Nonlinear Optical Engineering

Cross-Phase Modulation (2)
(NFO 5th ed: 7.4 ~ 7.5)

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Spectral and Temporal Effects (1)

Asymmetric spectral broadening:

Coupled NLSE:

$$\rightarrow \frac{\partial A_1}{\partial z} + \frac{i\beta_{21}}{2}\frac{\partial^2 A_1}{\partial T^2} = i\gamma_1(|A_1|^2 + 2|A_2|^2)A_1 \qquad \leftarrow T = t - \frac{z}{v_{g1}}, \quad d = \frac{v_{g1} - v_{g2}}{v_{g1}v_{g2}}$$

$$\rightarrow \frac{\partial A_2}{\partial z} + d\frac{\partial A_2}{\partial T} + \frac{i\beta_{22}}{2}\frac{\partial^2 A_2}{\partial T^2} = i\gamma_2(|A_2|^2 + 2|A_1|^2)A_2$$

Walk-off and dispersion lengths:

$$\rightarrow L_W = T_0 / |d|, \quad L_D = T_0^2 / |\beta_{21}|$$

Only ignoring the GVD effects: $\rightarrow L_W < L << L_D$

General solution at $z = L$:

$$\rightarrow A_1(L,T) = A_1(0,T)e^{i\phi_1}, \quad A_2(L,T) = A_2(0,T - dL)e^{i\phi_2}$$

$$\leftarrow \phi_1(T) = \gamma_1\left(L|A_1(0,T)|^2 + 2\int_0^L |A_2(0,T - zd)|^2\, dz\right)$$

$$\leftarrow \phi_2(T) = \gamma_2\left(L|A_2(0,T)|^2 + 2\int_0^L |A_1(0,T + zd)|^2\, dz\right)$$

Spectral and Temporal Effects (2)

Asymmetric spectral broadening:

Unchirped Gaussian pulses:

$$\rightarrow A_1(0,T) = \sqrt{P_1}\exp\left(-\frac{T^2}{2T_0^2}\right), \quad A_2(0,T) = \sqrt{P_2}\exp\left(-\frac{(T-T_d)^2}{2T_0^2}\right)$$

$$\rightarrow \phi_1(\tau) = \gamma_1 L\left(P_1 e^{-\tau^2} + P_2\frac{\sqrt{\pi}}{\delta}[\mathrm{erf}\,(\tau-\tau_d) - \mathrm{erf}\,(\tau-\tau_d-\delta)]\right)$$

$$\leftarrow \tau = \frac{T}{T_0}, \quad \tau_d = \frac{T_d}{T_0}, \quad \delta = \frac{dL}{T_0}$$

XPM-induced frequency chirp:

$$\rightarrow \Delta\nu_1(\tau) = \frac{-1}{2\pi}\frac{\partial\phi_1}{\partial T} = \frac{\gamma_1 L}{\pi T_0}\left[P_1\tau e^{-\tau^2} - \frac{P_2}{\delta}\left(e^{-(\tau-\tau_d)^2} - e^{-(\tau-\tau_d-\delta)^2}\right)\right]$$

$$\leftarrow \tau_d = 0, \quad |\delta| << 1\,(L << L_W)$$

$$\rightarrow \Delta\nu_1(\tau) \approx \frac{\gamma_1 L}{\pi T_0}e^{-\tau^2}\left[P_1\tau + P_2(2\tau-\delta)\right], \quad \Delta\nu_2(\tau) \approx \frac{\gamma_2 L}{\pi T_0}e^{-\tau^2}\left[P_2\tau + P_1(2\tau+\delta)\right]$$

