Continuous Energy Loss

- ✓ One method to account for the energy loss to sub-threshold (soft bremsstrahlung and soft collisions) is to assume that the energy is lost continuously along its path.
- ✓ The formalism that may be used is the Bethe-Bloch theory of charged particle energy loss as expressed by Berger and Seltzer and in ICRU 37.

[1] M. J. Berger, S. M. Seltzer, Stopping powers and ranges of electrons and positrons, (National Bureau of Standarts Report, NBSIR 82-2550 A, 1982).
[2] ICRU, Report No. 37, 1984, Stopping powers for electrons and positrons. (International Commission on Radiation Units and Measurements, Bethesda, MD, 1984).

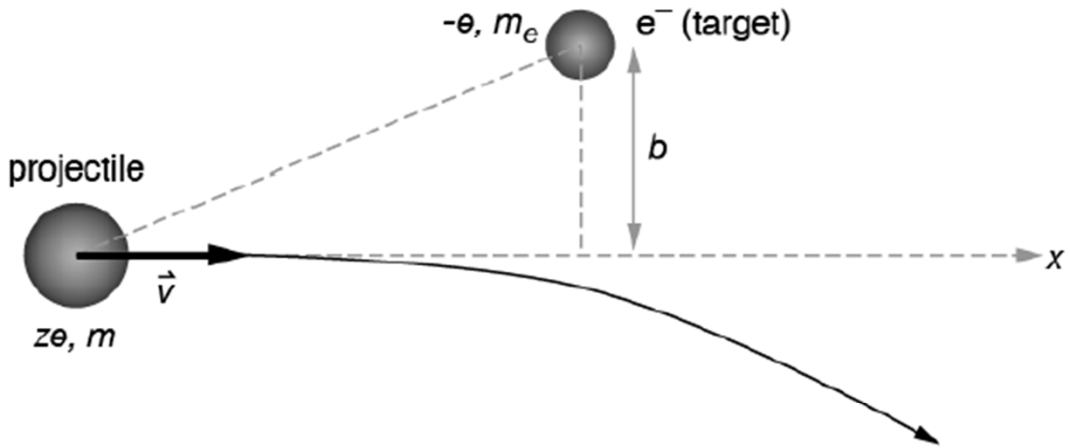
- ✓ This continuous energy loss scales with the Z of the medium for the collision contribution and Z^2 for the radiative part.

Energy loss of heavy charged particles

approach

Ansatz to derive classical formula of Bohr:

Energy loss *dE/dx* of a **heavy** ($m >> m_e$), charged particle through the scattering on an electron of the target atoms.



Assumptions:

- Electrons of the atoms are in rest, i.e. the original orbit movement and the recoil after collision are neglected.
- Binding of electrons to the nucleus is neglected. (i.e. energy transfer >> binding energy, but is still small fraction of its energy)

Bethe-Bloch formula for the average energy loss of heavy ($M \gg m_e$) charged particle: (interaction dominated by collision with electrons)

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] [\cdot \rho]$$

density

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV g^{-1} cm^2

$T_{max} = 2m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma\, m_e/M + (m_e/M)^2)$

[Max. energy transfer in single collision]

$N_A = 6.022 \cdot 10^{23}$

[Avogardo's number]

$r_e = e^2/4\pi\varepsilon_0 m_e c^2 = 2.8$ fm

[Classical electron radius]

$m_e = 511$ keV

[Electron mass]

$\beta = v/c$

[Velocity]

$\gamma = (1-\beta^2)^{-2}$

[Lorentz factor]

z : Charge of incident particle

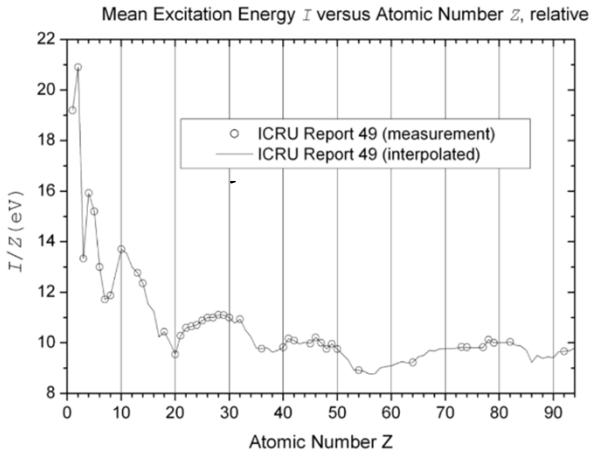
M : Mass of incident particle

Z : Charge number of medium

A : Atomic mass of medium

I : Mean excitation energy of medium

δ : Density correction [transv. extension of electric field]



- BB formula is valid only if $v_{\text{incident CP}} \gg v_{\text{orbital electron}}$ (~ 0.01c).

$mvr = \frac{nh}{2\pi}$ ⇒ the electron velocity in the 1st Bohr orbit = 2.18×10^6 m/sec

Review

(Inokuti and Turner, Report to DOE)

$$\ln I = \int \frac{df}{dE} \ln E \, dE \Big/ \int \frac{df}{dE} \, dE \,, \qquad (2)$$

where df/dE is the density of optical dipole oscillator strength f per unit energy of excitation E above the ground state and the integrals include summations over the discrete states. The oscillator-strength distribution satisfies various sum rules, such as Eq. (F-33), which in the present notation can be written

$$\int \frac{df}{dE} \, dE = Z \,. \qquad (3)$$

Review

Kamakura et al., J Appl Phys 100, 064905 (2006)

$$\ln I = \frac{\Sigma_n f_n \ln E_n + \int_{IP}^{\infty} (df/dE) \ln E dE}{S(0)}$$: mean excitation energy I for atoms and molecules

$$\frac{df}{dE} = \frac{mc}{2\pi^2 \hbar e^2} \sigma_{ph}(E)$$: oscillation strength vs. photon absorption cross section

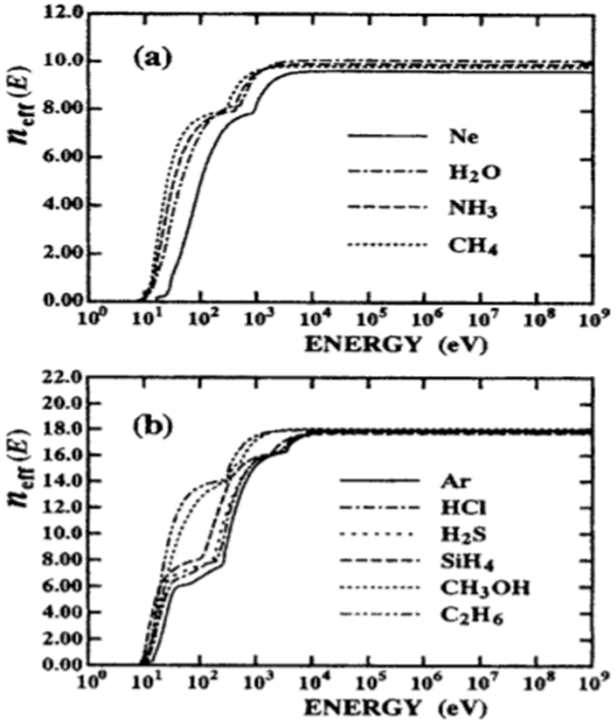


FIG. 1. Values of $n_{eff}(E)$ are shown as a function of the upper limit of absorbed energy E for 10-electron species (a) Ne, H_2O, NH_3, and CH_4 and 18-electron species (b) Ar, HCl, H_2S, SiH_4, CH_3OH, and C_2H_6.

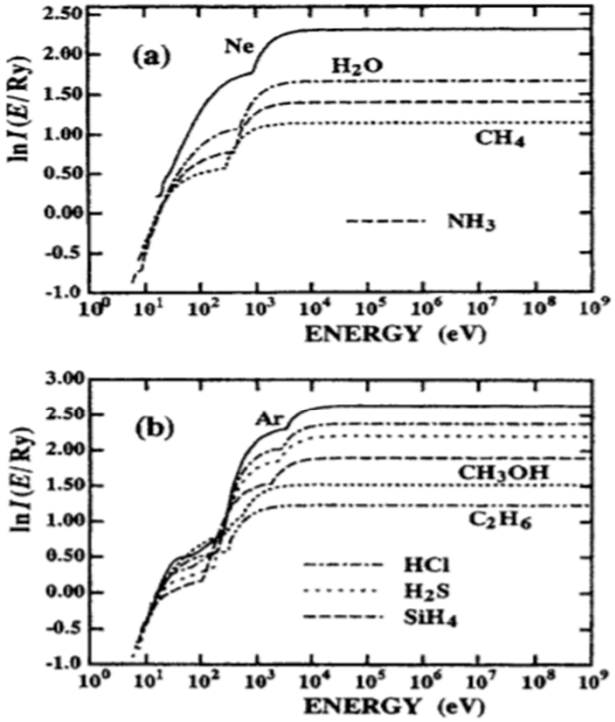


FIG. 2. Cumulative $I(E)$ values are shown as a function of the upper limit of absorbed energy E for 10-electron species (a) Ne, H_2O, NH_3, and CH_4 and 18-electron species (b) Ar, HCl, H_2S, SiH_4, CH_3OH, and C_2H_6.

