

# Nonlinear Optical Engineering

Parametric Processes (2)  
(NFO 5<sup>th</sup> ed: 10.3 ~ 10.4)

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# Phase-Matching Techniques (1)

Physical mechanisms:

Phase-matching condition:

$$\rightarrow \kappa = \Delta k_M + \Delta k_W + \Delta k_{NL} = 0$$

Effective indices:

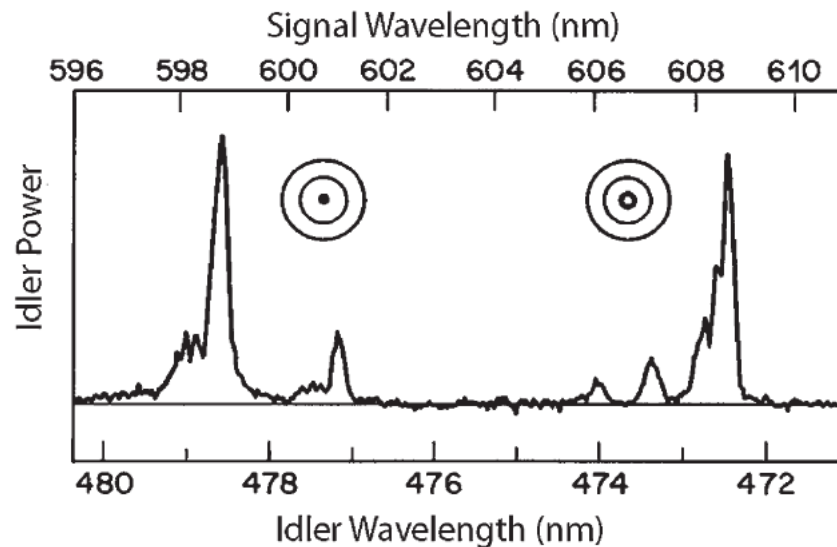
$$\rightarrow \tilde{n}_j = n_j + \Delta n_j \quad \rightarrow \Delta k_M = [n_3\omega_3 + n_4\omega_4 - n_1\omega_1 - n_2\omega_2] / c \approx \beta_2\Omega_s^2 + (\beta_4 / 12)\Omega_s^4$$

$$\rightarrow \Delta k_W = [\Delta n_3\omega_3 + \Delta n_4\omega_4 - \Delta n_1\omega_1 - \Delta n_2\omega_2] / c$$

$$\rightarrow \Delta k_{NL} = \gamma(P_1 + P_2)$$

Phase matching in multimode fibers:

$$\rightarrow \Delta k_W = [\Delta n_3\omega_3 + \Delta n_4\omega_4 - \Delta n_1\omega_1 - \Delta n_2\omega_2] / c$$



# Phase-Matching Techniques (2)

## Phase matching in single-mode fibers:

Nearly phase-matched FWM:

$$\rightarrow \kappa = \Delta k_M + \Delta k_W + \Delta k_{NL} \approx 0 \quad \rightarrow L_{coh} = 2\pi / |\Delta k_M| \approx 2\pi / \beta_2 \Omega_s^2 \quad \rightarrow L < L_{coh}$$

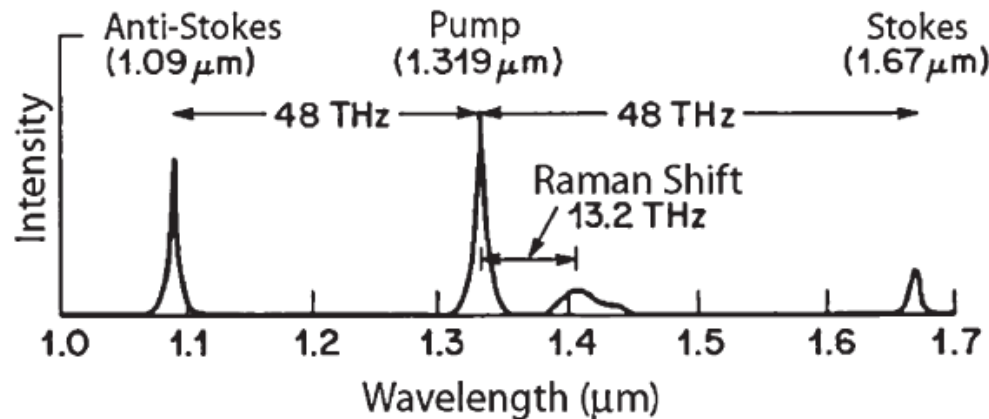
Phase matching near the zero-dispersion wavelength:

$$\rightarrow \beta_m = \left( \frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0} \quad (m = 1, 2, \dots)$$

