Nonlinear Optical Engineering

Parametric Processes (3)
(NFO 5th ed: 10.5 ~ 10.6)

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FWM Applications (1)

Wavelength conversion:

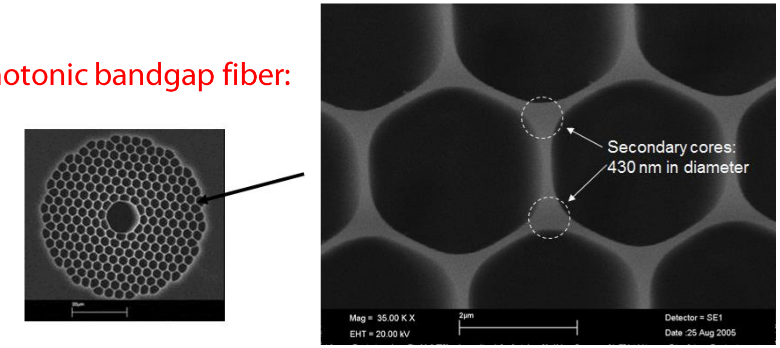
Modal birefringence:

$$\rightarrow \delta n = \Delta n_x - \Delta n_y$$

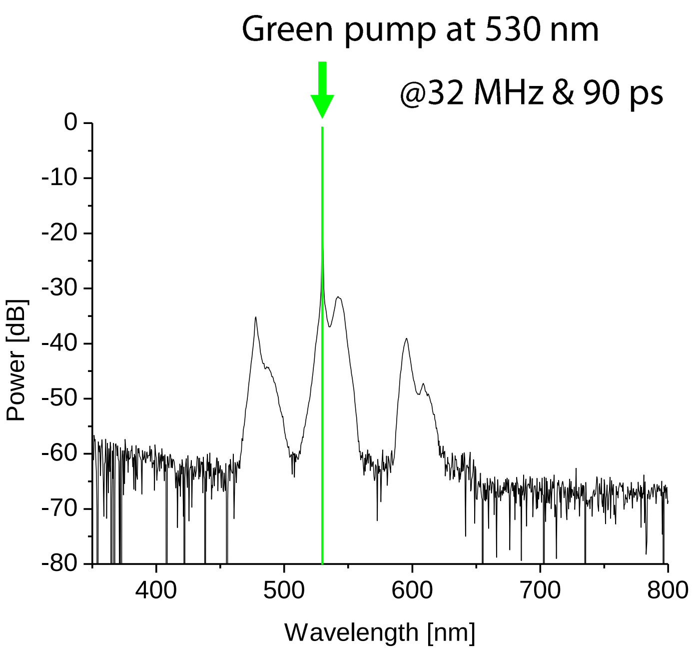
$$\rightarrow \Delta k_W = [\Delta n_y(\omega_3 + \omega_4) - 2\Delta n_x \omega_1] / c$$

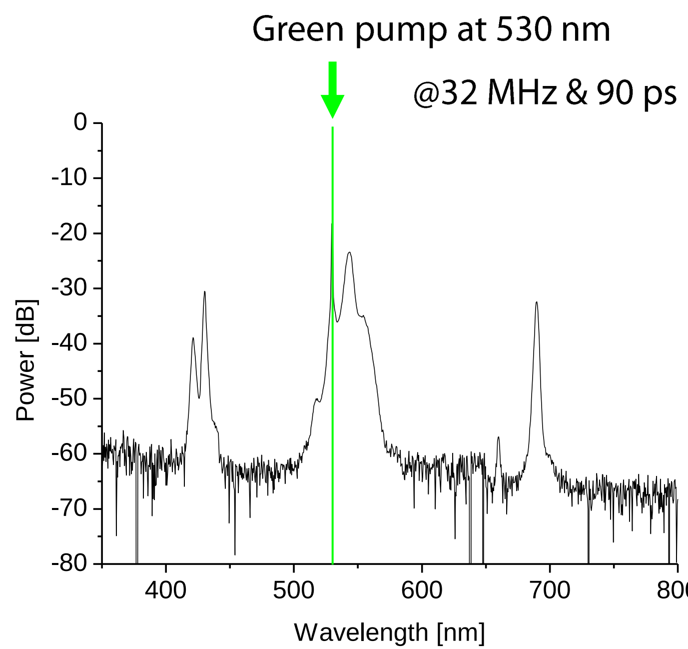
$$= -2\omega_1(\delta n) / c$$

Photonic bandgap fiber:



FWM by form birefringence:





Diffracted output RGB:

FWM Applications (2)

Phase conjugation:

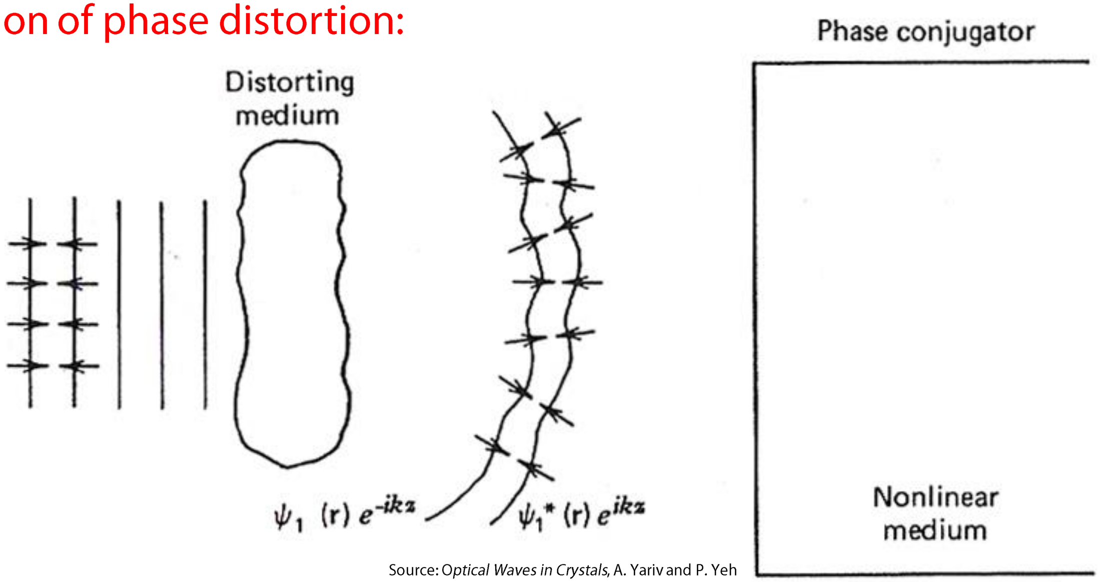
Undepleted-pump approximation:

$$\rightarrow A_1(z) = A_1(0)\exp(i\gamma P_1 z)$$

$$\rightarrow \frac{dA_3}{dz} = 2i\gamma P_1 A_3 + i\gamma[A_1(0)]^2 e^{-i\theta} A_4^* \qquad \leftarrow \theta = (\Delta k - 2\gamma P_1)z$$

$$\rightarrow \frac{dA_4^*}{dz} = -2i\gamma P_1 A_4^* - i\gamma[A_1^*(0)]^2 e^{i\theta} A_3 \qquad \rightarrow A_3 \Leftrightarrow A_4^*$$

Cancellation of phase distortion:



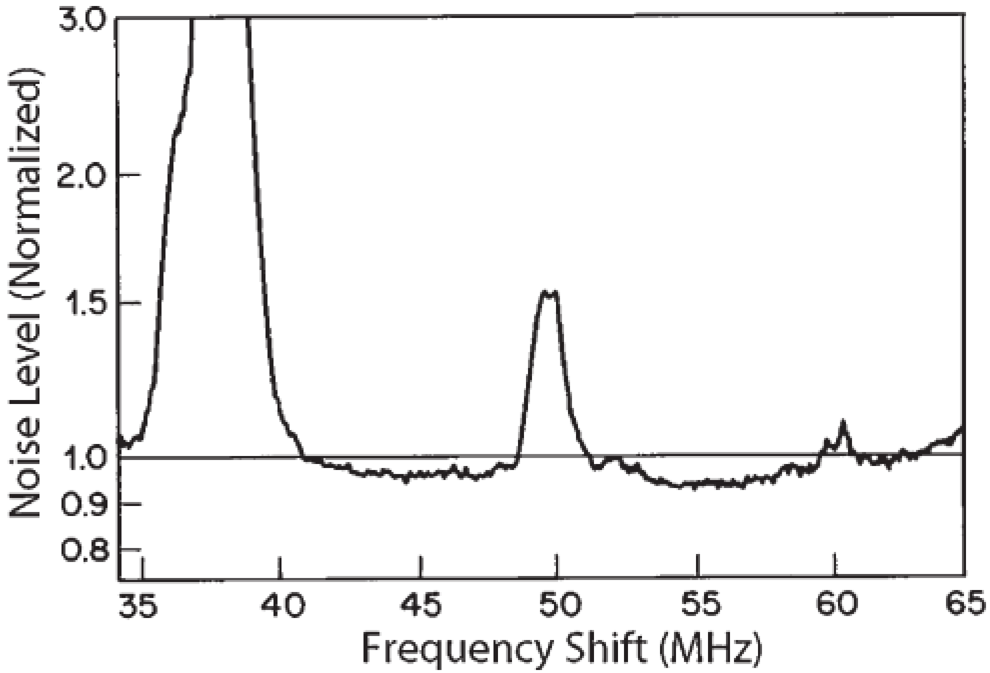
Source: *Optical Waves in Crystals*, A. Yariv and P. Yeh

$$\rightarrow e^{i\Delta\phi_{\Delta L}} = \prod_{j=1}^{N} e^{i\Delta\phi_j} \rightarrow e^{-i\Delta\phi_{\Delta L}} \prod_{j=1}^{N} e^{i\Delta\phi_j} = e^{j0}$$

FWM Applications (3)

Squeezing:

Noise reduction by phase-sensitive conversion:



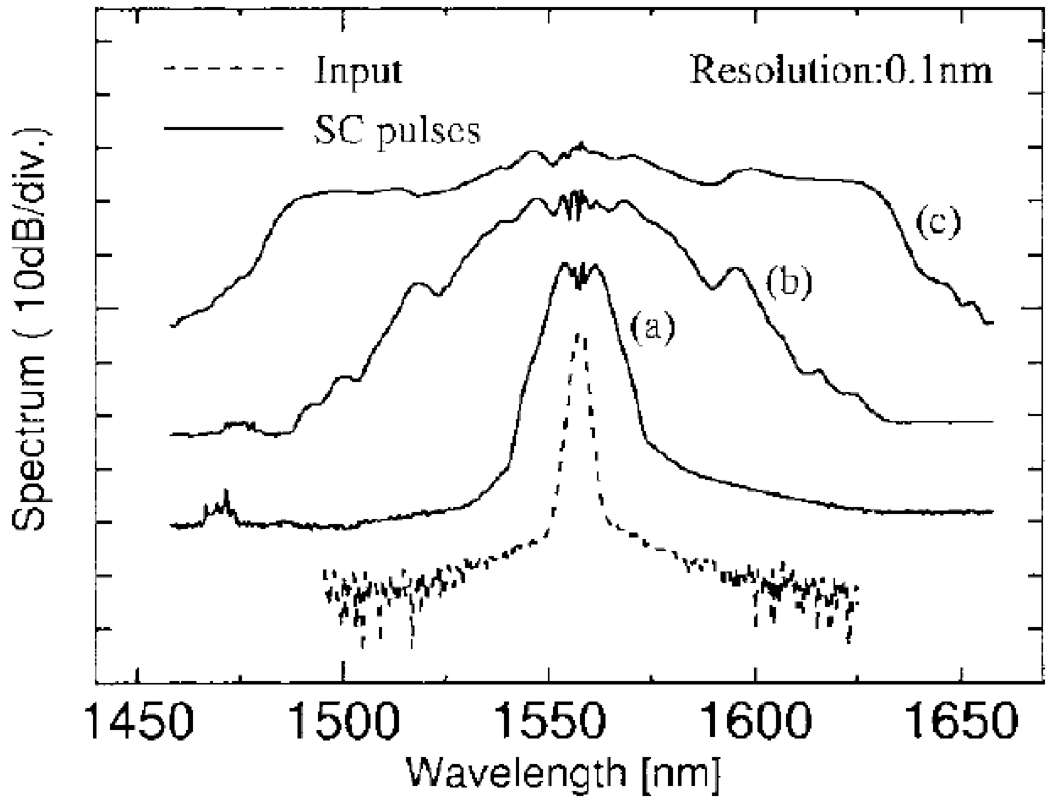
G. P. Agrawal, Nonlinear Fiber Optics, 5th ed.