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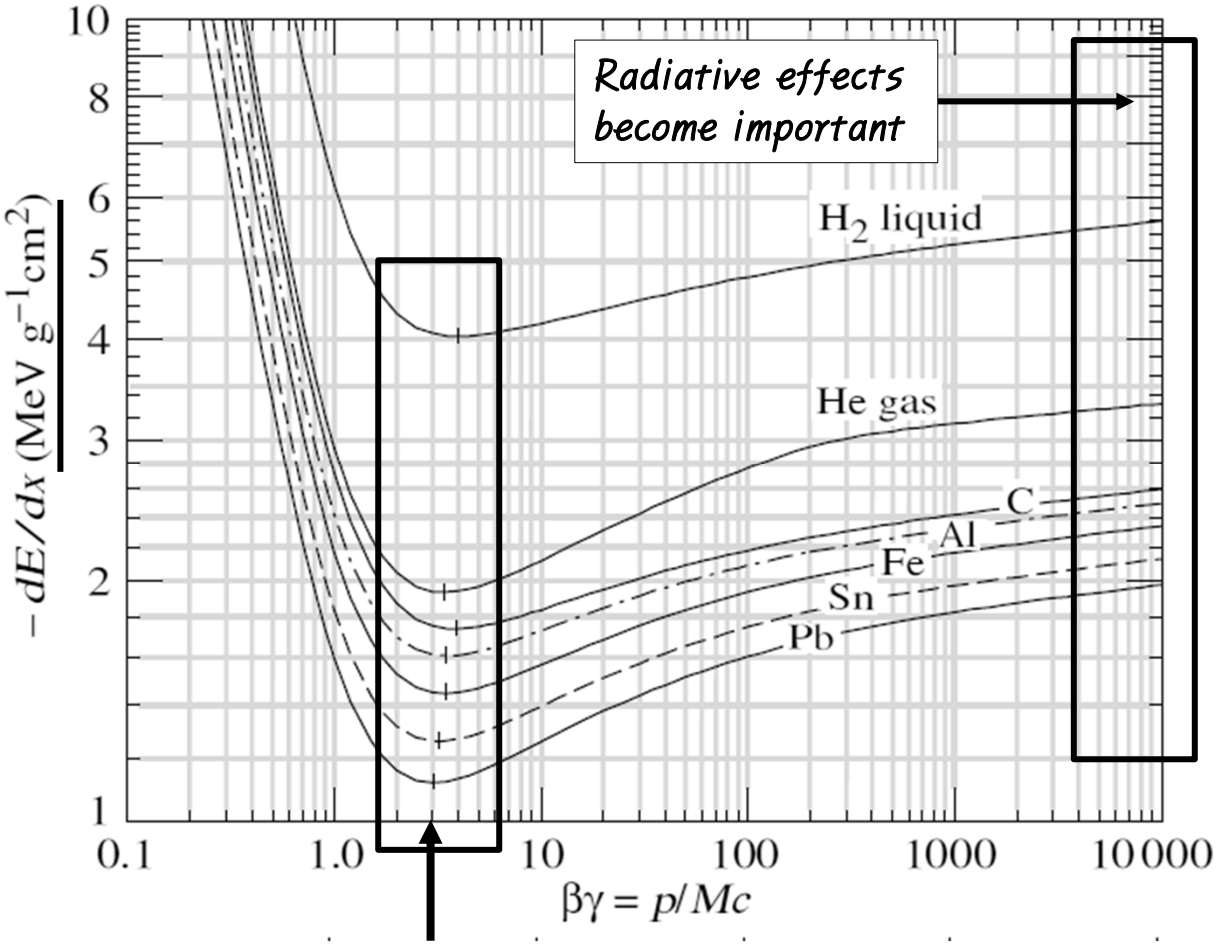
energy loss of heavy charged particles:

dependence on mass A and charge Z of target nucleus



source: Particle Data Group, Review of Particle Physics, Physics Letters B 592 (2004)

Bethe-Bloch formula for energy loss of electron
in collision with <u>undistinguishable</u> particle, electron:

$$-\left\langle \frac{dE}{dx} \right\rangle_{\text{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} + F(\gamma) \right]$$

T = kinetic energy of incoming electron ($T_{max} = T/2$)

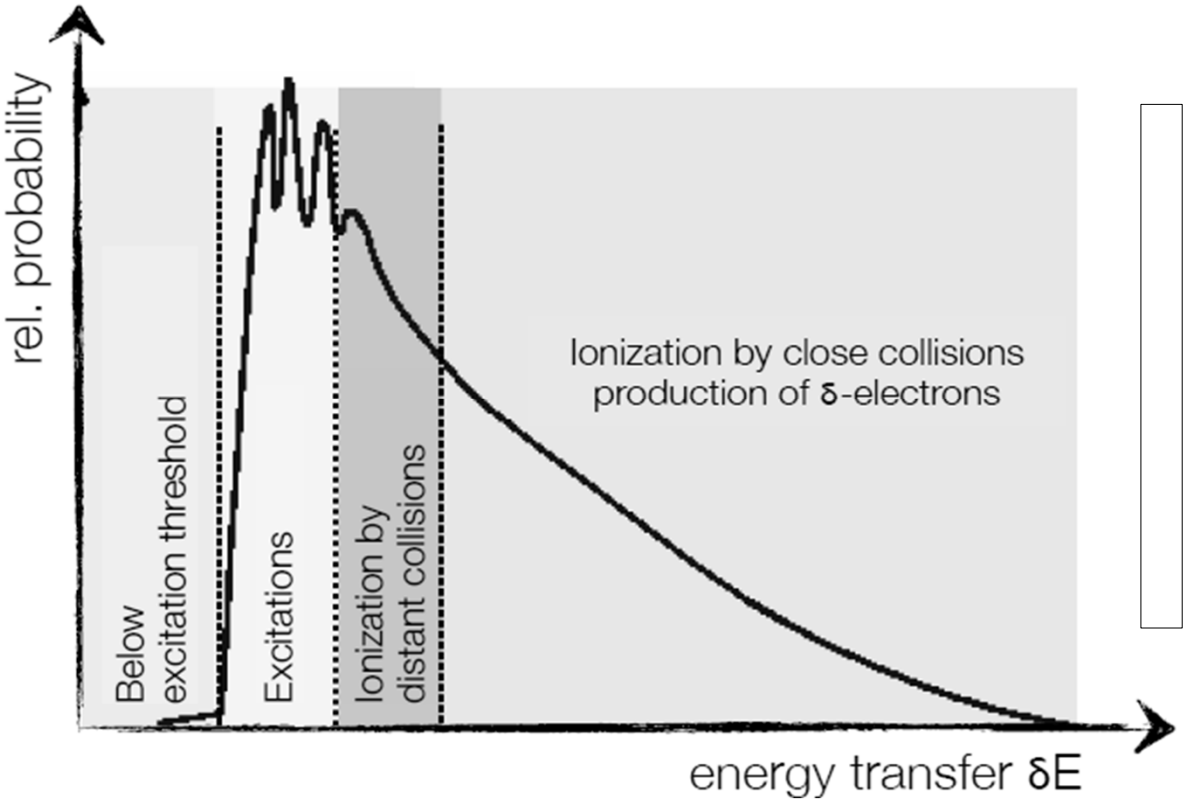
Note: different energy loss formula for electrons and positrons
at low energy as positrons are not identical with electrons.

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Statistics of energy loss

- ✓ The energy loss is a statistical process. Number of collisions and energy loss varies from particle to particle.
- ✓ The distribution is usually asymmetric. Collisions with a small energy transfer are more probable than those with a large energy transfer.
- ✓ The tail at very high energy loss values are caused by rare collisions with small impact parameters. In these collisions e^- with high energies (keV), are produced, so-called δ-electrons.
- ✓ A result of the asymmetric is that the mean energy loss is larger than the most probable energy loss.
- ✓ For thin absorber, the energy loss can be described by the Landau distribution.
- ✓ For thick absorbers, the Landau distributions goes slowly into a Gaussian distribution.

energy loss "straggling"



$$\Delta E = \sum_{n=1}^{N} \delta E_n$$: mean energy loss from Bethe-Bloch formula

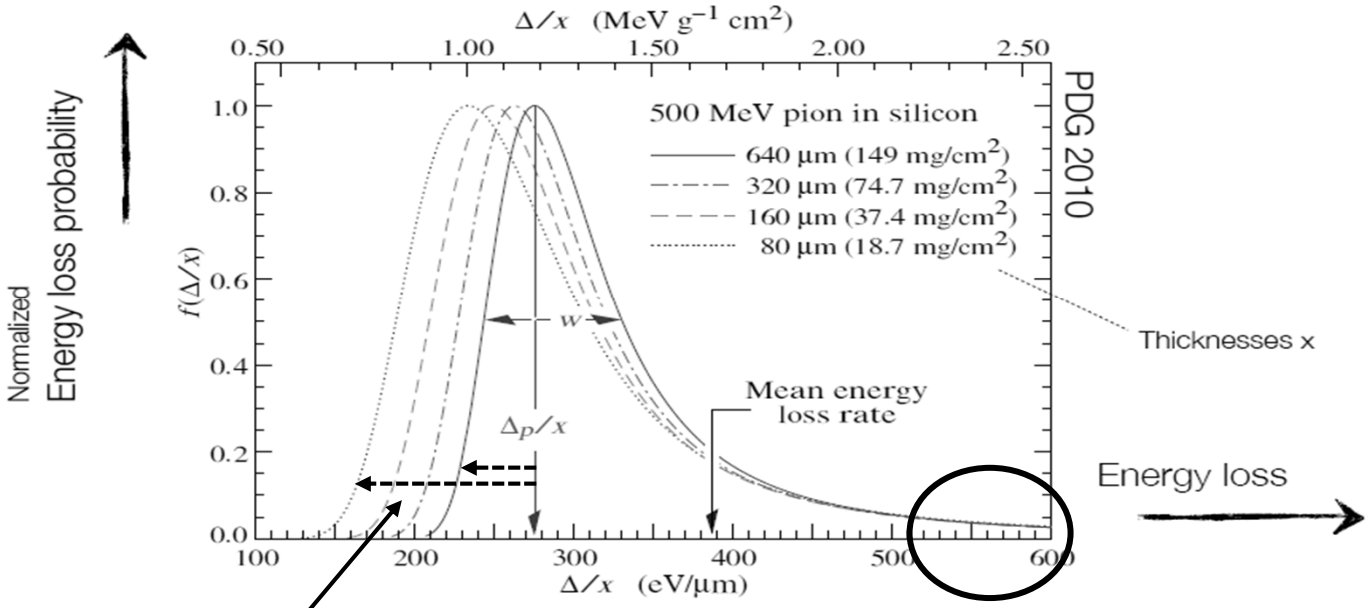
Landau distribution for thin absorber

✓ The fluctuations of energy loss by ionization of a charged particle in a thin layer of matter was theoretically described by Landau in 1944

Approximation:

$$f(\Delta/x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\Delta/x - a(\Delta/x)_{\text{mip}}}{\xi}\right) + e^{-\left(\frac{\Delta/x - a(\Delta/x)_{\text{mip}}}{\xi}\right)}\right]$$

ξ: material constant



- broader with a thinner absorber due to a greater statistical uncertainty

Limitation of Landau's theory (can not exceed its own energy)

Continuous Energy Loss (cont.)

