Nonlinear Optical Engineering

Parametric Processes (1)
(NFO 5th ed: 10.1 ~ 10.2)

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Origin of Four-Wave Mixing (1)

Parametric processes:

Characteristic features:

$\rightarrow$ Involvement of modulation of a medium parameter such as the refractive index

$\rightarrow$ Second- or third-order processes: $\leftarrow \chi^{(2)}$ or $\chi^{(3)}$

$\rightarrow$ Four-wave mixing, harmonic generation, etc.

Third-order nonlinear polarization:

$$\rightarrow \mathbf{P}_{NL} = \varepsilon_0 \chi^{(3)} \vdots \mathbf{EEE}$$

Total electric field:

$$\rightarrow \mathbf{E} = \frac{1}{2}\hat{x}\sum_{j=1}^{4} E_j \exp[i(k_j z - \omega_j t)] + c.c.$$

$$\rightarrow \mathbf{P}_{NL} = \frac{1}{2}\hat{x}\sum_{j=1}^{4} P_j \exp[i(k_j z - i\omega_j t)] + c.c.$$

For P_4:

$$\rightarrow P_4 = \frac{3\varepsilon_0}{4}\chi^{(3)}_{xxxx}[|E_4|^2 E_4 + 2(|E_1|^2 + |E_2|^2 + |E_3|^2)E_4$$

$$+ 2E_1E_2E_3 \exp(i\theta_+) + 2E_1E_2E_3^* \exp(i\theta\_) + \cdots]$$

$$\leftarrow \theta_+ = (k_1 + k_2 + k_3 - k_4)z - (\omega_1 + \omega_2 + \omega_3 - \omega_4)t$$ $\leftarrow$ 3rd-harmonic gen.

$$\leftarrow \theta\_ = (k_1 + k_2 - k_3 - k_4)z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)t$$ $\leftarrow$ Easy phase-matching

Origin of Four-Wave Mixing (2)

Parametric processes:

Conservation of energy: $\rightarrow \omega_3 + \omega_4 = \omega_1 + \omega_2$

Conservation of momentum: $\rightarrow \Delta k = k_3 + k_4 - k_1 - k_2$

$$= (n_3\omega_3 + n_4\omega_4 - n_1\omega_1 - n_2\omega_2) / c \ = 0$$

$\leftarrow$ Phase-matching condition

Partially degenerate FWM: $\rightarrow \omega_1 = \omega_2$

$\rightarrow \Omega_s = \omega_1 - \omega_3 = \omega_4 - \omega_1 \quad for\ \omega_3 < \omega_4$ $\leftarrow$ Signal & idler

Theory of Four-Wave Mixing (1)

Coupled amplitude equations:

Spatial dependence of the electric field: $\rightarrow E_j(\mathbf{r}) = F_j(x, y) A_j(z)$

In the paraxial approximation:

$$\left.\begin{aligned}
\frac{dA_1}{dz} &= \frac{in_2\omega_1}{c}[(f_{11}|A_1|^2 + 2\sum_{j\neq 1} f_{1j}|A_j|^2)A_1 + 2f_{1234}A_2^* A_3 A_4 e^{i\Delta kz} \\
\frac{dA_2}{dz} &= \frac{in_2\omega_2}{c}[(f_{22}|A_2|^2 + 2\sum_{j\neq 2} f_{2j}|A_j|^2)A_2 + 2f_{2134}A_1^* A_3 A_4 e^{i\Delta kz}
\end{aligned}\right\} \textit{Pump fields}$$

$$\left.\begin{aligned}
\frac{dA_3}{dz} &= \frac{in_2\omega_3}{c}[(f_{33}|A_3|^2 + 2\sum_{j\neq 3} f_{3j}|A_j|^2)A_3 + 2f_{3412}A_1 A_2 A_4^* e^{-i\Delta kz} \\
\frac{dA_4}{dz} &= \frac{in_2\omega_4}{c}[(f_{44}|A_4|^2 + 2\sum_{j\neq 4} f_{4j}|A_j|^2)A_4 + 2f_{4312}A_1 A_2 A_3^* e^{-i\Delta kz}
\end{aligned}\right\} \textit{Signal \& idler}$$