Limitation: Inelastic scattering effects, e.g., SBS & SRS

FWM Applications (4)

Supercontinuum generation:

FWM + SPM + XPM + SRS + etc.:



G. P. Agrawal, Nonlinear Fiber Optics, 3rd ed.

Second-Harmonic Generation (1)

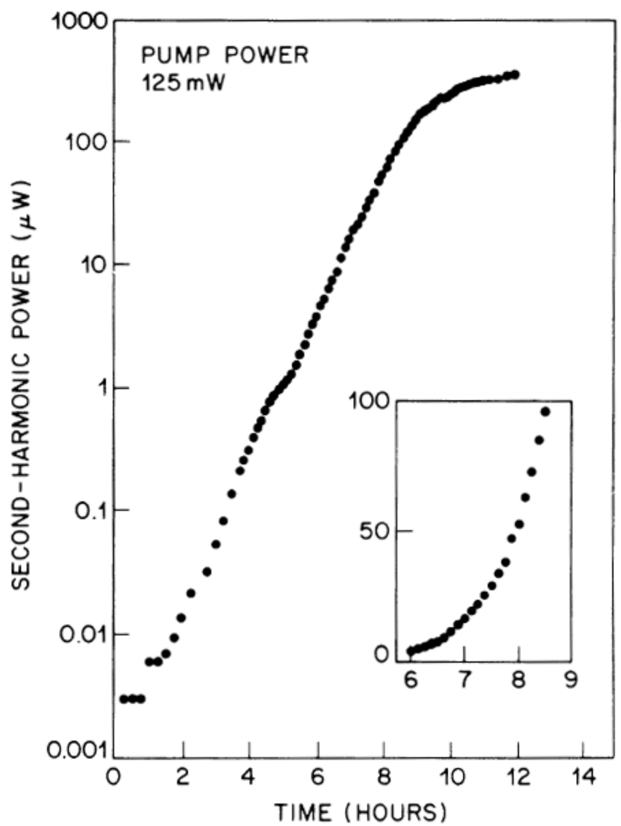
Physical mechanism:

Induced static polarization:

$$\rightarrow P_{dc} = (3\varepsilon_0/4)\,\mathrm{Re}[\chi^{(3)} E_p^* E_p^* E_{SH} \exp(i\Delta k_p z) \qquad \leftarrow \Delta k_p = [n(2\omega_p) - 2n(\omega_p)]\omega_p / c$$

Effective second-order nonlinearity:

$$\rightarrow \chi^{(2)} = \alpha_{SH} P_{dc} = (3\alpha_{SH}/4)\varepsilon_0 \chi^{(3)} \left|E_p\right|^2 \left|E_{SH}\right| \cos(\Delta k_p + \phi_p)$$



G. P. Agrawal, Nonlinear Fiber Optics, 3rd ed.

Second-Harmonic Generation (2)

Simple theory:

Coupled amplitude equation:

$$\rightarrow \frac{dA_1}{dz} = i\gamma_1(|A_1|^2 + 2|A_2|^2)A_1 + \frac{i}{2}\gamma_{SH}^* A_2 A_1^* \exp(-i\kappa z)$$

$$\rightarrow \frac{dA_2}{dz} = i\gamma_2(|A_2|^2 + 2|A_1|^2)A_2 + \frac{i}{2}\gamma_{SH} A_1^2 \exp(i\kappa z)$$

$$\leftarrow \gamma_{SH} = (3\omega_1 / 4n_1 c)\varepsilon_0^2 \alpha_{SH} f_{112}\chi^{(3)} |E_p|^2 |E_{SH}|^2$$

$$\leftarrow \kappa = \Delta k_p - \Delta k$$

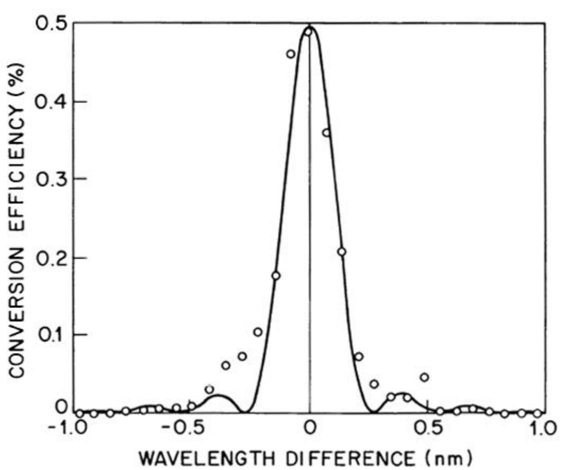
Undepleted pump approximation:

$$\rightarrow A_1(z) = \sqrt{P_1}\exp(i\gamma_1 P_1 z) \qquad \rightarrow A_2 = B_2 \exp(2i\gamma_1 P_1 z)$$

$$\rightarrow \frac{dB_2}{dz} = i\gamma_{SH} P_1 \exp(i\kappa z) + 2i(\gamma_2 - \gamma_1)P_1 B_2$$

$$\rightarrow P_2(L) = |B_2(L)|^2 = |\gamma_{SH} P_1 L|^2 \frac{\sin^2(\kappa' L/2)}{(\kappa' L/2)^2}$$