- ✓ Charged particles can also polarize the medium in which they travel. This "density effect" is important at high energies and for dense media.
- ✓ Default density effect parameters are available from a 1982 compilation by Sternheimer, Seltzer and Berger and state-of-the-art compilations (as defined by the stopping-power guru Berger who distributes a PC-based stopping power program)

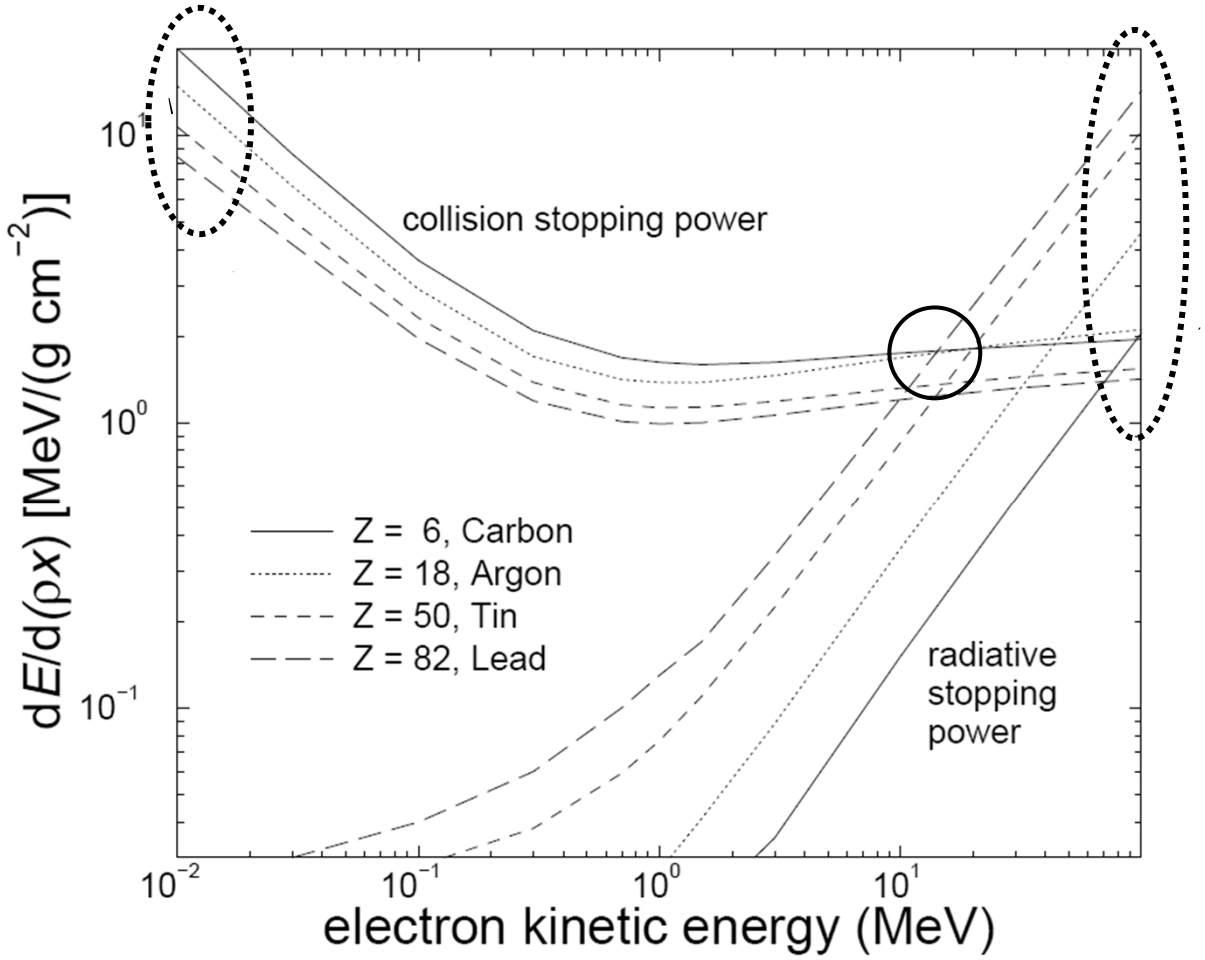
Density effect

- In condensed media, the dipole distortion of the atom near the track of the passing charged particle weakens the Coulomb force field experienced by the more distant atoms, thus decreasing the energy loss to them.

Continuous Energy Loss (cont.)

- ✓ Again, atomic binding effects are treated rather crudely by the Bethe–Bloch formalism. It assumes that each electron can be treated as if it were bound by an average binding potential.
- ✓ The use of more refined theories does not seem advantageous unless one wants to study electron transport below the K-shell binding energy of the highest atomic number element in the problem.

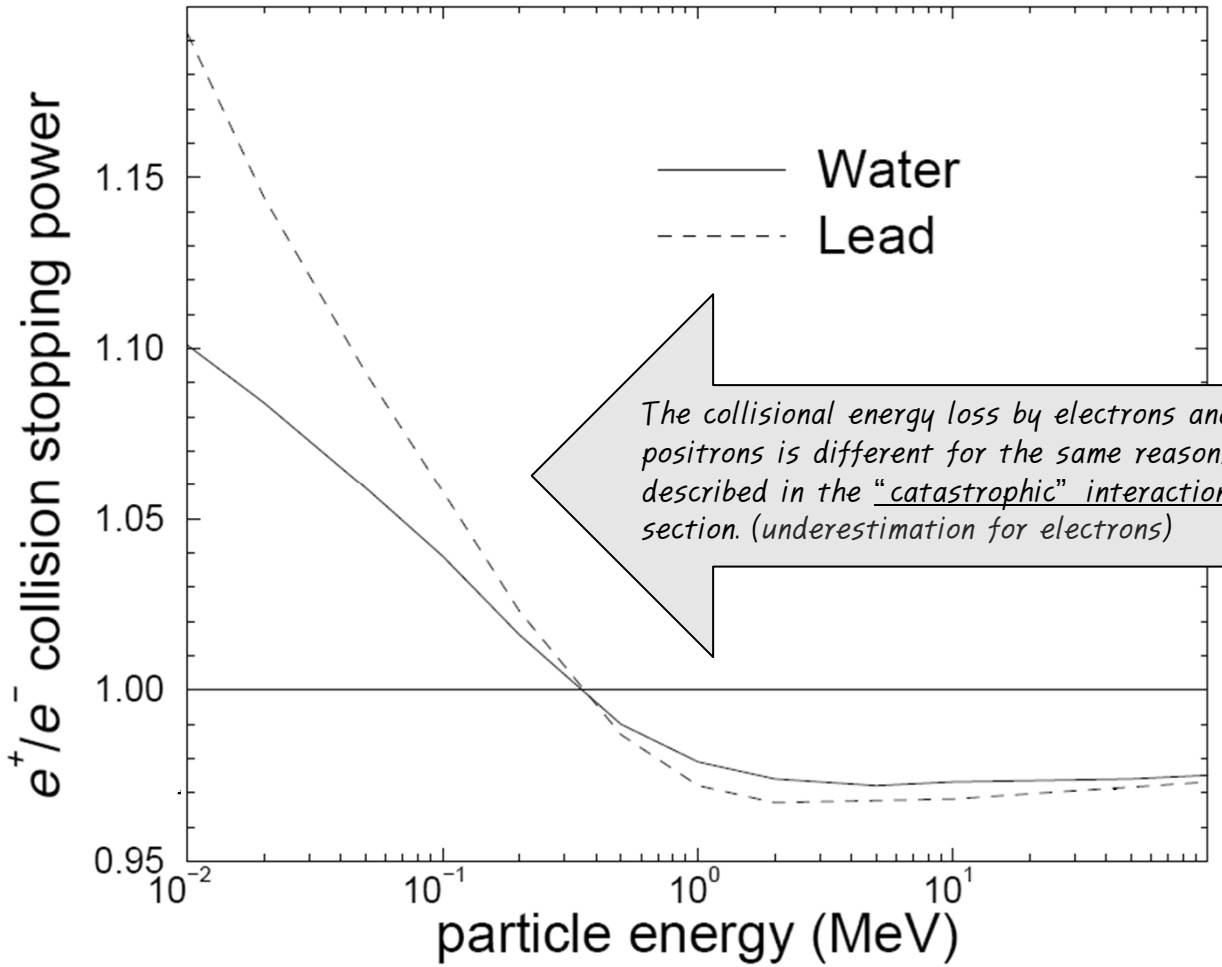
stopping power versus electron energy



Continuous Energy Loss (cont.)