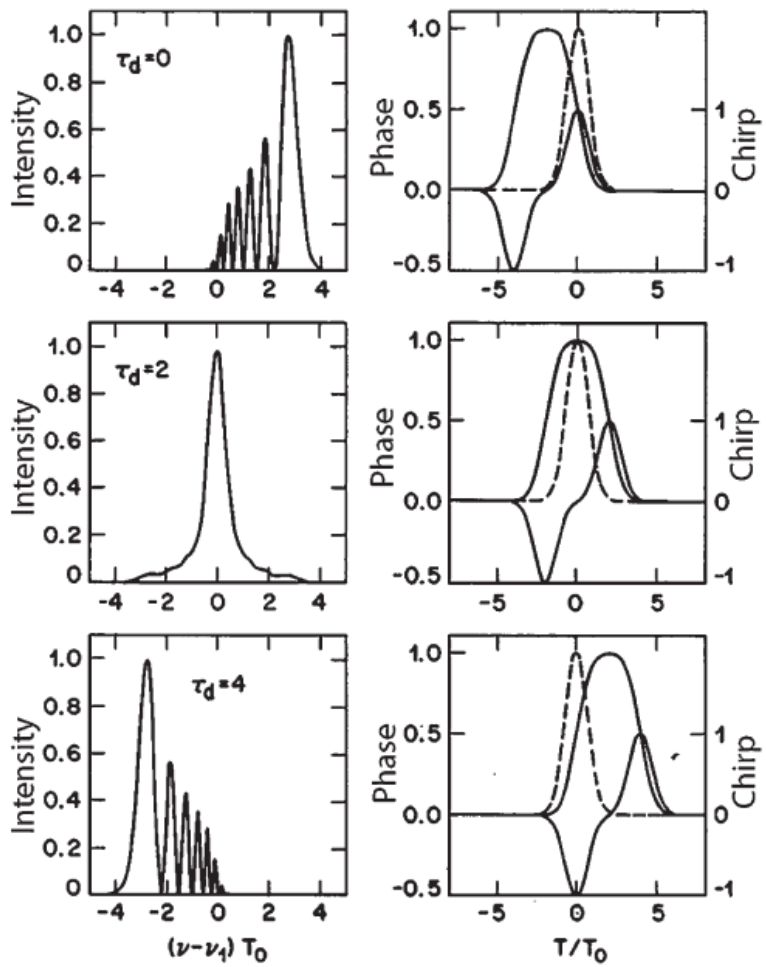
For pump and probe:

$$\rightarrow P_1 << P_2 \qquad \rightarrow \Delta\nu_1(\tau) = \frac{\gamma_1 L}{\pi T_0}\left[\cancel{P_1\tau e^{-\tau^2}} - \frac{P_2}{\delta}\left(e^{-(\tau-\tau_d)^2} - e^{-(\tau-\tau_d-\delta)^2}\right)\right]$$

Spectral and Temporal Effects (3)

Asymmetric spectral broadening:

Optical spectra and XPM-induced phase and chirp:



← Faster-moving pump

G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Spectral and Temporal Effects (4)

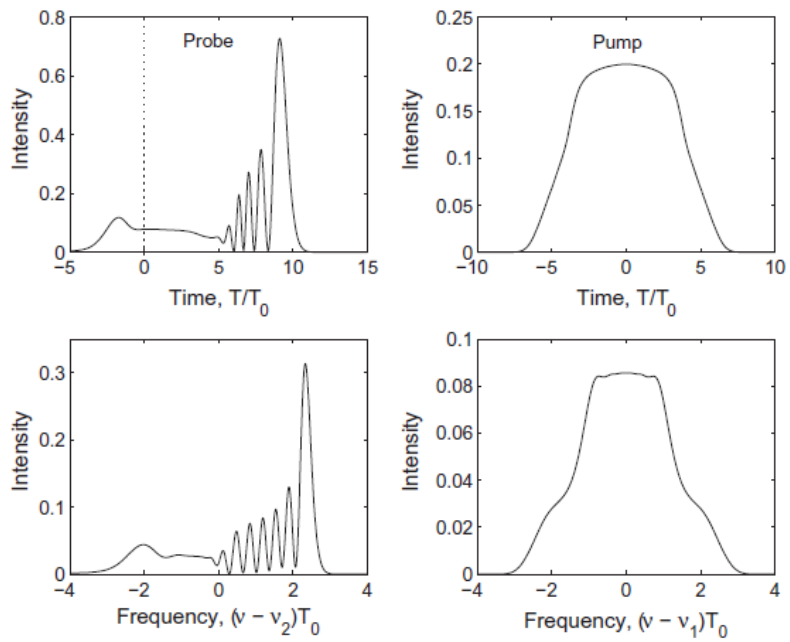
Asymmetric temporal changes:

