

# Nonlinear Optical Engineering

## Stimulated Raman Scattering (1) (NFO 5<sup>th</sup> ed: 8.1)

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# Basic Concepts (1)

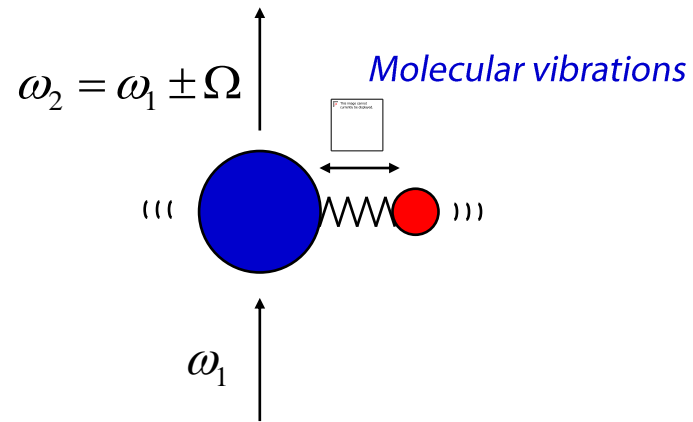
## Discovery of Raman scattering:

→ C. V. Raman in 1928

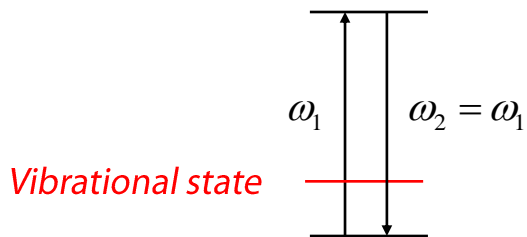
*Inelastic scattering ↔ molecular vibrations*



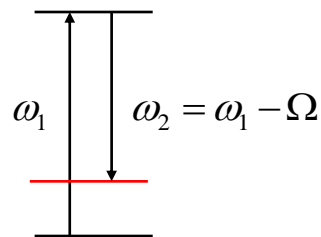
C. V. Raman  
(Chandrasekara Venkata Raman, 1888-1970)



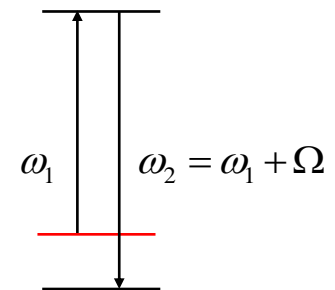
*Does every kind of molecule tend to vibrate?*



*Elastic scattering:  
Rayleigh scattering*



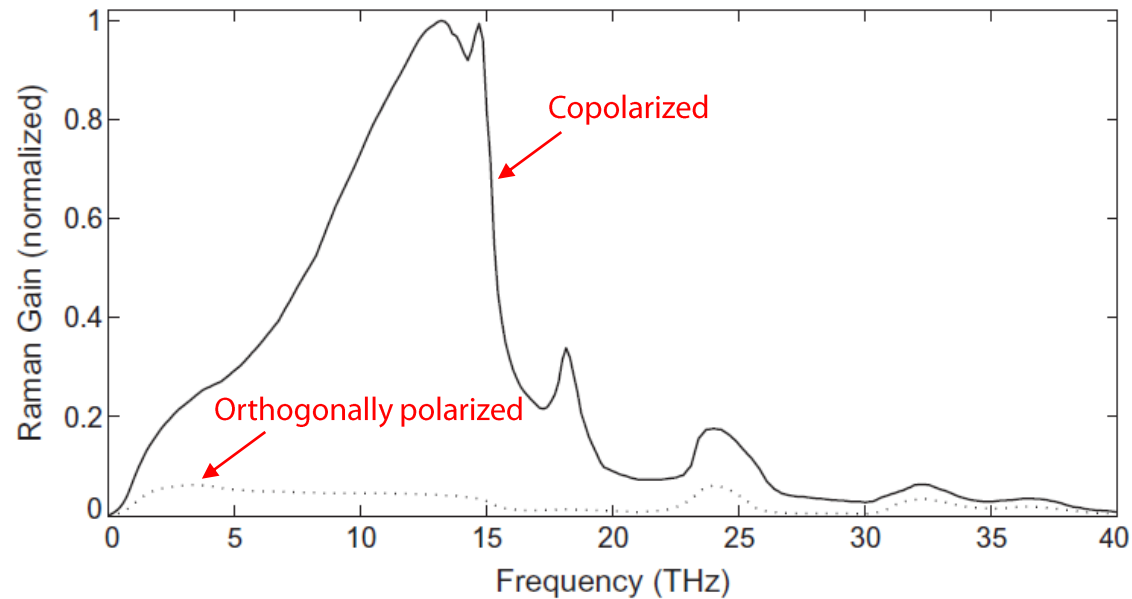
*Inelastic scattering:  
Stokes scattering*



