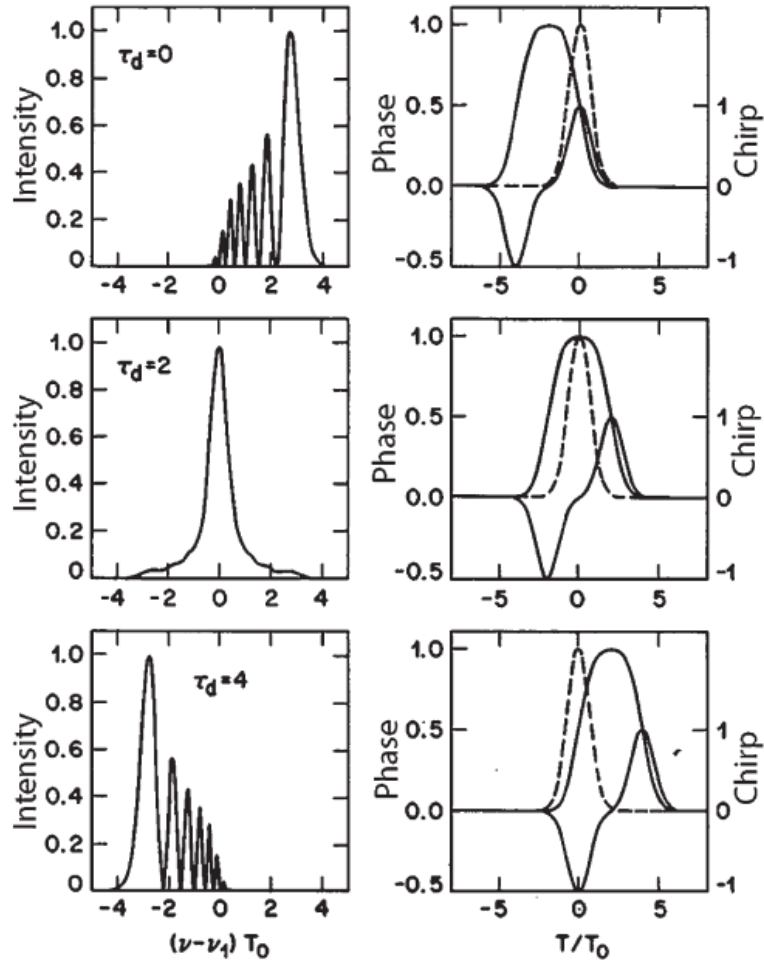
For pump and probe:

$$\rightarrow P_1 \ll P_2 \quad \rightarrow \Delta \nu_1(\tau) = \frac{\gamma_1 L}{\pi T_0} \left[\cancel{P_1 \tau} e^{-\tau^2} - \frac{P_2}{\delta} \left(e^{-(\tau - \tau_d)^2} - e^{-(\tau - \tau_d - \delta)^2} \right) \right]$$

Spectral and Temporal Effects (3)

Asymmetric spectral broadening:

Optical spectra and XPM-induced phase and chirp:



← Faster-moving pump

Spectral and Temporal Effects (4)

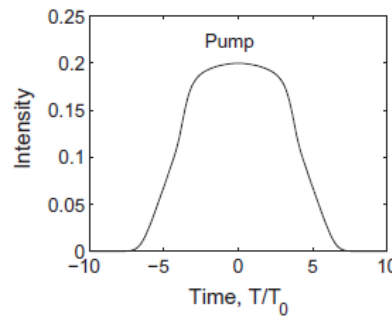
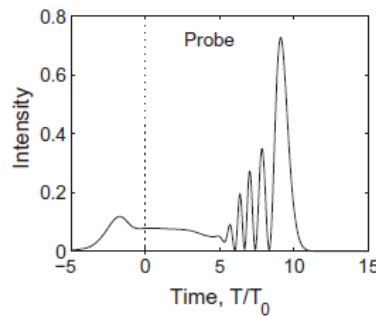
Asymmetric temporal changes:

Coupled NLSE:

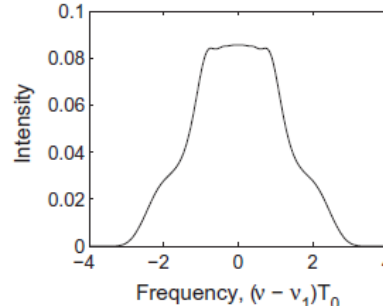
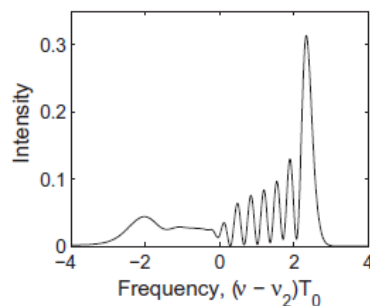
$$\rightarrow \frac{\partial U_1}{\partial \xi} + \text{sgn}(\beta_{21}) \frac{i}{2} \frac{\partial^2 U_1}{\partial \tau^2} = iN^2 (|U_1|^2 + 2|U_2|^2) U_1$$