Phase matching due to self-phase modulation:

$$\rightarrow \kappa = \Delta k_M + \Delta k_W + \Delta k_{NL} \approx \Delta k_M + \Delta k_{NL} \approx \beta_2 \Omega_s^2 + 2\gamma P_0 \quad \rightarrow \Omega_s = (2\gamma P_0 / |\beta_2|)^{1/2}$$

$$\rightarrow (\beta_4 / 12) \Omega_s^4 + \beta_2 \Omega_s^2 + 2\gamma P_0 = 0 \quad \rightarrow \Omega_s^2 = \frac{6}{|\beta_4|} \left( \sqrt{\beta_2^2 + 2|\beta_4| \gamma P_0 / 3} + \beta_2 \right)$$



# Phase-Matching Techniques (3)

## Phase matching in birefringent fibers:

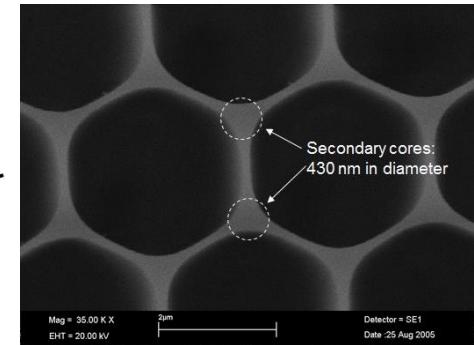
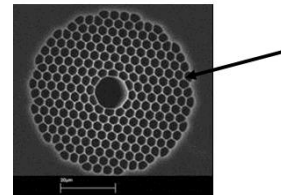
Modal birefringence:

$$\rightarrow \delta n = \Delta n_x - \Delta n_y$$

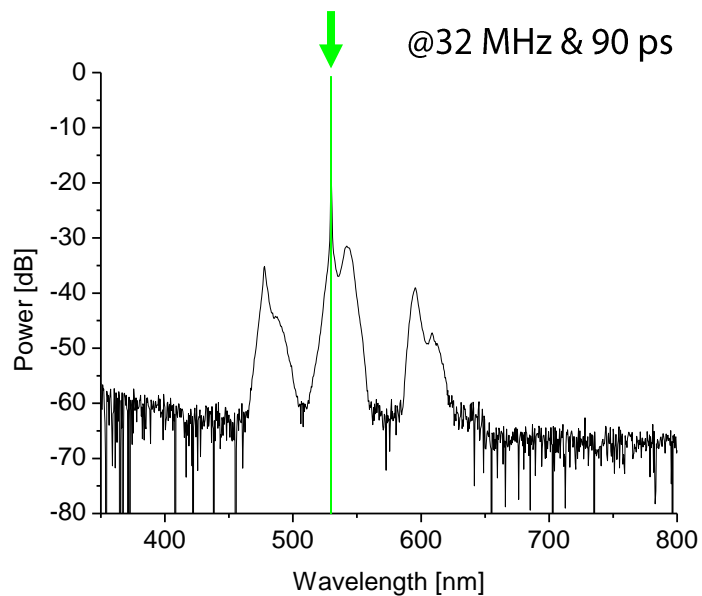
$$\rightarrow \Delta k_W = [\Delta n_y (\omega_3 + \omega_4) - 2\Delta n_x \omega_1] / c$$
$$= -2\omega_1 (\delta n) / c$$

FWM by form birefringence:

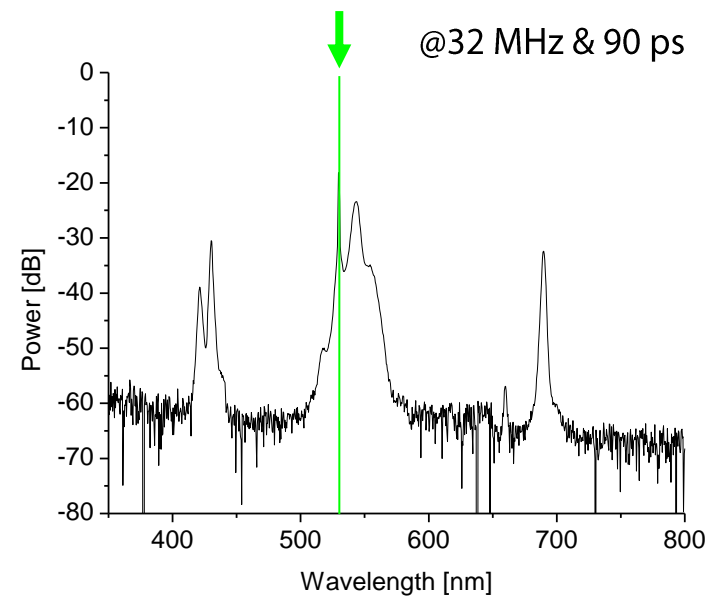
Photonic bandgap fiber:



Green pump at 530 nm



Green pump at 530 nm



Diffracted output RGB:



# Parametric Amplification (1)

## Gain and bandwidth:

Signal and idler powers:

$$\rightarrow P_3 = |B_3|^2, \quad P_4 = |B_4|^2, \quad B_3(0) \neq 0, \quad B_4(0) = 0 \quad \rightarrow \frac{d}{dz}(P_3 - P_4) = 0$$

Conservation of photons:

Signal and idler powers:

$$\rightarrow P_3(z) = P_3(0)[1 + (1 + \kappa^2 / 4g^2) \sinh^2(gz)]$$

$$\rightarrow P_4(z) = P_3(0)[(1 + \kappa^2 / 4g^2) \sinh^2(gz)]$$

Amplification factor:

$$\rightarrow G_p = P_3(L) / P_3(0) = 1 + (\gamma P_0 r / g)^2 \sinh^2(gL) \quad \leftarrow g = \sqrt{(\gamma P_0 r)^2 - (\kappa / 2)^2}$$

$$\rightarrow \kappa \gg \gamma P_0 r \quad \rightarrow G_p \approx 1 + (\gamma P_0 r L)^2 \frac{\sin^2(\kappa L / 2)}{(\kappa L / 2)^2}$$

$$\rightarrow \kappa = 0, \quad gL \gg 1 \quad \rightarrow G_p \approx \frac{1}{4} \exp(2\gamma P_0 r L)$$

Amplifier bandwidth:

$$\rightarrow \Delta\Omega_A = \frac{1}{|\beta_2| \Omega_s} \left[ \left( \frac{\pi}{L} \right)^2 + (\gamma P_0 r)^2 \right]^{1/2} \quad \leftarrow L_{NL} = \frac{1}{\gamma P_0}$$