*Inelastic scattering:  
Anti-Stokes scattering*

# Basic Concepts (2)

Raman-gain spectrum:



G. P. Agrawal, Nonlinear Fiber Optics, 5<sup>th</sup> ed.

Initial growth of the Stokes wave:

$$\rightarrow \frac{dI_s}{dz} = g_R I_p I_s \quad \leftarrow g_R \sim 10^{-13} \text{ m/W}, \quad \Omega_R \sim 13.2 \text{ THz} \quad \leftarrow \text{For silica}$$

# Basic Concepts (3)

## Raman threshold:

Coupled equations in the CW case:

$$\begin{aligned} \rightarrow \frac{dI_s}{dz} &= g_R I_p I_s - \alpha_s I_s \\ \rightarrow \frac{dI_p}{dz} &= -\frac{\omega_p}{\omega_s} g_R I_p I_s - \alpha_p I_p \end{aligned}$$

In the absence of losses:

$$\rightarrow \frac{d}{dz} \left( \frac{I_s}{\omega_s} + \frac{I_p}{\omega_p} \right) = 0$$

For non-depleted pump:

$$\begin{aligned} \rightarrow \frac{dI_s}{dz} &= g_R I_0 \exp(-\alpha_p z) I_s - \alpha_s I_s \\ \rightarrow I_s(L) &= I_s(0) \exp(g_R I_0 L_{eff} - \alpha_s L) \quad \leftarrow L_{eff} = [1 - \exp(-\alpha_p L)] / \alpha_p \end{aligned}$$

Raman-gain spectrum:

$$\rightarrow P_s(L) = \int_{-\infty}^{\infty} h\nu \exp[g_R (\nu_p - \nu) I_0 L_{eff} - \alpha_s L] d\nu$$

← Method of steepest descent, assuming a Lorentzian gain profile

$$\rightarrow P_s(L) \approx P_{s0}^{eff} \exp[g_R (\nu_R) I_0 L_{eff} - \alpha_s L] \quad \leftarrow \nu_R = \nu_p - \nu_s$$

$$\leftarrow P_{s0}^{eff} = h\nu_s B_{eff}, \quad B_{eff} = \frac{\sqrt{\pi}}{2} \frac{\Delta\nu_{FWHM}}{[g_R(\nu_R) I_0 L_{eff}]^{1/2}} = \left( \frac{2\pi}{I_0 L_{eff}} \right)^{1/2} \left| \frac{\partial^2 g_R}{\partial \nu^2} \right|_{\nu=\nu_R}^{-1/2}$$

# Basic Concepts (4)

## Raman threshold:

Critical pump power:

$$\rightarrow P_s(L) = P_p(L) \equiv P_0 \exp(-\alpha_p L)$$

$$\rightarrow P_{s0}^{eff} \exp[g_R(\nu_R) I_0 L_{eff} - \alpha_s L] = P_0 \exp(-\alpha_p L) \quad \leftarrow I_0 = P_0 / A_{eff}, \quad \alpha_s \approx \alpha_p$$

$$\rightarrow P_{s0}^{eff} \exp[g_R(\nu_R) P_0 L_{eff} / A_{eff}] = P_0$$

$$\rightarrow \exp[g_R(\nu_R) P_0 L_{eff} / A_{eff}] = P_0 / P_{s0}^{eff}$$

$$\rightarrow \boxed{\frac{g_R P_0^{cr} L_{eff}}{A_{eff}} \approx 16} \quad \leftarrow \frac{P_0}{P_{s0}^{eff}} \approx 70 \text{ dB}$$

# Basic Concepts (5)

Coupled amplitude equations:

Optical field:

$$\rightarrow \mathbf{E}(\mathbf{r}, t) = \frac{1}{2} \hat{x} \left\{ A_p \exp[i(\beta_{0p}z - \omega_p t)] + A_s \exp[i(\beta_{0s}z - \omega_s t)] \right\} + c.c.$$