$$\rightarrow \frac{\partial U_2}{\partial \xi} + \text{sgn}(d) \frac{L_D}{L_W} \frac{\partial U_2}{\partial \tau} + \frac{i}{2} \frac{\beta_{22}}{\beta_{21}} \frac{\partial^2 U_2}{\partial \tau^2} = iN^2 \frac{\omega_2}{\omega_1} (|U_2|^2 + 2|U_1|^2) U_2$$

$$\leftarrow \xi = \frac{z}{L_D}, \quad \tau = \frac{t - z/v_{g1}}{T_0}, \quad U_j = \frac{A_j}{\sqrt{P_1}}, \quad N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma_1 P_1 T_0^2}{|\beta_{21}|}$$



← Faster-moving pump



Spectral and Temporal Effects (5)

Higher-order nonlinear effects:

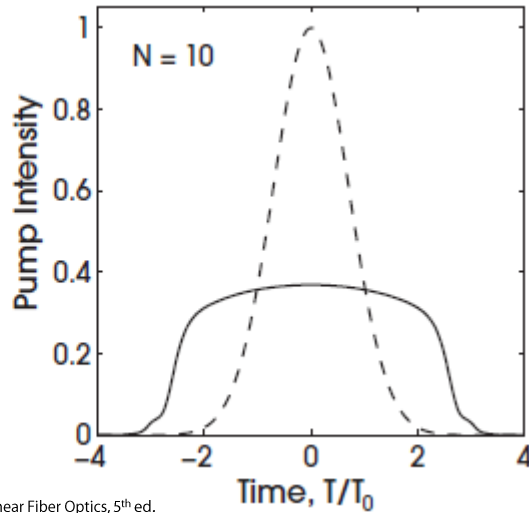
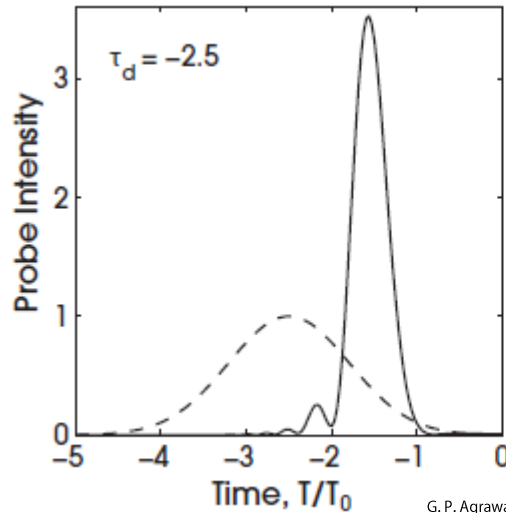
Coupled NLSE:

$$\begin{aligned} \rightarrow \frac{\partial A_j}{\partial z} + \frac{1}{v_{gj}} \frac{\partial A_j}{\partial t} + \frac{i\beta_{2j}}{2} \frac{\partial^2 A_j}{\partial t^2} + \frac{\alpha_j}{2} A_j &= i\gamma_j(1 - f_R)(|A_j|^2 + 2|A_m|^2)A_j \\ &+ i\gamma_j f_R \int_0^\infty ds h_R(s) \left\{ [|A_j(z, t-s)|^2 + |A_m(z, t-s)|^2] A_j(z, t) \right. \\ &\left. + A_j(z, t-s) A_m^*(z, t-s) \exp[i(\omega_j - \omega_m)s] A_m(z, t) \right\} \end{aligned}$$

→ Intrapulse and interpulse Raman scattering

Applications of XPM (1)

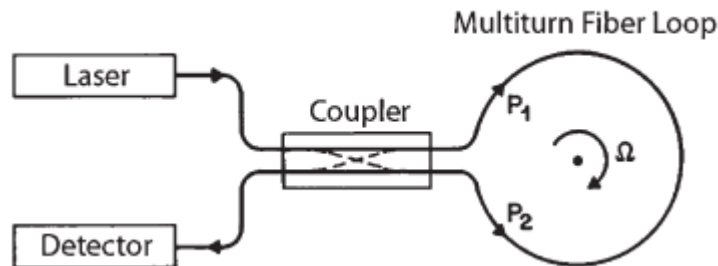
XPM-induced pulse compression:



← For normal GVD

G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

XPM-induced nonreciprocity:



$$\rightarrow \phi_j = \gamma z (P_j + 2P_{3-j})$$

$$\rightarrow \Delta\phi = \gamma L (P_2 - P_1) + S\Omega$$

← Fiber gyroscope

G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Applications of XPM (2)

XPM-induced optical switching via LPFG:

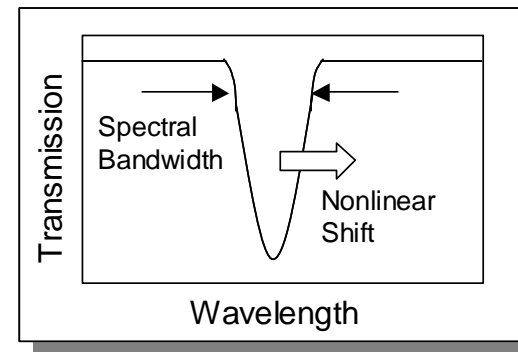
Coupled-mode theory with nonlinear perturbation:

$$\nabla \cdot (E' \times H_p^* + E_p^* \times H') = -i\omega E_p^* \cdot (\Delta\epsilon_L + \Delta\epsilon_{NL})E', \quad (p = 1, 2, \dots)$$

$$\Delta\epsilon_{NL(q)} = \epsilon_o \frac{3}{4} \chi^{(3)}(r, \phi, z) \cdot \sum_s \alpha_{(q,s)} |E_s(t, z)|^2 |\hat{e}_s|^2, \quad \alpha_{(q,s)} = \begin{cases} 1 & (q = s), \\ 2 & (q \neq s). \end{cases}$$

Nonlinear spectral shift:

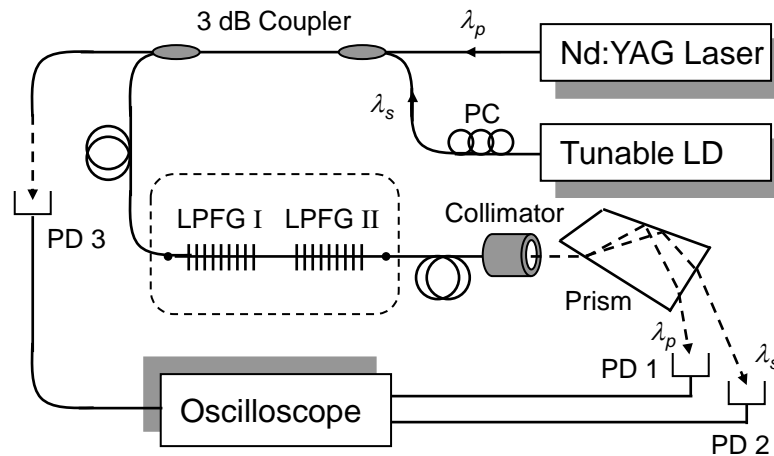
$$\rightarrow \frac{\Delta\lambda_s}{\lambda} \approx \frac{n_{2,eff} I_{eff}}{\Delta n_g}$$



Applications of XPM (3)

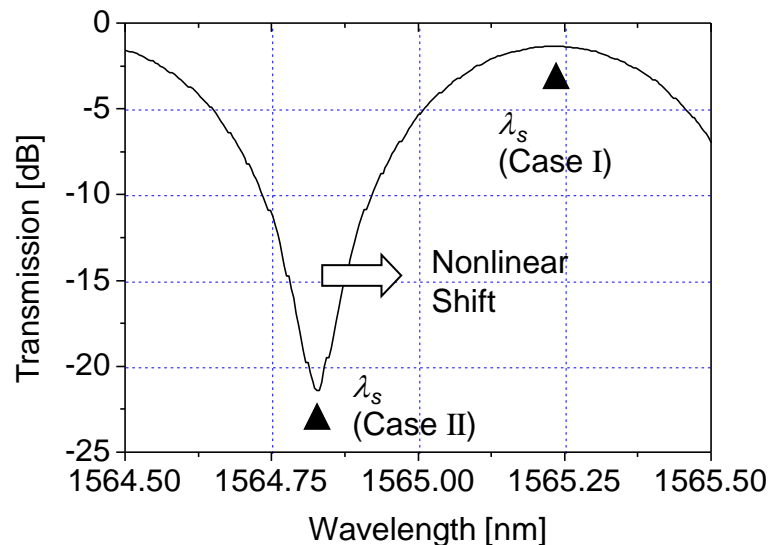
XPM-induced optical switching via LPFG:

Experimental setup: Signal wave: Tunable LD @1565.2 nm (Case I), @1564.8 nm (Case II)
Pump wave: Q-switched Nd:YAG laser (1 kHz)