- ✓ From the stopping power versus energy for different materials, the difference in the collision part is due mostly to the difference in ionisation potentials of the various atoms and partly to a Z/A difference, because the vertical scale is plotted in MeV/(g/cm^2), a normalisation by atomic weight rather than electron density.
- ✓ Note that at high energy the argon line rises above the carbon line. Argon, being a gas, is reduced less by the density effect at this energy.
- ✓ The radiative contribution reflects mostly the relative Z^2 dependence of bremsstrahlung production.

positron/electron collisional stopping power ratio



The collisional energy loss by electrons and positrons is different for the same reasons described in the "catastrophic" interaction section. (underestimation for electrons)

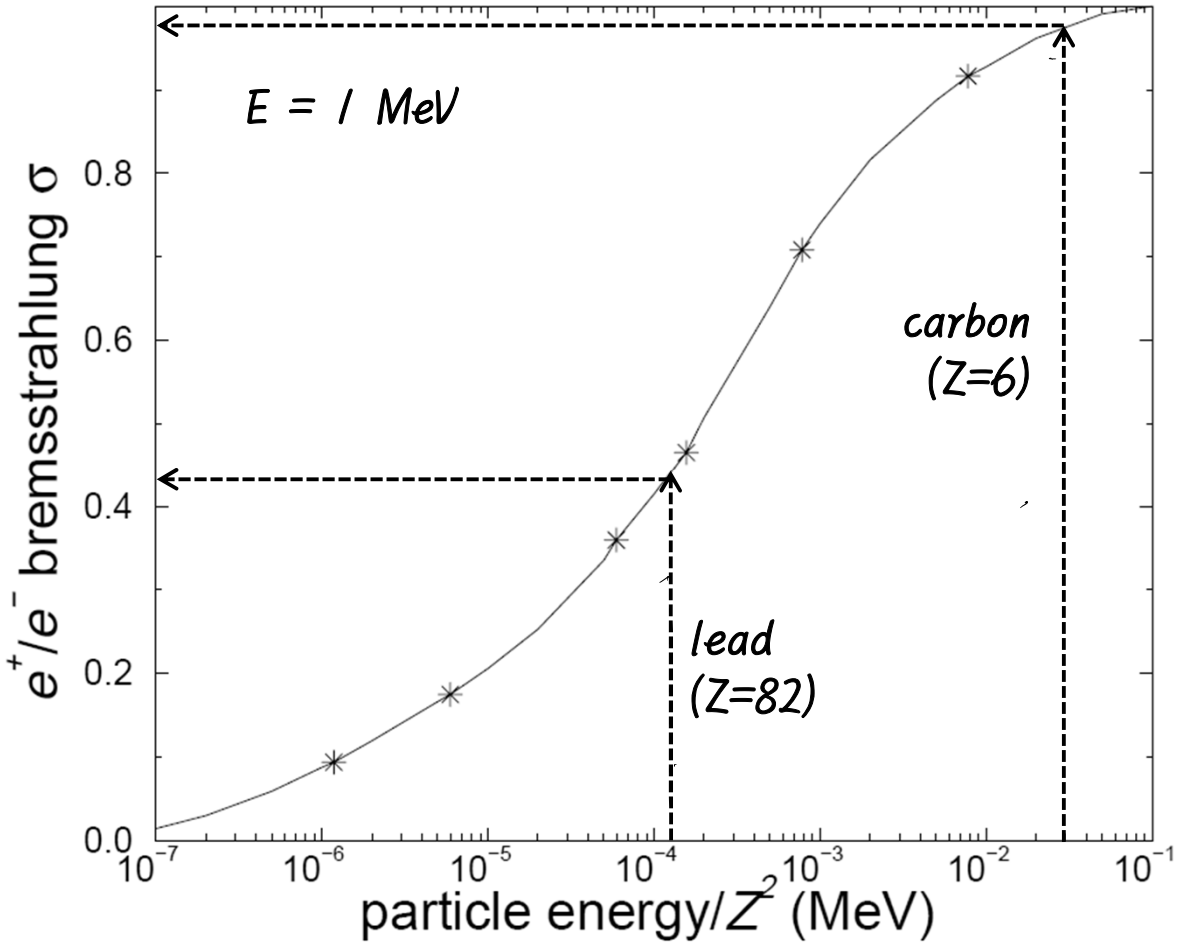
more electrons in a lead atom

energy loss by positrons via annihilation

Continuous Energy Loss (cont.)

- ✓ The collisional energy loss by electrons and positrons is different for the same reasons described in the "catastrophic" interaction section.
- ✓ Annihilation is generally not treated as part of the positron slowing down process and is treated discretely as a "catastrophic" event.
- ✓ The positron radiative stopping power is reduced with respect to the electron radiative stopping power. At 1 MeV, this difference is a few percent in carbon and 60% in lead.

positron/electron bremsstrahlung cross section ratio



bremsstrahlung energy loss

$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. proportional to $1/m^2$ → main relevance for electrons
($\alpha \sim 1/137$, fine structure constant)

bremsstrahlung energy loss of electron (by Rossi): for E >~10 MeV, below which energy loss by ionization is dominant.

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$-\frac{dE}{dx} = \frac{E}{X_0} \qquad \text{with } \rho X_0 = \frac{\rho A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm²]

vs. (collisional) mean free path

$$\Rightarrow E = E_0 e^{-x/X_0}$$

Rossi

After passage of one X_0 electron has lost all but (1/e)th of its energy

[i.e. 63%]

Material	X_0 (g/cm^2) $= \rho X_o$	X_0 (cm)
H_2O	36.1	36.1
Air (NTP)	36.2	30050
H_2	63	$7 \cdot 10^5$
C	43	18.8
Polystyrol	43.8	42.9
Fe	13.8	1.76
Pb	6.4	0.56

– C. Grupen, *Teilchendetektoren*, BI-Wissenschaftsverlag, 1993
– W.R. Leo, *Techniques for Nuclear and Particle Physics Experiments*, Springer, 1987
– K. Kleinknecht, *Detektoren für Teilchenstrahlung*, B.G. Teubner, 1992
– D.H. Perkins, *Introduction to High Energy Physics*, Addison-Wesley, 1987

positron/electron total (collisional+radiative) stopping power

$$\left(\frac{dE}{dx}\right)_{\text{Tot}} = \left(\frac{dE}{dx}\right)_{\text{Ion}} + \left(\frac{dE}{dx}\right)_{\text{Brems}}$$

critical energy:

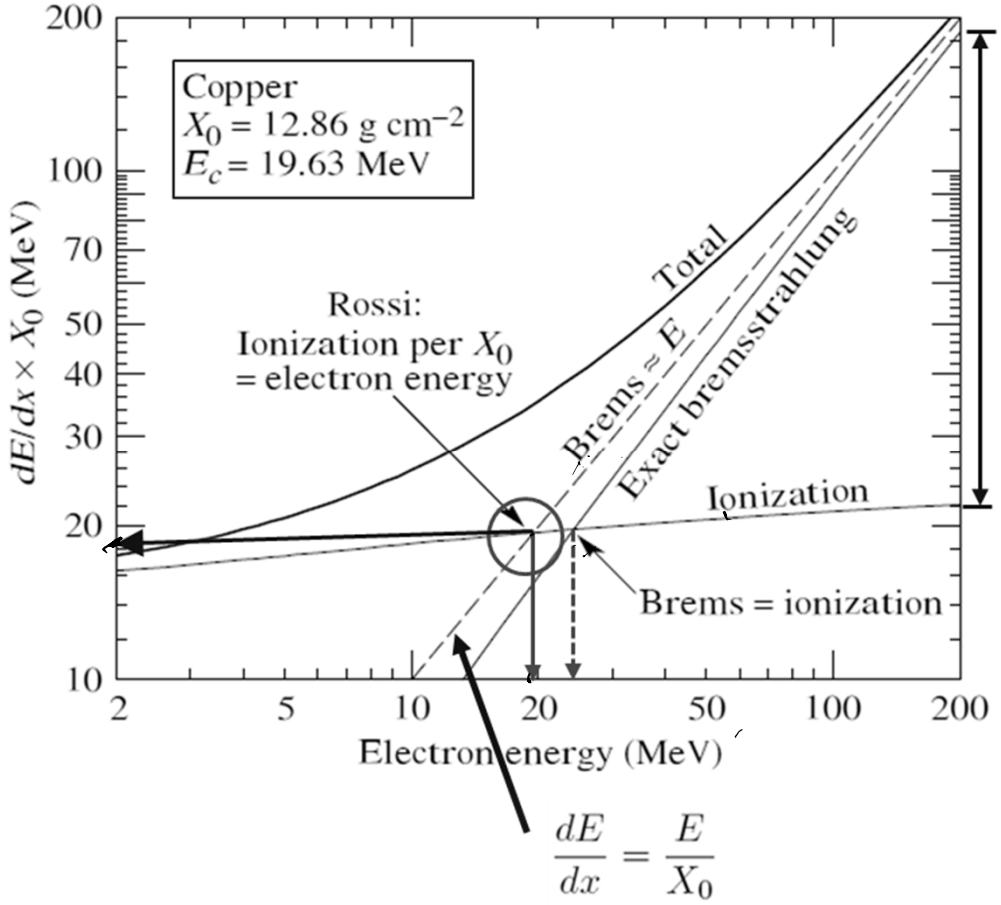
$$\left.\frac{dE}{dx}(E_c)\right|_{\text{Brems}} = \left.\frac{dE}{dx}(E_c)\right|_{\text{Ion}}$$