Coupled NLSE:

$$\rightarrow \frac{\partial A_p}{\partial z} + \frac{1}{v_{gp}} \frac{\partial A_p}{\partial t} + \frac{i\beta_{2p}}{2} \frac{\partial^2 A_p}{\partial t^2} + \frac{\alpha_p}{2} A_p = i\gamma_p (1 - f_R) (|A_p|^2 + 2|A_s|^2) A_p + R_p(z, t)$$

$$\rightarrow \frac{\partial A_s}{\partial z} + \frac{1}{v_{gs}} \frac{\partial A_s}{\partial t} + \frac{i\beta_{2s}}{2} \frac{\partial^2 A_s}{\partial t^2} + \frac{\alpha_s}{2} A_s = i\gamma_s (1 - f_R) (|A_s|^2 + 2|A_p|^2) A_s + R_s(z, t)$$

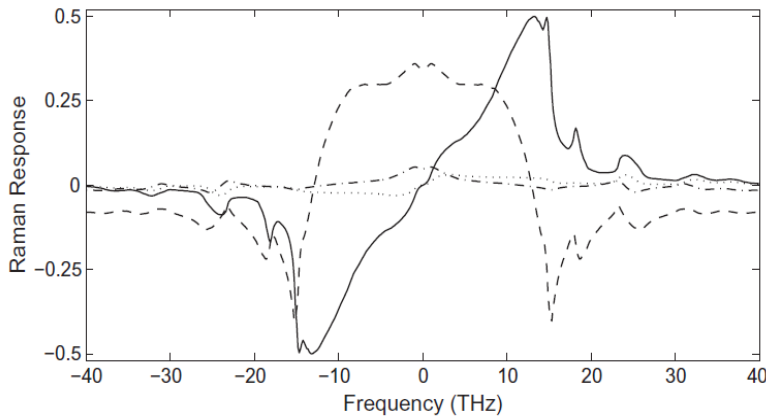
$$\leftarrow R_j(z, t) = i\gamma_j f_R A_j \int_{-\infty}^t h_R(t-t') \left[ |A_j(z, t')|^2 + |A_k(z, t')|^2 \right] dt'$$

$$+ i\gamma_j f_R A_k \int_{-\infty}^t h_R(t-t') A_j(z, t') A_k^*(z, t') \exp[\pm i\Omega(t-t')] dt'$$

$$\leftarrow j, k = p \text{ or } s, \quad j \neq k, \quad \Omega_R = \omega_p - \omega_s$$

$$\rightarrow R_j = i\gamma_j f_R \left[ (|A_j|^2 + |A_k|^2) A_j + \tilde{h}_R(\pm\Omega_R) |A_k|^2 A_j \right]$$

$$\rightarrow \delta_R = f_R \text{Re}[\tilde{h}_R(\Omega)], \quad g_j = 2\gamma_j f_R \text{Im}[\tilde{h}_R(\Omega)]$$



# Basic Concepts (6)

Coupled amplitude equations:

Coupled NLSE:

$$\begin{aligned} \rightarrow \frac{\partial A_p}{\partial z} + \frac{1}{v_{gp}} \frac{\partial A_p}{\partial t} + \frac{i\beta_{2p}}{2} \frac{\partial^2 A_p}{\partial t^2} + \frac{\alpha_p}{2} A_p \\ = i\gamma_p \left[ |A_p|^2 + (2 + \delta_R - f_R) |A_s|^2 \right] A_p - \frac{g_p}{2} |A_s|^2 A_p \\ \rightarrow \frac{\partial A_s}{\partial z} + \frac{1}{v_{gs}} \frac{\partial A_s}{\partial t} + \frac{i\beta_{2s}}{2} \frac{\partial^2 A_s}{\partial t^2} + \frac{\alpha_s}{2} A_s \\ = i\gamma_s \left[ |A_s|^2 + (2 + \delta_R - f_R) |A_p|^2 \right] A_s + \frac{g_s}{2} |A_p|^2 A_s \end{aligned}$$

For silica fibers:  $\leftarrow f_R \approx 0.18$

Walk-off length:

$$\rightarrow L_W = T_0 / \left| v_{gp}^{-1} - v_{gs}^{-1} \right|$$