Nonlinear Optical Engineering

Stimulated Brillouin Scattering (2)
(NFO 5th ed: 9.4 ~ 9.5)

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SBS Dynamics (1)

Coupled amplitude equations:

Local density change by electrostriction:

$$\rightarrow \frac{\partial^2 \rho'}{\partial t^2} - \Gamma_A \nabla^2 \frac{\partial \rho'}{\partial t} - v_A^2 \nabla^2 \rho' = -\varepsilon_0 \gamma_e \nabla^2 (\mathbf{E}\cdot\mathbf{E}) \;\leftarrow \rho' = \rho - \rho_0$$ ← Local density change

$$\leftarrow \gamma_e = \rho_0 (\partial \varepsilon / \partial \rho)_{\rho=\rho_0}$$ ← Electrostrictive constant

Nonlinear polarization:

$$\rightarrow \mathbf{P}_{NL} = \varepsilon_0 [\chi^{(3)} \vdots \mathbf{EEE} + (\gamma_e / \rho_0) \rho' \mathbf{E}] \quad \leftarrow \Delta\varepsilon = (\partial \varepsilon / \partial \rho)_{\rho=\rho_0} \rho'$$

$$\rightarrow \mathbf{E}(\mathbf{r},t) = \tfrac{1}{2}\{F_p(x,y) A_p(z,t) \exp[i(k_p z - \omega_p t)]$$

$$+ F_s(x,y) A_s(z,t) \exp[-i(k_p z + \omega_p t)] + c.c.\}$$

$$\rightarrow \rho'(\mathbf{r},t) = \tfrac{1}{2}\{F_A(x,y) Q(z,t) \exp[i(k_A z - \Omega t)] + c.c.\} \quad \leftarrow \Omega = \omega_p - \omega_s$$

Coupled amplitude equations:

$$\rightarrow \frac{\partial A_p}{\partial z} + \frac{1}{v_g}\frac{\partial A_p}{\partial t} = -\frac{\alpha}{2} A_p + i\gamma(|A_p|^2 + 2|A_s|^2) A_p + i\kappa_1 A_s Q$$

$$\rightarrow -\frac{\partial A_s}{\partial z} + \frac{1}{v_g}\frac{\partial A_s}{\partial t} = -\frac{\alpha}{2} A_p + i\gamma(|A_s|^2 + 2|A_p|^2) A_s + i\kappa_1 A_p Q^*$$

$$\rightarrow \frac{\partial Q}{\partial t} + v_A \frac{\partial Q}{\partial z} = -\left[\frac{1}{2}\Gamma_B + i(\Omega - \Omega_B)\right] Q + i\kappa_2 A_p A_s^*$$ ← Acoustic mode

$$\leftarrow \Gamma_B = k_A^2 \Gamma_A \quad \leftarrow \kappa_1 = \frac{\omega_p \gamma_e \langle F_p^2 F_A \rangle}{2 n_p c \rho_0 \langle F_p^2 \rangle}, \quad \kappa_2 = \frac{\omega_p \gamma_e \langle F_p^2 F_A \rangle}{2 c^2 v_A \langle F_p^2 \rangle A_{eff}} 2$$

SBS Dynamics (2)

Coupled amplitude equations:

For broad pump pulse widths: $\rightarrow T_0 >> T_B = \Gamma_B^{-1}$

$$\rightarrow \frac{\partial P_p}{\partial z} + \frac{1}{v_g}\frac{\partial P_p}{\partial t} = -\frac{g_B(\Omega)}{A_{eff}} P_p P_s - \alpha P_p$$

$$\rightarrow -\frac{\partial P_s}{\partial z} + \frac{1}{v_g}\frac{\partial P_s}{\partial t} = \frac{g_B(\Omega)}{A_{eff}} P_p P_s - \alpha P_s \qquad \leftarrow g_B(\Omega_B) = \frac{4\pi^2 \gamma_e^2 f_A}{n_p c \lambda_p^2 \rho_0 v_A \Gamma_B}$$

Acoustic overlap factor: $\leftarrow f_A = \frac{\left\langle F_p^2 F_A \right\rangle^2}{\left\langle F_p^2 \right\rangle \left\langle F_A^2 \right\rangle} = \frac{\iint F_p^2(x,y) F_A(x,y)\,dxdy}{\left(\iint F_p^2(x,y)\,dxdy\right)\left(\iint F_A^2(x,y)\,dxdy\right)}$