approximately,

$$E_c^{\text{Gas}} = \frac{710\ \text{MeV}}{Z + 0.92}$$

$$E_c^{\text{Sol/Liq}} = \frac{610\ \text{MeV}}{Z + 1.24}$$

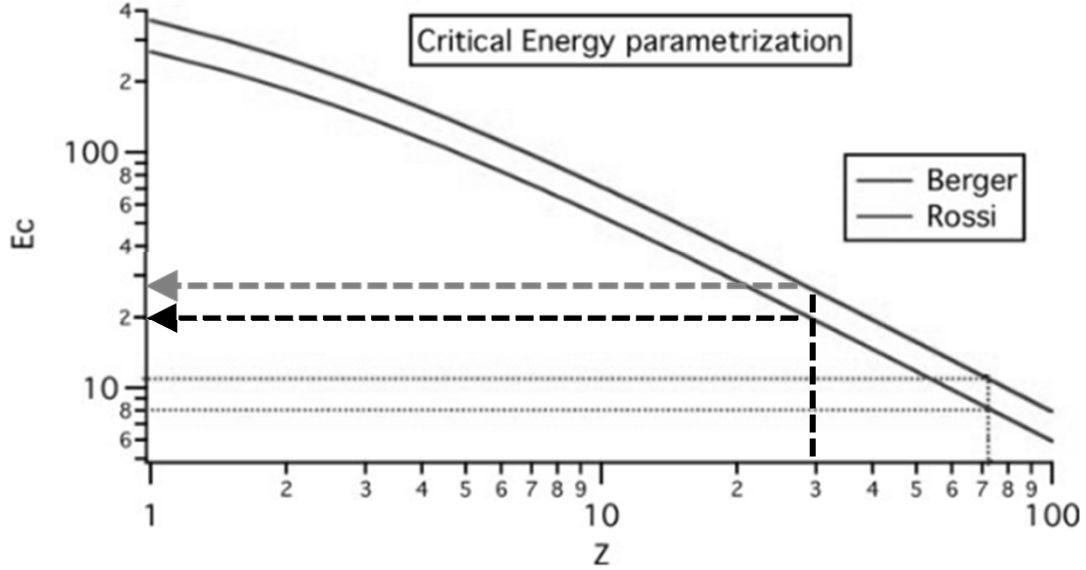
$$-\frac{dE}{dx} = \frac{E}{X_0} \quad \Rightarrow \quad -\frac{1}{E}\frac{dE}{dx} = \frac{1}{X_0}$$

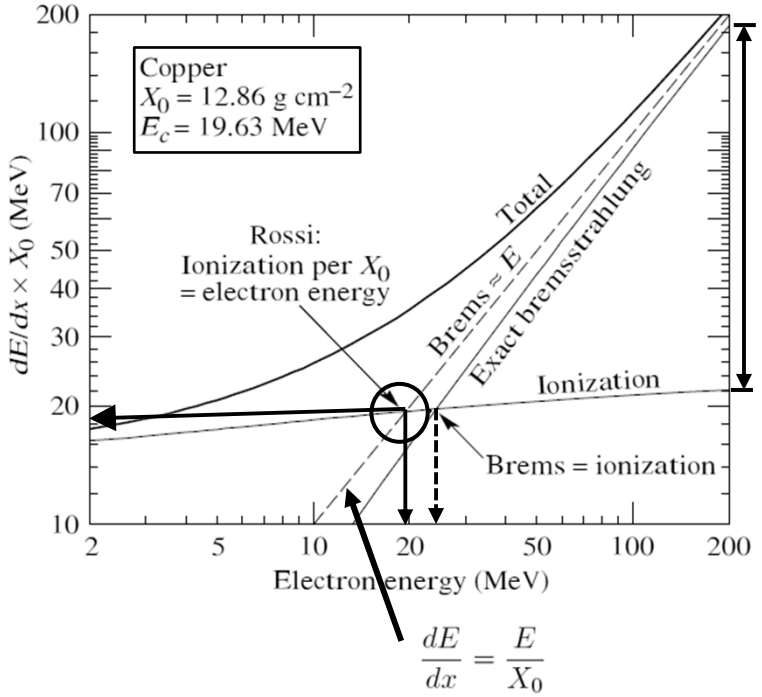


$$Berger\ amd\ Seltzer\colon\ E_C \approx \frac{800\ MeV}{(Z+1.2)}$$

$$Rossi\colon\ E_C \approx \frac{610\ MeV}{(Z+1.24)}$$

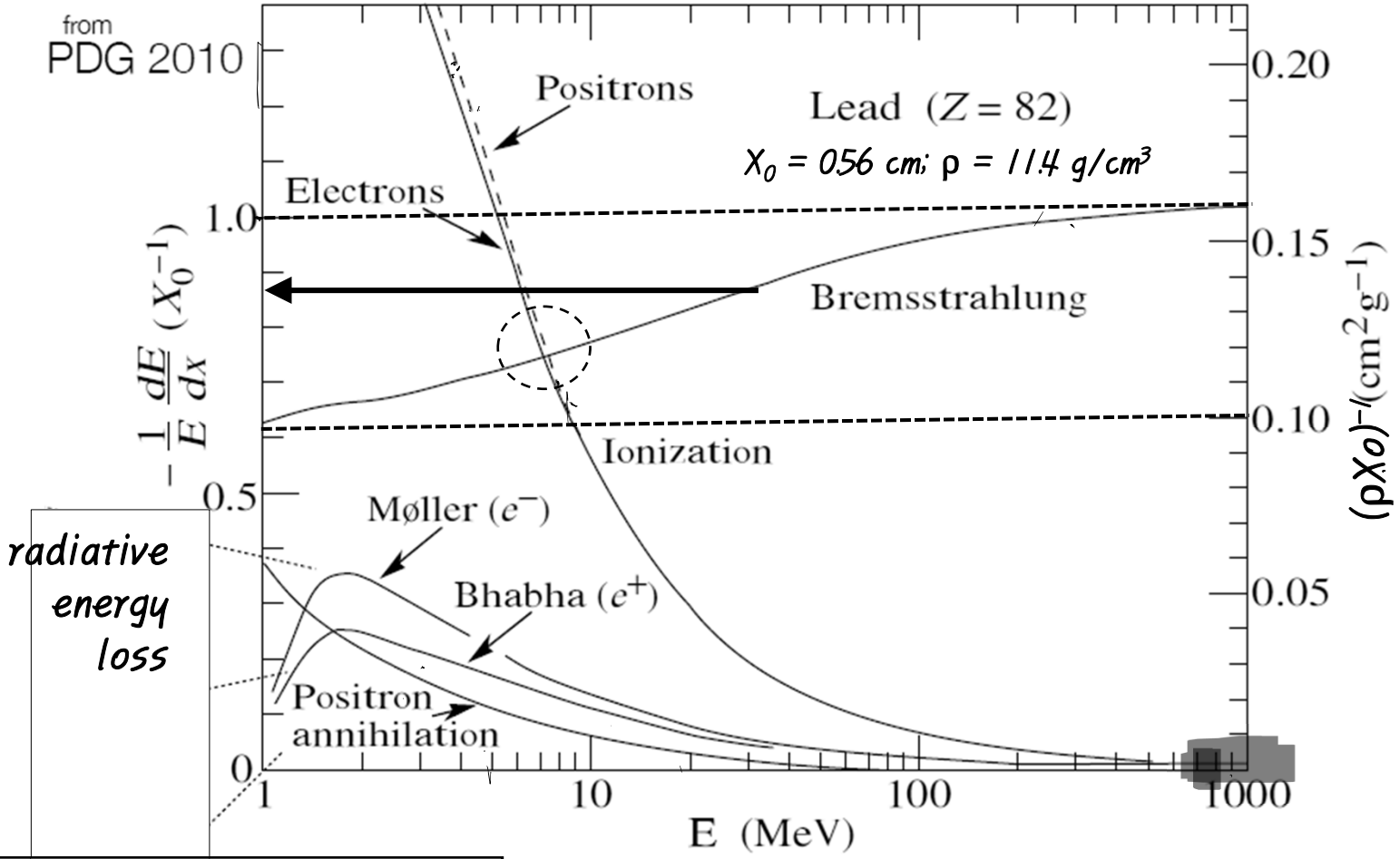
Ec: based on Rossi vs. Berger





total energy loss of electrons/positrons

fractional energy loss per radiation length in lead as a function of electron or positron energy



radiative energy loss

$$-\frac{dE}{dx} = \frac{E}{X_0} \quad \Rightarrow \quad -\frac{1}{E}\frac{dE}{dx} = \frac{1}{X_0}$$

Particle Data Group, *Review of Particle Physics*, Physics Letters B **592** (2004)