$$\leftarrow \kappa' = \kappa - 2(\gamma_2 - \gamma_1)P_1$$

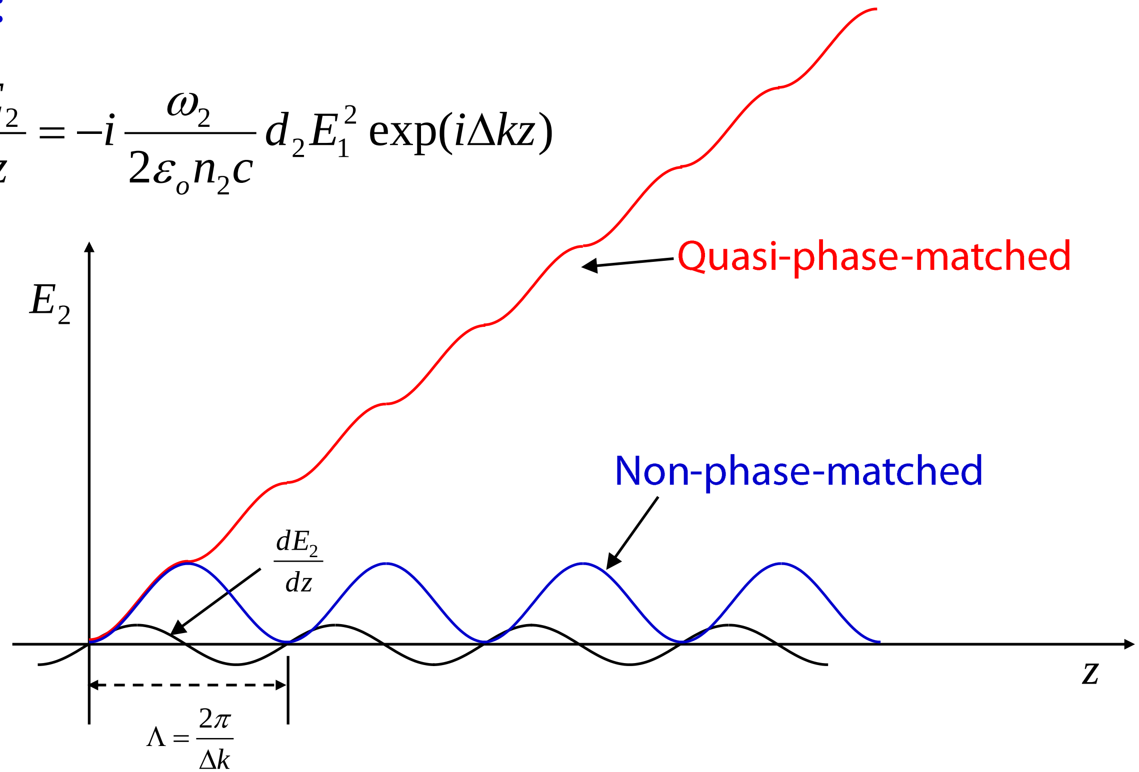


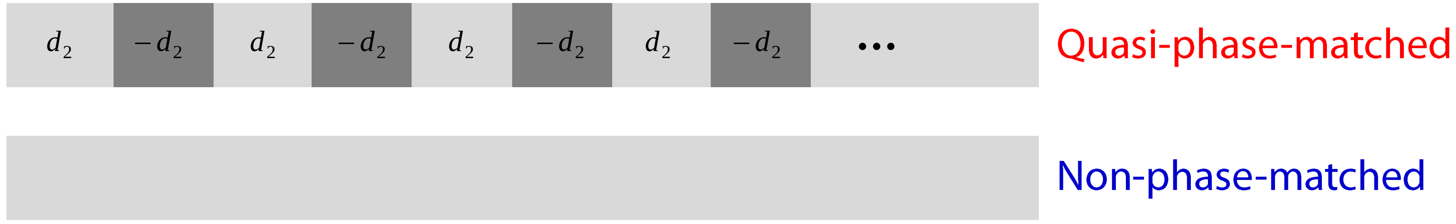
G. P. Agrawal, Nonlinear Fiber Optics, 3rd ed.

Quasi-Phase Matching

SH field:

$$\frac{dE_2}{dz} = -i\,\frac{\omega_2}{2\varepsilon_o n_2 c}\,d_2 E_1^2 \exp(i\Delta k z)$$





→ *Periodic spatial modulation of the nonlinear coefficient:*
e.g. PPLN (periodically-poled lithium niobate), PP glass, etc.

CM Equations with 2nd and 3rd Nonlinearities (1)

Coupled equations for the complex amplitudes:

$$\rightarrow \frac{dA_{\omega_1}}{dz} = -i\kappa_{\varepsilon,\omega_1} A_{\omega_1} - i\kappa_{d,\omega_1} A_{\omega_2} A^*_{\omega_1} \exp(-i\Delta\beta z) - i(\kappa_{\chi,\omega_1,\omega_1} \left|A_{\omega_1}\right|^2 + \kappa_{\chi,\omega_1,\omega_2} \left|A_{\omega_2}\right|^2) A_{\omega_1}$$

$$\rightarrow \frac{dA_{\omega_2}}{dz} = -i\kappa_{\varepsilon,\omega_2} A_{\omega_2} - i\kappa_{d,\omega_2} A^2_{\omega_1} \exp(i\Delta\beta z) - i(\kappa_{\chi,\omega_2,\omega_1} \left|A_{\omega_1}\right|^2 + \kappa_{\chi,\omega_2,\omega_2} \left|A_{\omega_2}\right|^2) A_{\omega_2}$$

$$\leftarrow \Delta\beta = \beta_{\omega_2} - 2\beta_{\omega_1}$$

Coupled equations for the amplitudes and phases:

$$\rightarrow \frac{du}{dz} = -\kappa_{d,\omega_1} \sqrt{P_o}\, uv \sin\theta, \quad \frac{dv}{dz} = \kappa_{d,\omega_2} \sqrt{P_o}\, u^2 \sin\theta,$$

$$\rightarrow \frac{d\varphi_f}{dz} = \kappa_{\varepsilon,\omega_1} + \kappa_{d,\omega_1} \sqrt{P_o}\, v \cos\theta + P_o(\kappa_{\chi,\omega_1,\omega_1} u^2 + \kappa_{\chi,\omega_1,\omega_2} v^2),$$

$$\rightarrow \frac{d\varphi_s}{dz} = \kappa_{\varepsilon,\omega_2} + \kappa_{d,\omega_2} \sqrt{P_o}\, \frac{u^2}{v} \cos\theta + P_o(\kappa_{\chi,\omega_2,\omega_1} u^2 + \kappa_{\chi,\omega_2,\omega_2} v^2),$$

$$\rightarrow \frac{d\theta}{dz} = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{u^2 v} \cdot \frac{d}{dz}(u^2 v) + f_o + g_o v^2.$$

Invariant constant: $\rightarrow \Gamma_o = u^2 v \cos\theta + \sigma_f v^2 + \sigma_g v^4$

$$\rightarrow \left(\frac{dv^2}{dz}\right)^2 = (2\kappa_{d,\omega_2} \sqrt{P_o})^2 \cdot \left\{(1 - v^2)^2 v^2 - (\Gamma_o - \sigma_f v^2 - \sigma_g v^4)^2\right\}$$

CM Equations with 2nd and 3rd Nonlinearities (2)

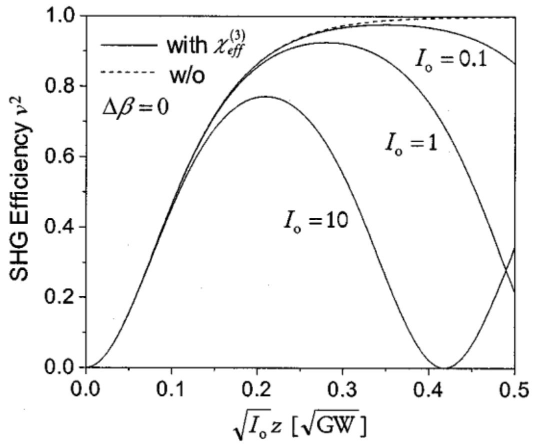
Analytic solutions:

$$\rightarrow u^2 = 1 - \frac{v_b^2(v_c^2 - v_a^2) - v_c^2(v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)}{v_c^2 - v_a^2 - (v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)}$$

$$\rightarrow v^2 = \frac{v_b^2(v_c^2 - v_a^2) - v_c^2(v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)}{v_c^2 - v_a^2 - (v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)}$$

$$\rightarrow \varphi_f = \varphi_{f,0} + \psi_{f,0} z + \frac{1}{\alpha}\Big(\psi_{f,1}\left\{\Pi(\phi, \rho_{f,1}, \gamma^2) - \Pi(\phi_o, \rho_{f,1}, \gamma^2)\right\}$$
$$+ \psi_{f,2}\left\{\Pi(\phi, \rho_{f,2}, \gamma^2) - \Pi(\phi_o, \rho_{f,2}, \gamma^2)\right\}\Big)$$

$$\rightarrow \varphi_s = \varphi_{s,0} + \psi_{s,0} z + \frac{1}{\alpha}\Big(\psi_{s,1}\left\{\Pi(\phi, \rho_{s,1}, \gamma^2) - \Pi(\phi_o, \rho_{s,1}, \gamma^2)\right\}$$
$$+ \psi_{s,2}\left\{\Pi(\phi, \rho_{s,2}, \gamma^2) - \Pi(\phi_o, \rho_{s,2}, \gamma^2)\right\}\Big)$$



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