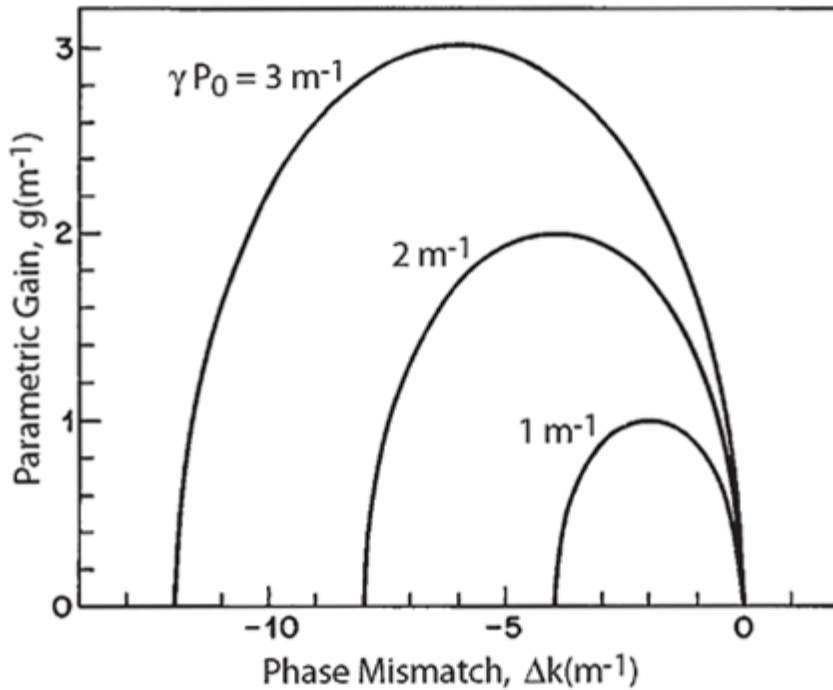
$$\rightarrow L_{NL} \gg L \quad \rightarrow \Delta\Omega_A \approx \frac{\pi}{|\beta_2| \Omega_s L}$$

$$\rightarrow L_{NL} \ll L \quad \rightarrow \Delta\Omega_A \approx \frac{\gamma P_0 r}{|\beta_2| \Omega_s} = \frac{1}{|\beta_2| \Omega_s L_{NL}} \quad \leftarrow r = 1$$

# Parametric Amplification (2)

Gain and bandwidth:

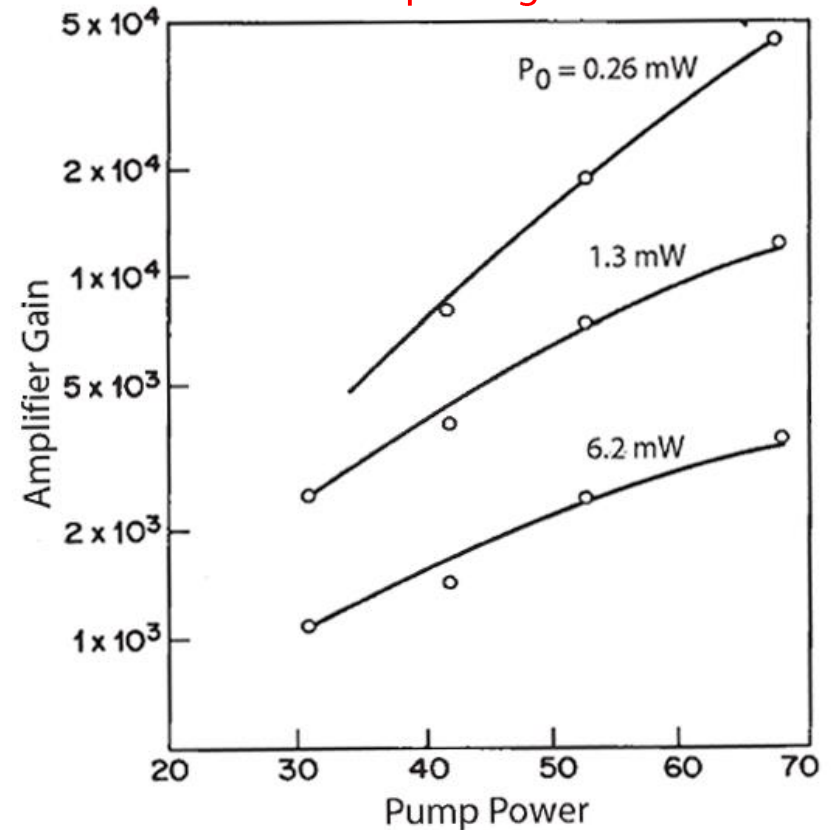
Amplifier bandwidth:



G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

$$\rightarrow \Delta\Omega_A \approx \frac{2\gamma P_0}{2|\beta_2|\Omega_s} = \frac{1}{|\beta_2|\Omega_s L_{NL}}$$

Amplifier gain:



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→ Parametric amplifiers & parametric oscillators