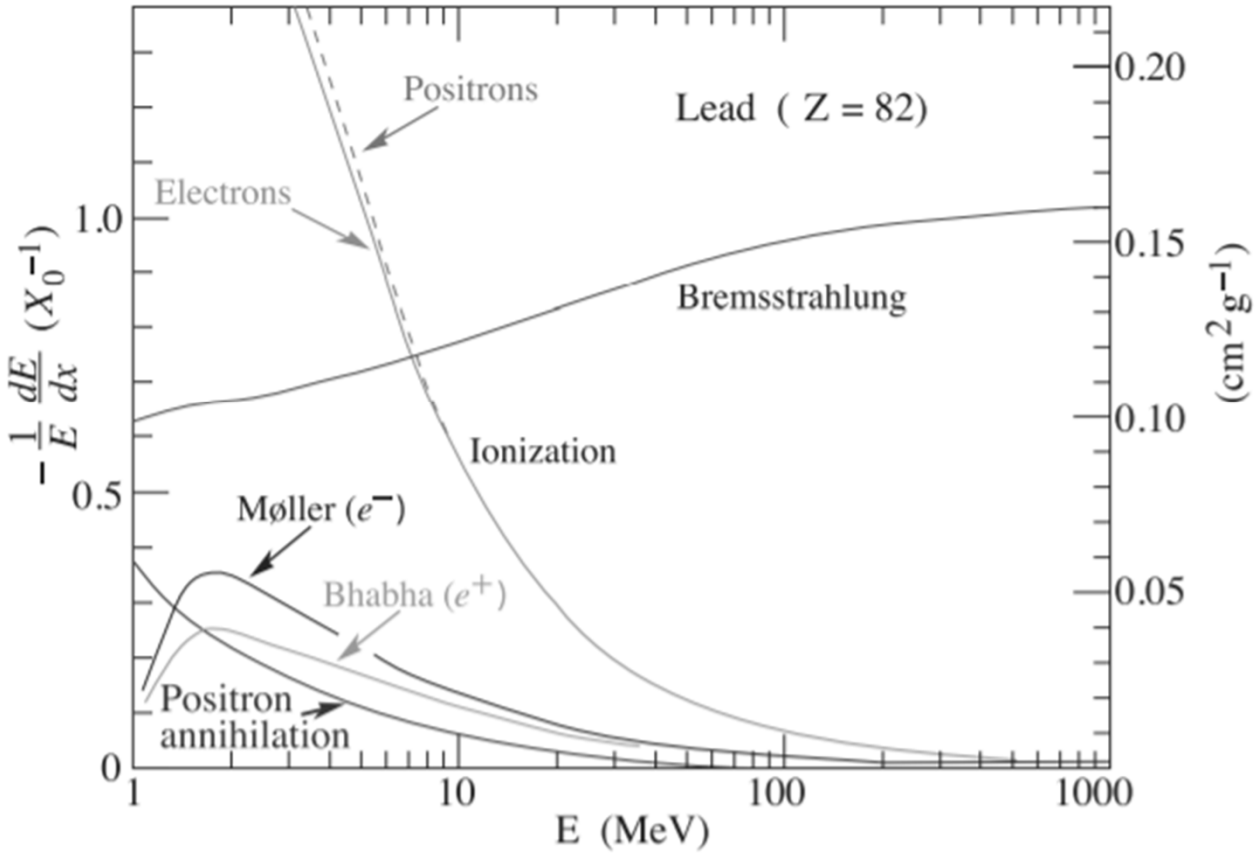


Figure 33.11: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Møller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, *Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers*, Pergamon Press, 1970. Messel and Crawford use $X_0(\mathrm{Pb}) = 5.82$ g/cm^2, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials ($X_0(\mathrm{Pb}) = 6.37$ g/cm^2).

Multiple Scattering

- ✓ Elastic scattering of electrons and positrons from nuclei is predominantly small angle with the occasional large-angle scattering event.
- ✓ If it were not for screening by the atomic electrons, the cross section would be infinite. The cross sections are, nonetheless, very large.
- ✓ It is impractical to model all individual interactions discretely. There are several statistical theories that deal with multiple scattering. Some of these theories describe these "weak" interactions by accounting for them in a cumulative sense. These are the so-called "statistically grouped" interactions.

Multiple Scattering (cont.)

- ✓ The most popular such theory is the Fermi–Eyges theory, a small angle theory. This theory neglects large angle scattering and is unsuitable for accurate electron transport unless large angle scattering is somehow included.
- ✓ The most accurate theory is that of Goudsmit and Saunderson.
- ✓ A fixed step-size scheme permits an efficient implementation of Goudsmit–Saunderson theory and this has been done in ETRAN, ITS (E&P) and MCNP.

Multiple Scattering (cont.)

- ✓ The Moli"ere theory, although originally designed as a small angle theory, has been shown with small modifications to predict large angle scattering quite successfully.
- ✓ Owing to analytic approximations made by Molie're theory, this theory requires a minimum step-size. as
- ✓ The Moli"ere theory ignores differences in the scattering of electrons and positrons, and uses the screened Rutherford cross sections instead of the more accurate Mott cross sections. However, the differences are known to be small.
- ✓ EGS4 uses the Moliere theory which produces results as good as Goudsmit-Saunderson for many applications and is much easier to implement in EGS4's transport scheme

Rutherford elastic scattering cross section

- ✓ Rutherford cross section describe the interaction probability in the Coulombic collisions b/w the ion and the point-like target nucleus.
- ✓ Rutherford cross section in LAB system is

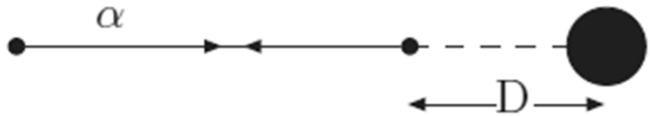
$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{16\pi\varepsilon_0 E}\right)^2 \frac{4}{\sin^4\theta} \frac{\left[\sqrt{1-\left(\frac{m_1}{m_2}\right)^2 \sin^2\theta} + \cos\theta\right]^2}{\sqrt{1-\left(\frac{m_1}{m_2}\right)^2 \sin^2\theta}}$$

where Z_1 and Z_2 are atomic numbers and m_1 and m_2 are the masses of ion and target, respectively. E is the incident laboratory energy of the ion and θ is the laboratory scattering angle.

Review

Derivation of Rutherford formula

- Step 1. head-on collision of a charged particle (+ze) with the nucleus (+Ze),



- ✓ The closest approach D is obtained by equating the initial kinetic energy T to the Coulomb energy:

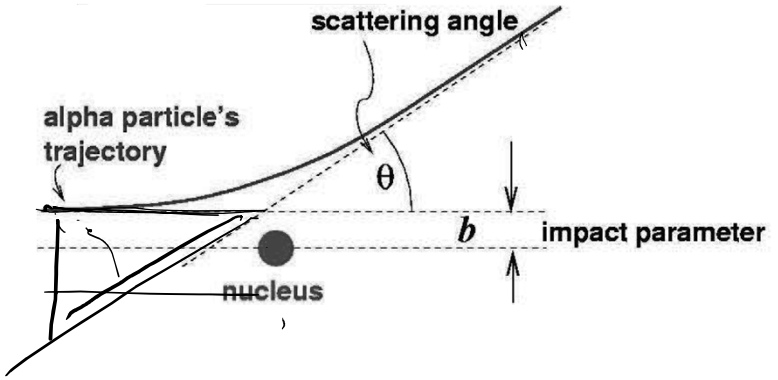
$$T = \frac{z\,Z\,e^2}{4\pi\epsilon_0 D} \quad \text{(Eq. 1)}; \qquad \boxed{D = \frac{z\,Z\,e^2}{4\pi\epsilon_0 T}} \quad \text{(Eq. 2)} \quad (m << M).$$

at which point the incident-particle (z << Z) would reverse direction, i.e. the scattering angle θ would equal π.

- ✓ With an impact parameter b, the scattering angle θ would be smaller.

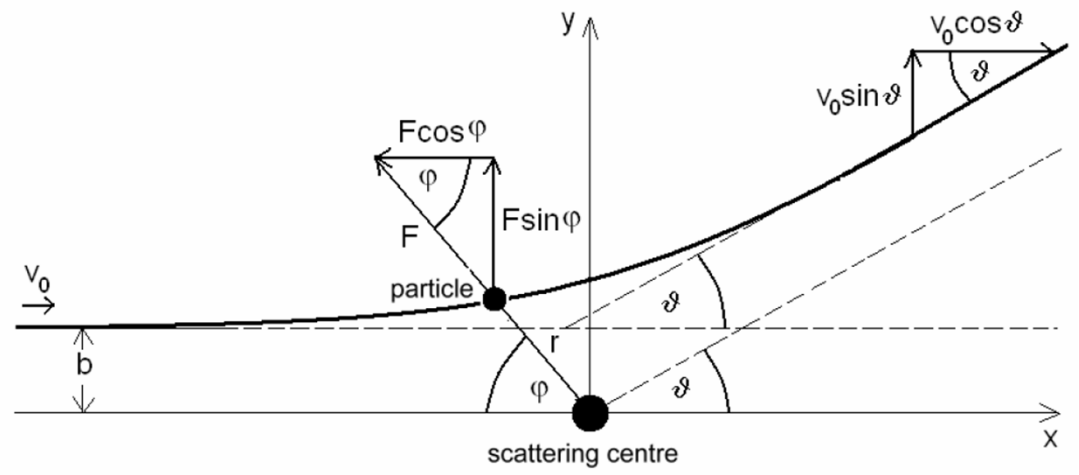
$$\tan\left(\frac{\theta}{2}\right) = \frac{D}{2b} \qquad \text{(Eq. 3)}$$

  - θ = π when b = 0.
  - As b increases the particle 'glances' the nucleus so that the scattering angle decreases.



Derivation of Rutherford formula (cont.)