Y. Jeong et al., IEEE PTL 12, 1216 (2000).

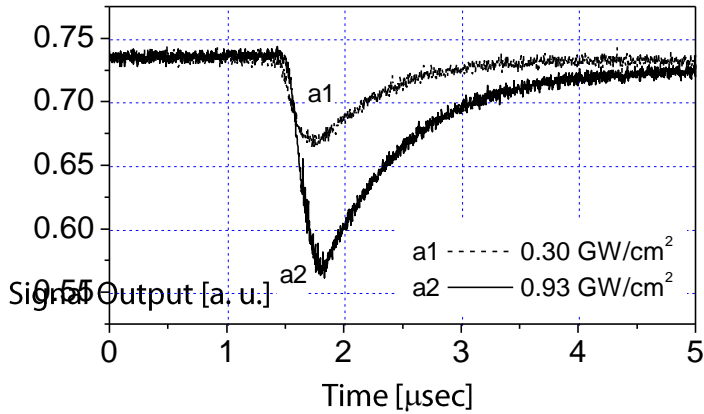
Initial transmission spectra:



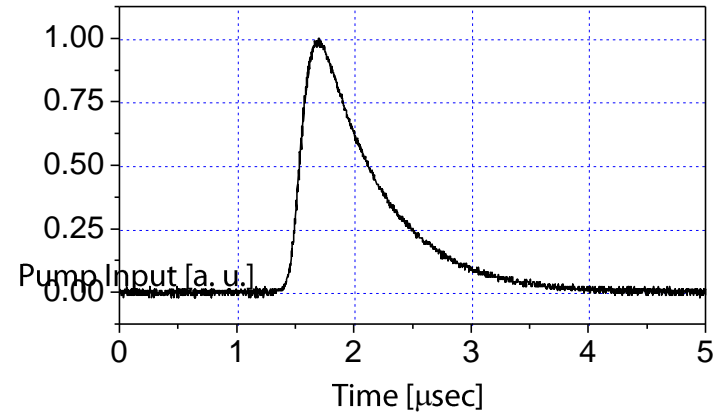
Applications of XPM (4)

XPM-induced optical switching via LPFG:

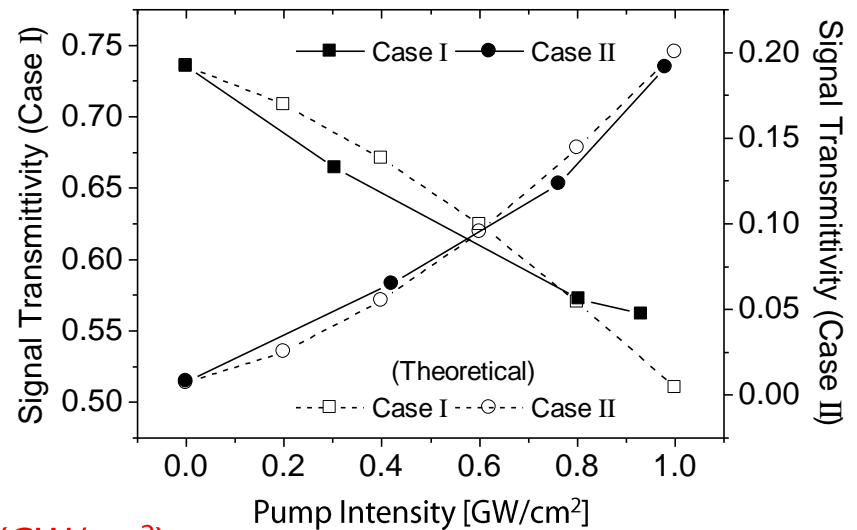
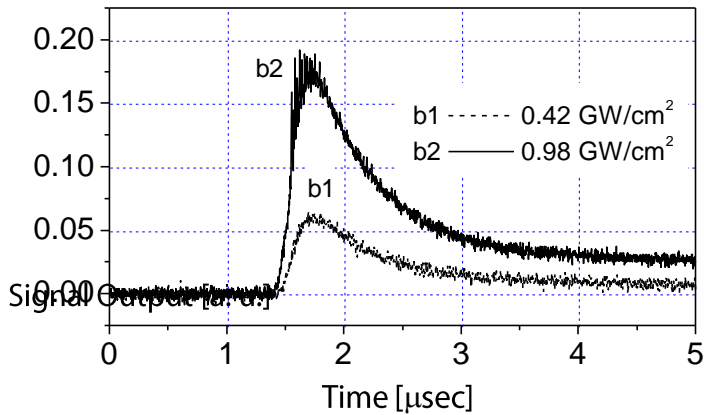
Signal (Case I):



Pump:



Signal (Case II):



$\rightarrow \Delta\lambda_s \sim 0.12 \text{ nm}/(\text{GW}/\text{cm}^2)$