SBS Dynamics (3)

SBS with pulses:

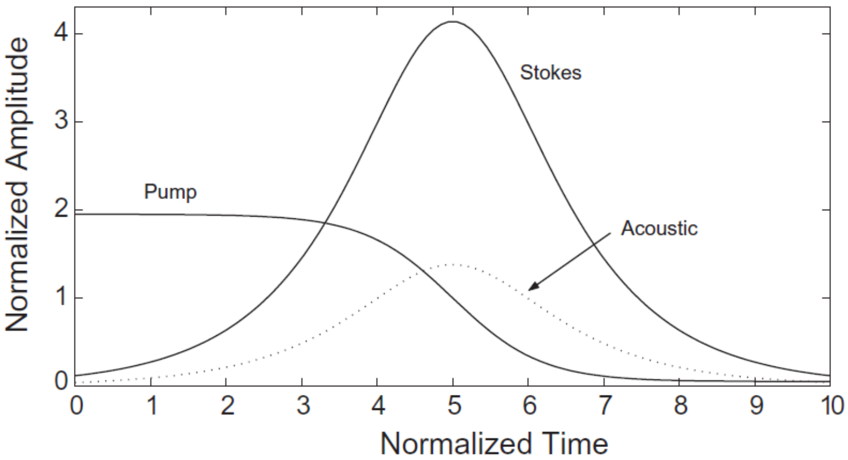
Solitary-wave solutions:

$$\rightarrow Z = z/v_g, \quad B_a = -iv_g\kappa_1 Q, \quad B_j = (\kappa_1\kappa_2 v_g)^{1/2} A_j \quad \leftarrow j = p, s$$

$$\rightarrow \frac{\partial B_p}{\partial t} + \frac{\partial B_p}{\partial Z} + \frac{\alpha v_g}{2} B_p = -B_s B_a \qquad \rightarrow B_p(Z,t) = C_p\{1 - b\tanh[p(Z+Vt)]\}$$

$$\rightarrow \frac{\partial B_s}{\partial t} - \frac{\partial B_s}{\partial Z} + \frac{\alpha v_g}{2} B_s = B_p B_a^* \qquad \rightarrow B_s(Z,t) = C_s \operatorname{sech}[p(Z+Vt)]$$

$$\rightarrow \frac{\partial B_a}{\partial t} + \frac{\Gamma_B}{2} B_a = B_p B_s^* \qquad \rightarrow B_a(Z,t) = C_a \operatorname{sech}[p(Z+Vt)]$$



$$\leftarrow C_p = \frac{1}{2}\Gamma_B\sqrt{\mu}, \quad C_s = b\sqrt{2/\mu - 1}C_p, \quad C_a = \sqrt{2-\mu}$$

$$\leftarrow V = (1-\mu)^{-1}, \quad p = (b\Gamma_B/2)(1-\mu), \quad \mu = \alpha v_g/\Gamma_B$$

$\rightarrow$ Brillouin solitons $\leftarrow$ Dissipative solitons

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SBS Dynamics (4)

SBS-induced index change:

Stokes wave with the steady-state solution of *Q*:

$$\rightarrow -\frac{\partial A_s}{\partial z} + \frac{1}{v_g}\frac{\partial A_s}{\partial t} + \frac{\alpha}{2}A_p = i\gamma(|A_s|^2 + 2|A_p|^2)A_s + \frac{\kappa_1\kappa_2}{\frac{1}{2}\Gamma_B(1-i\delta)}|A_p|^2 A_s$$

$\frac{g_c}{2}A_s$

$$\leftarrow \delta = 2(\Omega - \Omega_B)/\Gamma_B$$

Complex gain:

$$\rightarrow g_c = \frac{g_p|A_p|^2}{A_{eff}}\left(\frac{1}{1-i\delta}\right) \qquad \leftarrow g_p \equiv g_B(\Omega_B)$$

SBS gain:

$$\rightarrow g_B = \mathrm{Re}(g_c)$$

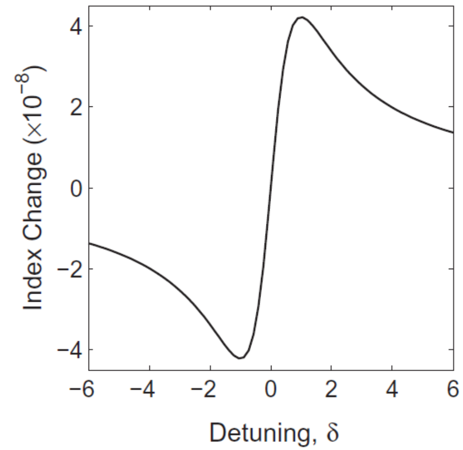
SBS-induced index change:

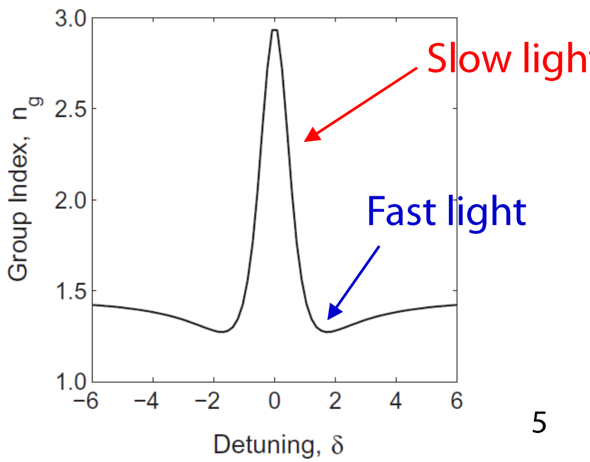
$$\rightarrow \Delta n_B = (c/2\omega_s)\,\mathrm{Im}(g_c) = \frac{cg_p|A_p|^2}{2\omega_s A_{eff}}\left(\frac{\delta}{1+\delta^2}\right)$$

Group index:

$$\rightarrow \beta_1 = d\beta/d\omega = n_g/c \quad \rightarrow n_g = n_t + \omega(dn_t/d\omega)$$

$$\rightarrow n_g = n_{g0} + \left(\frac{cg_pP_p}{\Gamma_B A_{eff}}\right)\frac{1-\delta^2}{(1+\delta^2)^2} \qquad \leftarrow P_p = |A_p|^2$$





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SBS Dynamics (5)

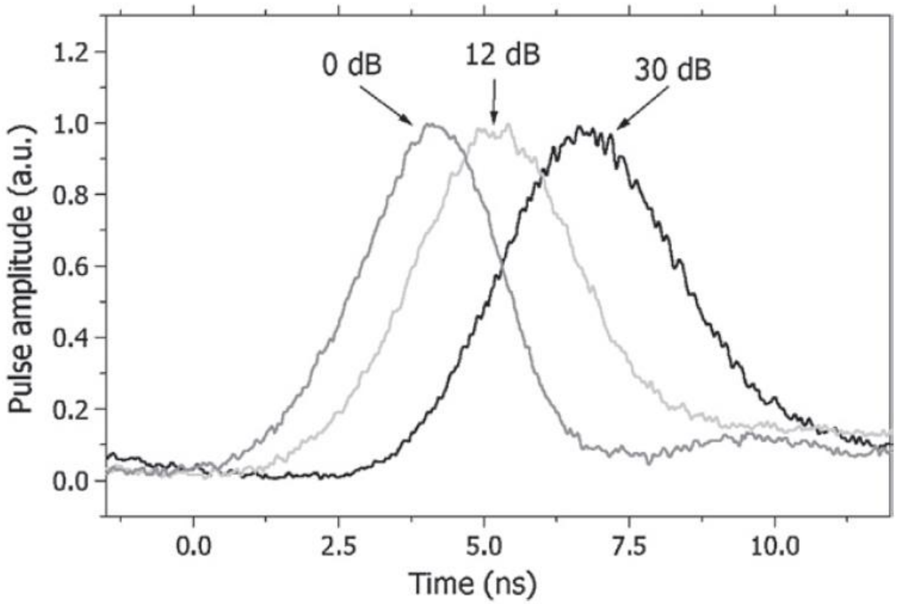
SBS-induced index change:

Transit time:

$$\rightarrow T_r = L/v_g = T_f + \left(\frac{c g_p P_p L}{\Gamma_B A_{eff}}\right) \frac{1-\delta^2}{(1+\delta^2)^2} \qquad \leftarrow T_f = n_{g0} L / c$$

SBS-induced pulse delay:

$$\rightarrow T_{d,\max} = T_r - T_f = \frac{g_p P_p L}{\Gamma_B A_{eff}} = \frac{\ln G_A}{\Gamma_B} \qquad \leftarrow \text{Slow light}$$



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