- Step 2. scattering of an ion ($+Z_1e$) by a single nucleus ($+Z_2e$) under the action of Coulomb force



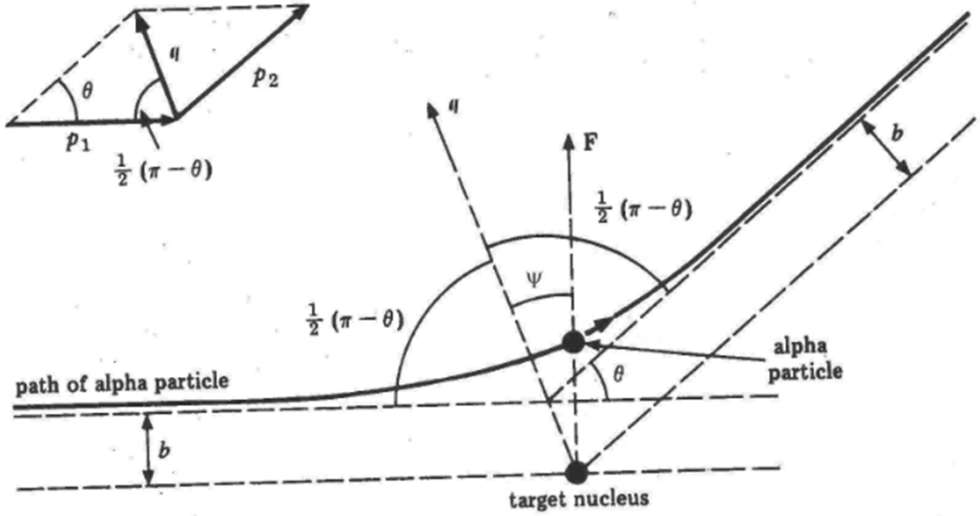
$$b(\vartheta) = \frac{zZe^2}{8\pi\varepsilon_o E_{kin}} \cot\left(\frac{\vartheta}{2}\right) \quad \text{(Eq. 4)}$$

The particle will be scattered more

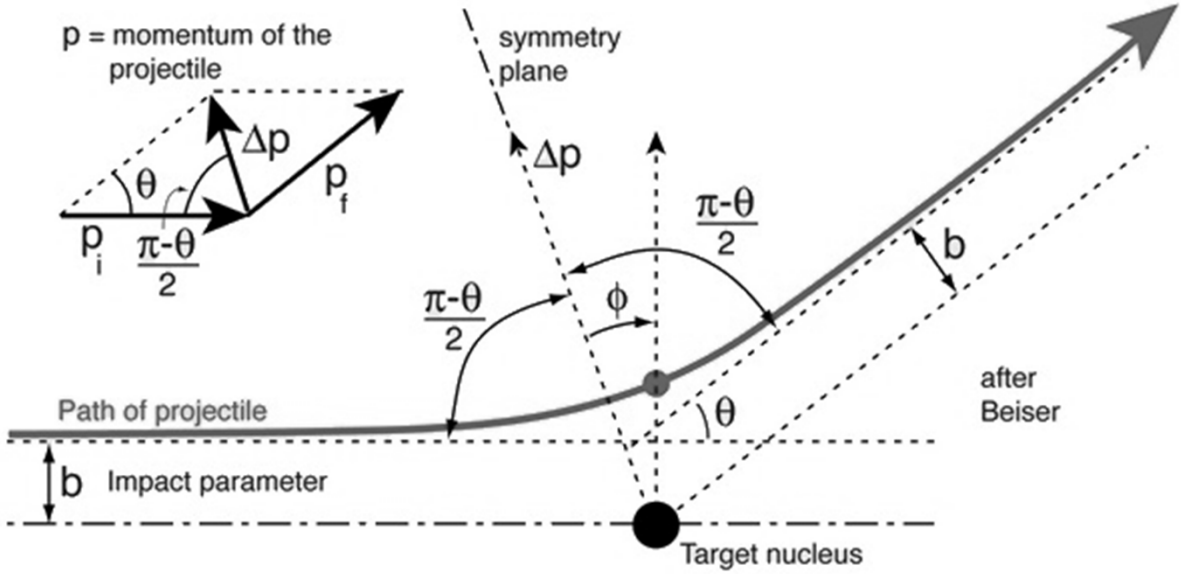
- if the impact parameter (b) is smaller (this particle is passing the scattering center in a smaller distance and thus feeling a stronger Coulomb force),
- if its velocity v_0 is smaller (the Coulomb force is acting for a longer period),
- if the mass of the particle is small,
- if the atomic number z of the particle is larger or if the atomic number Z of the scattering center is larger (due to larger Coulomb forces).

Review

Rutherford elastic scattering



Geometrical relationships in Rutherford scattering.



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Review

Screened Rutherford formula

- Coulomb scattering of a charged particle ($+Z_1e$) in a Coulomb potential

$$\phi(r) = \frac{Z_2 e}{r} e^{-r/a} \; ; \quad a \text{ (screening factor)} \rightarrow \underline{\infty \text{ without screening}}$$

given by an atomic nuclei ($+Z_2e$) under the screening of atomic electrons.

- The potential energy is then

$$V(r) = \frac{Z_1 Z_2 e^2}{r} e^{-r/a}$$

- The closest approach $D_{screened}$ is obtained by equating the initial kinetic energy T to the Coulomb potential energy:

$$\frac{d\sigma}{d\Omega} = \left| \frac{Z_1 Z_2 e^2}{\frac{\hbar^2}{2\mu a^2} + 4E \sin^2 \frac{\theta}{2}} \right|^2 ; \quad = \left| \frac{Z_1 Z_2 e^2}{4E \sin^2 \frac{\theta}{2}} \right|^2 \quad \underline{\text{without screening}}$$

where $\mu = m_\alpha \cdot M_Z/(m_\alpha + M_Z)$.

Mott cross section

- ✓ The Mott cross section formula is the mathematical description of the elastic scattering of a high energy electron beam from an atomic nucleus-sized positively charged point in space.

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E}\right) \frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}$$

  - Assumptions

  1. The energy of the electron must be such that $\beta \approx 1$ so that β^4 can safely be set to unity. (a good approximation for E> 10 MeV)
  2. The target is light nuclei as defined by $\frac{Z}{137} = \frac{Ze^2}{\hbar c} \ll 1$. (reasonable up to Calcium)

Mott cross section (cont.)

- ✓ The Mott cross section formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E}\right) \frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}$$

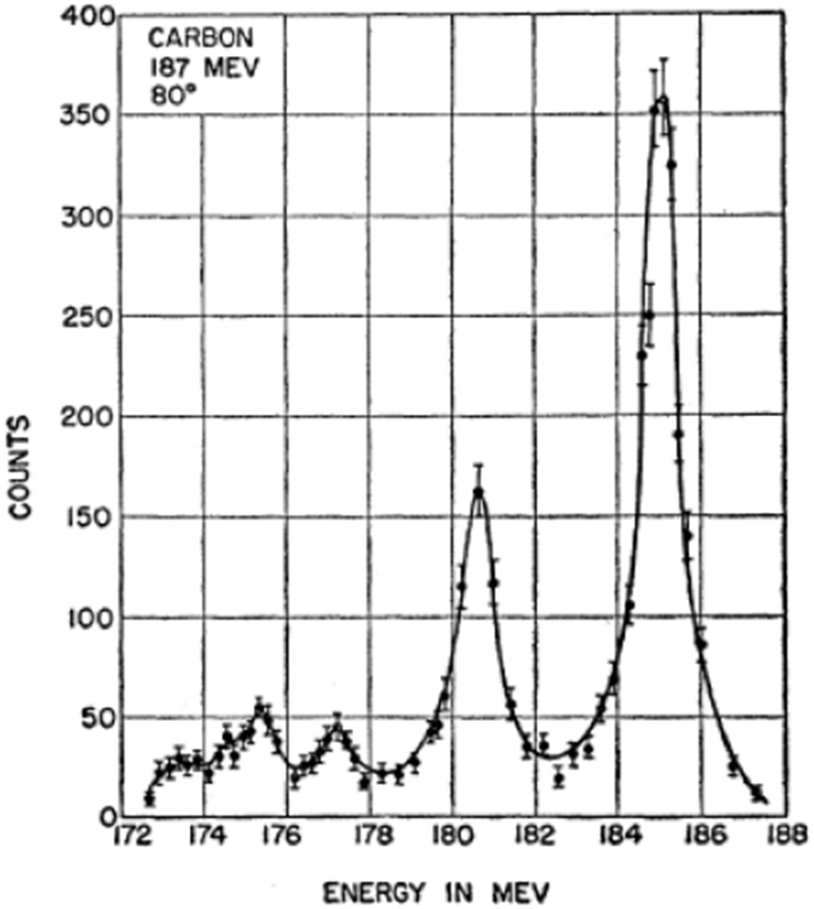
resembles the Rutherford scattering cross section.

- ✓ If an extended charge distribution (vs. a point target) is considered, it becomes

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E}\right) \frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)} |F(q)|^2$$

where F(q) is the form factor for the electron screening.

Electron Inelastic scattering and Nucleus Structure



A plot of **electron intensity vs. scattered electron energy of 187 MeV electrons scattered off a carbon-12 target**. *Note the distinct peaks with energy differences corresponding to the energies associated with excited states of the target nucleus.*

Electron Transport Mechanics:

CSDA (continuously slowing-down approximation)

1. Energy is being lost "continuously" to sub-threshold knock-on electrons and bremsstrahlung (i.e., no secondaries are created). The rate of energy loss at every point along the track is assumed to be equal to the same as the total stopping power.
2. Energy-loss fluctuations are neglected (i.e., no energy-loss straggling).