Coupled NLSE:

$$\rightarrow \frac{\partial U_1}{\partial \xi} + \mathrm{sgn}(\beta_{21})\frac{i}{2}\frac{\partial^2 U_1}{\partial \tau^2} = iN^2(|U_1|^2 + 2|U_2|^2)U_1$$

$$\rightarrow \frac{\partial U_2}{\partial \xi} + \mathrm{sgn}(d)\frac{L_D}{L_W}\frac{\partial U_2}{\partial \tau} + \frac{i}{2}\frac{\beta_{22}}{\beta_{21}}\frac{\partial^2 U_2}{\partial \tau^2} = iN^2\frac{\omega_2}{\omega_1}(|U_2|^2 + 2|U_1|^2)U_2$$

$$\leftarrow \xi = \frac{z}{L_D}, \quad \tau = \frac{t - z/v_{g1}}{T_0}, \quad U_j = \frac{A_j}{\sqrt{P_1}}, \quad N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma_1 P_1 T_0^2}{|\beta_{21}|}$$



← Faster-moving pump

G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Spectral and Temporal Effects (5)

Higher-order nonlinear effects:

Coupled NLSE:

$$\rightarrow \frac{\partial A_j}{\partial z} + \frac{1}{v_{gj}}\frac{\partial A_j}{\partial t} + \frac{i\beta_{2j}}{2}\frac{\partial^2 A_j}{\partial t^2} + \frac{\alpha_j}{2}A_j = i\gamma_j(1-f_R)(|A_j|^2 + 2|A_m|^2)A_j$$

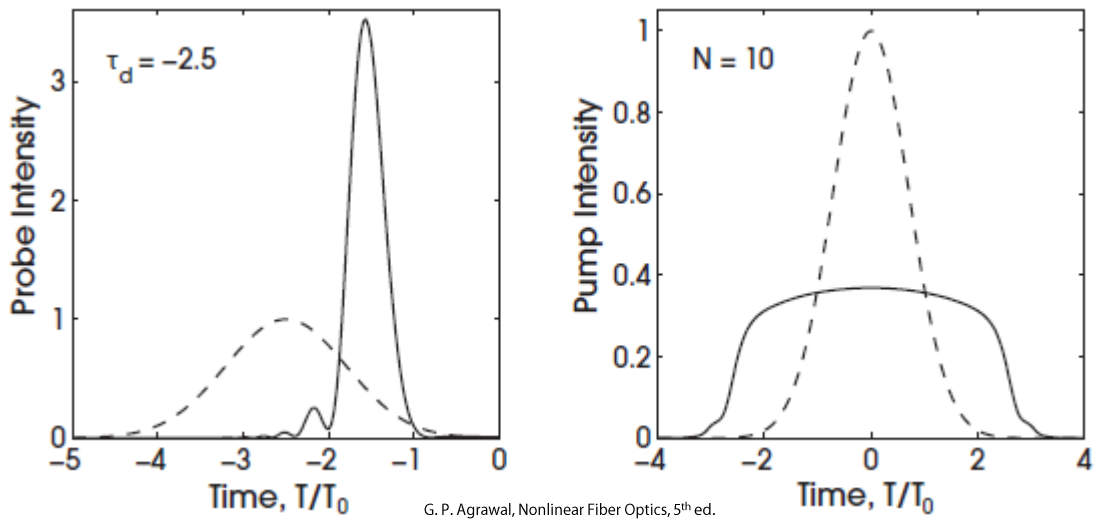
$$+ i\gamma_j f_R \int_0^\infty ds h_R(s)\Big\{[|A_j(z,t-s)|^2 + |A_m(z,t-s)|^2]A_j(z,t)$$

$$+ A_j(z,t-s)A_m^*(z,t-s)\exp[i(\omega_j-\omega_m)s]A_m(z,t)\Big\}$$

$\rightarrow$ Intrapulse and interpulse Raman scattering

Applications of XPM (1)

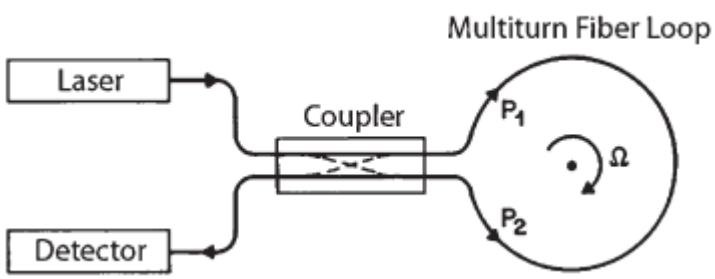
XPM-induced pulse compression:



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

← For normal GVD

XPM-induced nonreciprocity:



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

$$\rightarrow \phi_j = \gamma z(P_j + 2P_{3-j})$$

$$\rightarrow \Delta\phi = \gamma L(P_2 - P_1) + S\Omega$$

← Fiber gyroscope