Wave-vector mismatch: $\rightarrow \Delta k = (\tilde{n}_3\omega_3 + \tilde{n}_4\omega_4 - \tilde{n}_1\omega_1 - \tilde{n}_2\omega_2)/c$

Overlap integral: $\rightarrow f_{jk} = \dfrac{\langle |F_j|^2 |F_k|^2 \rangle}{\langle |F_j|^2 \rangle \langle |F_k|^2 \rangle}$

$$\rightarrow f_{ijkl} = \frac{\langle F_i^* F_j^* F_k F_l \rangle}{[\langle |F_i|^2 \rangle \langle |F_j|^2 \rangle \langle |F_k|^2 \rangle \langle |F_l|^2 \rangle]^{1/2}}$$

Theory of Four-Wave Mixing (2)

Approximate solution:

Overlap integrals: $\rightarrow f_{ijkl} \approx f_{ij} \approx 1/A_{eff}$

Nonlinear parameter: $\rightarrow \gamma_j = n_2\omega_j/(cA_{eff}) \approx \gamma$

Undepleted-pump approximation:

$$\rightarrow A_1(z) = A_1(0)\exp[i\gamma(P_1 + 2P_2)z]$$

$$\rightarrow A_2(z) = A_2(0)\exp[i\gamma(P_2 + 2P_1)z]$$

$$\rightarrow \frac{dA_3}{dz} = 2i\gamma[(P_1 + P_2)A_3 + A_1(0)A_2(0)e^{-i\theta}A_4^*]$$

$$\rightarrow \frac{dA_4^*}{dz} = -2i\gamma[(P_1 + P_2)A_4^* + A_1^*(0)A_2^*(0)e^{i\theta}A_3] \quad \leftarrow \theta = [\Delta k - 3\gamma(P_1 + P_2)]z$$

Parameter exchange: $\rightarrow B_j = A_j \exp[-2i\gamma(P_1 + P_2)z]$

$$\rightarrow \frac{dB_3}{dz} = 2i\gamma A_1(0)A_2(0)\exp(-i\kappa z)B_4^*$$

$$\rightarrow \frac{dB_4^*}{dz} = -2i\gamma A_1^*(0)A_2^*(0)\exp(i\kappa z)B_3 \quad \leftarrow \kappa = \Delta k + \gamma(P_1 + P_2)$$

Theory of Four-Wave Mixing (3)

Approximate solution:

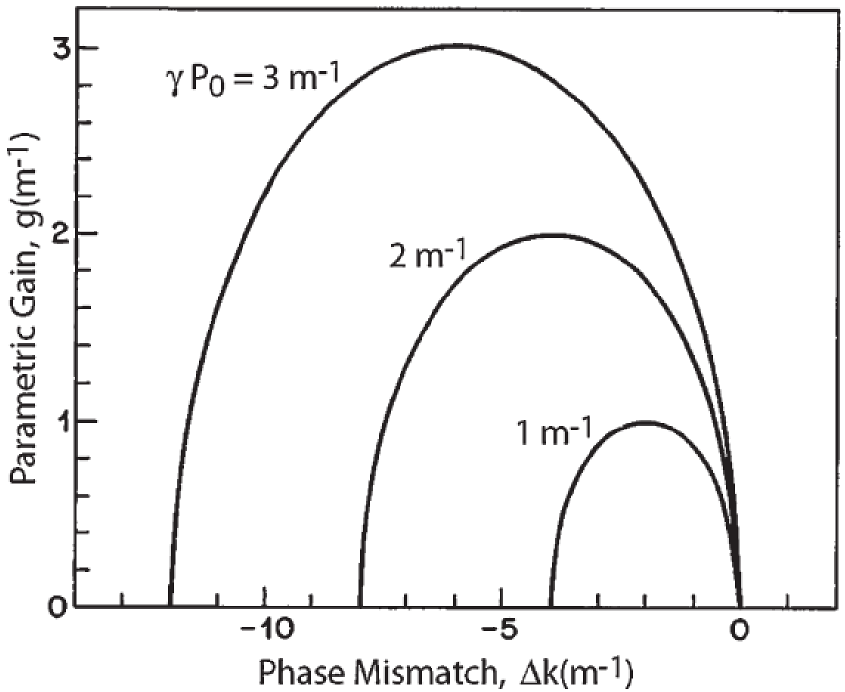
General solution:

$$\rightarrow B_3(z) = (a_3 e^{gz} + b_3 e^{-gz}) \exp(-i\kappa z/2)$$

$$\rightarrow B_4^*(z) = (a_4 e^{gz} + b_4 e^{-gz}) \exp(i\kappa z/2)$$

$$\leftarrow g = \sqrt{(\gamma P_0 r)^2 - (\kappa/2)^2}$$

$$\leftarrow r = 2(P_1 P_2)^{1/2}/P_0, \quad P_0 = P_1 + P_2$$



$$\leftarrow \kappa = \Delta k + \gamma(P_1 + P_2)$$

G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Theory of Four-Wave Mixing (4)

Effect of phase matching:

Partially degenerate FWM: $\rightarrow \omega_1 = \omega_2$

$\leftarrow$ Indistinguishable in terms of freq., pol, or spatial mode

Undepleted-pump approximation:

$$\rightarrow A_1(z) = A_1(0)\exp(i\gamma P_1 z)$$

$$\rightarrow \frac{dA_3}{dz} = 2i\gamma P_1 A_3 + i\gamma[A_1(0)]^2 e^{-i\theta} A_4^*$$

$$\rightarrow \frac{dA_4^*}{dz} = -2i\gamma P_1 A_4^* - i\gamma[A_1^*(0)]^2 e^{i\theta} A_3 \qquad \leftarrow \theta = (\Delta k - 2\gamma P_1)z$$

Parameter exchange: $\rightarrow B_i = A_j \exp(-2i\gamma P_1 z)$

$$\rightarrow \frac{dB_3}{dz} = i\gamma[A_1(0)]^2 \exp(-i\kappa z) B_4^*$$

$$\rightarrow \frac{dB_4^*}{dz} = -i\gamma[A_1^*(0)]^2 \exp(i\kappa z) B_3 \qquad \leftarrow \kappa = \Delta k + 2\gamma P_1$$

Maximum gain: $\rightarrow g = \sqrt{(\gamma P_1)^2 - (\kappa/2)^2}$

$\rightarrow g_{\max} = \gamma P_1$

Coherence length: $\rightarrow L_{coh} = 2\pi/|\kappa|$ $\rightarrow$ Significant FWM: $\rightarrow L < L_{coh}$

Theory of Four-Wave Mixing (5)

Ultrafast FWM:

Coupled amplitude equations:

$$\rightarrow \frac{dA_j}{dz} \rightarrow \frac{\partial A_j}{\partial z} + \beta_{1j}\frac{\partial A_j}{\partial t} + \frac{i}{2}\beta_{2j}\frac{\partial^2 A_j}{\partial t^2} + \frac{1}{2}\alpha_j A_j$$

CW pumping with an undepleted-pump approximation: $> 10\ \mathrm{THz}$

$$\rightarrow \frac{\partial A_3}{\partial z} + \beta_{13}\frac{\partial A_3}{\partial t} + \frac{i}{2}\beta_{23}\frac{\partial^2 A_3}{\partial t^2} + \frac{1}{2}\alpha_3 A_3 = i\gamma(|A_3|^2 + 2|A_4|^2 + 2P_0)A_3 + i\gamma P_0 e^{-i\theta}A_4^*$$

$$\rightarrow \frac{\partial A_4}{\partial z} + \beta_{14}\frac{\partial A_4}{\partial t} + \frac{i}{2}\beta_{24}\frac{\partial^2 A_4}{\partial t^2} + \frac{1}{2}\alpha_4 A_4 = i\gamma(|A_4|^2 + 2|A_3|^2 + 2P_0)A_4 + i\gamma P_0 e^{-i\theta}A_3^*$$

$$\leftarrow \theta = \Delta k - 2\gamma P_0 \qquad \leftarrow P_0 = P_1$$

Symbiotic soliton pairs:

$\rightarrow \beta_{13} = \beta_{14}, \quad \beta_{23} = -\beta_{24}$ → Balance between the parametric gain and fiber losses

→ Dissipative solitons

Single NLSE approach: $< 1\ \mathrm{THz}$

$$\rightarrow A(0,t) = A_1(0,t) + A_3(0,t)\exp(-i\Omega_s t) + A_4(0,t)\exp(i\Omega_s t)$$