✓Charged Particles: range

- continuous slowing down approximation (CSDA) range, $\mathscr{R}_{CSDA}$

$$R_{CSDA} = \int_0^{T_0} \left(\frac{dT}{dx} \right)^{-1} dT \qquad [\mu m]$$

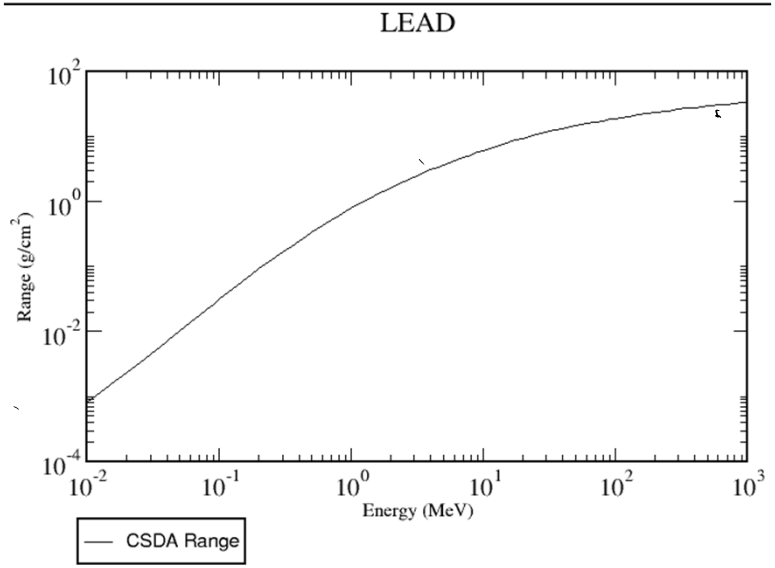
- Range is inversely proportional to the <u>density</u> of the absorbing material.

- Mass ranges in different elements are practically identical.

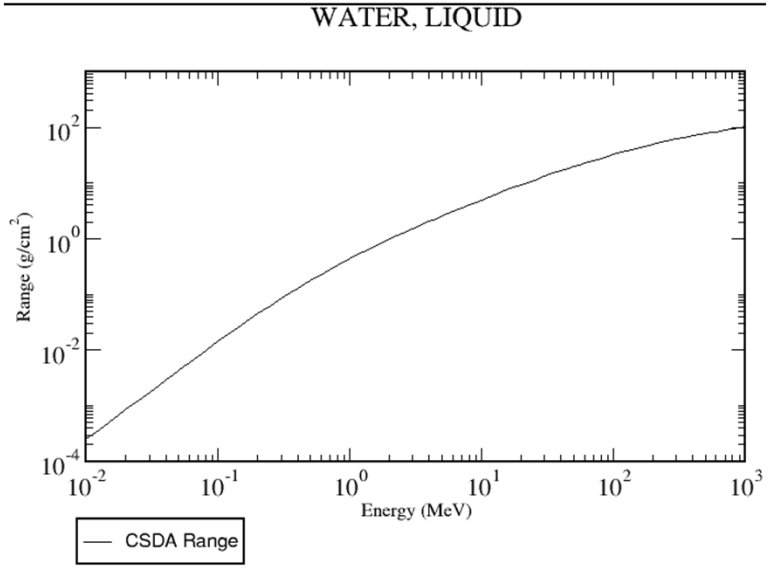
$$\rho_m R_m \propto \rho_{air} R_{air} \qquad [g/\mu m^2]$$

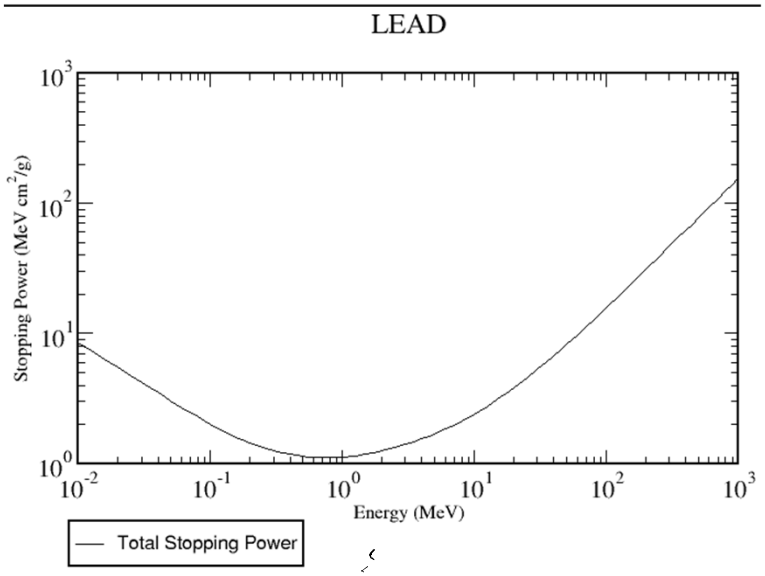
$$\text{where} \quad \rho_m R_m = \int_0^{T_0} \left(\frac{dT}{\rho_m dx} \right)^{-1} dT \qquad [g/\mu m^2]$$

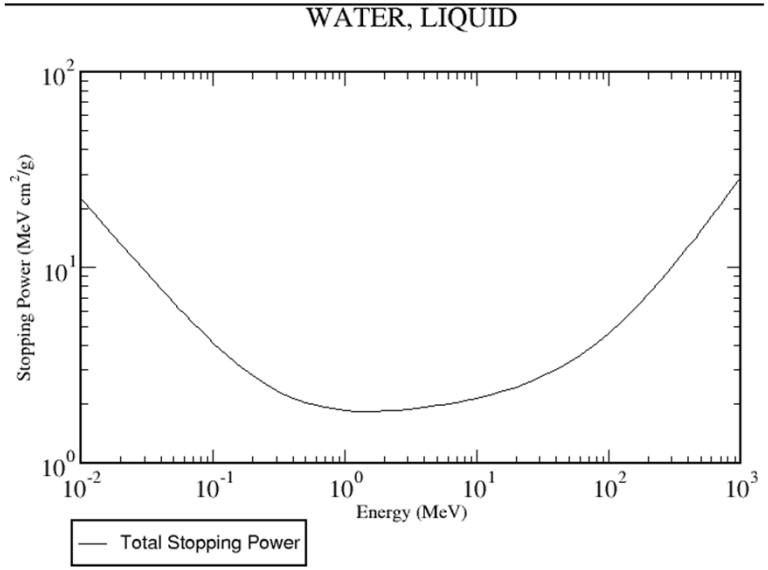
ρ_{Pb} = ***11.34 g/cm³***



ρ_{H2O} = ***1.0 g/cm³***







source: NIST Physical Measurement Laboratory

Electron Transport Mechanics:

stopping power for continuos energy loss

- ✓ In continuous slowing down approximation (CSDA), the projectile is assumed to lose energy continuously along its path and the slowing-down process is completely characterized by the (linear) stopping power S(E), which defined as the average energy loss per unit path length,

$$S(E) = -\frac{\mathrm{d}E}{\mathrm{d}s}$$

- ✓ CSDA completely neglects energy straggling, i.e., fluctuations in the energy loss due to the discreteness of the energy transfers in inelastic and radiative interactions and to the randomness of the number of these interactions.

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

- ✓ The CSDA range of an electron with kinetic energy E is given by

$$R(E) = \int_{E_{\text{abs}}}^{E} \frac{\mathrm{d}E'}{S(E')} \qquad R(E) = \int_{E}^{E_{abs}} ds(E') = \int_{E}^{E_{abs}} \frac{-dE'}{S(E')} = \int_{E_{abs}}^{E} \frac{dE'}{S(E')}$$

where E_{abs} is the 'absorption' energy, i.e., the energy at which the electron is assumed to be effectively absorbed in the medium

- ✓ If an electron starts its trajectory with kinetic energy E, the energy loss W after a path length s (would be randomly chosen) is determined by

$$s = \int_{E-\mathrm{W}}^{E} \frac{\mathrm{d}E'}{S(E')} = R(E) - R(E - W)$$

- ✓ To calculate the energy loss as a function of the path length, W(s), we only need to know the CSDA range as a function of energy, R(E).

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

- ✓ **Step 1**. Set the energy E_1 of an electron from STOCK at start
- ✓ **Step 2**. Select s_1 from $s_1 = -[\ln \xi/\Sigma_t(E_1)]$
- ✓ **Step 3**. Calculate $R_2(E_1-W_1) = R_1(E_1) - s_1$, read the energy $E_2 = E_1-W_1$ corresponding to R_2 from Table and determine W_1.
- ✓ **Step 4**. If $W_1 <$ ECUT or PCUT, then $E_{dep} = W_1$ and go to Step 6. Or If $W_1 >$ AE or AP(=PCUT), then stock the secondary particle at AE or AP and go to Step 6. (Otherwise, go to Step 5 with ECUT$<W_1<$AE)
- ✓ **Step 5**. Operate the routine of (CSDA and multiple scattering) for subsets of $t_{1,i}$ $(i = 1, 2, \ldots, W_1/\text{ESTEPE})$ → n

  $t_{1,1} + t_{1,2} + \ldots + t_{1,(W1/ESTEPE)} = s_1$; $t_{1,i} = s_1/(W_1/\text{ESTEPE})$

  *ESTEPE = the max. fractional electron energy loss per electron step

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

- ✓ **Step 6**. If $E_2 <$ ECUT, then $E_{dep} = E_2$ and go to Step 1.

  Or If $E_2 >$ AE, $E_1 = E_2$ and go to Step 2.

  Otherwise (ECUT $< E_2 >$ AE), $W_1 = E_2$. (Go to Step 7)

- ✓ **Step 7**. Operate the routine of (CSDA and multiple scattering) for subsets of $t_{1,i}$ ($i = 1, 2, \ldots, W_1/\text{ESTEPE}$)

  $t_{1,1} + t_{1,2} + \ldots + t_{1,(W1/ESTEPE)} = s_1$; $t_{1,i} = s_1/(W_1/\text{ESTEPE})$

  *ESTEPE = the max. fractional electron energy loss per electron step

- ✓ **Step 9**. Go to Step 1.

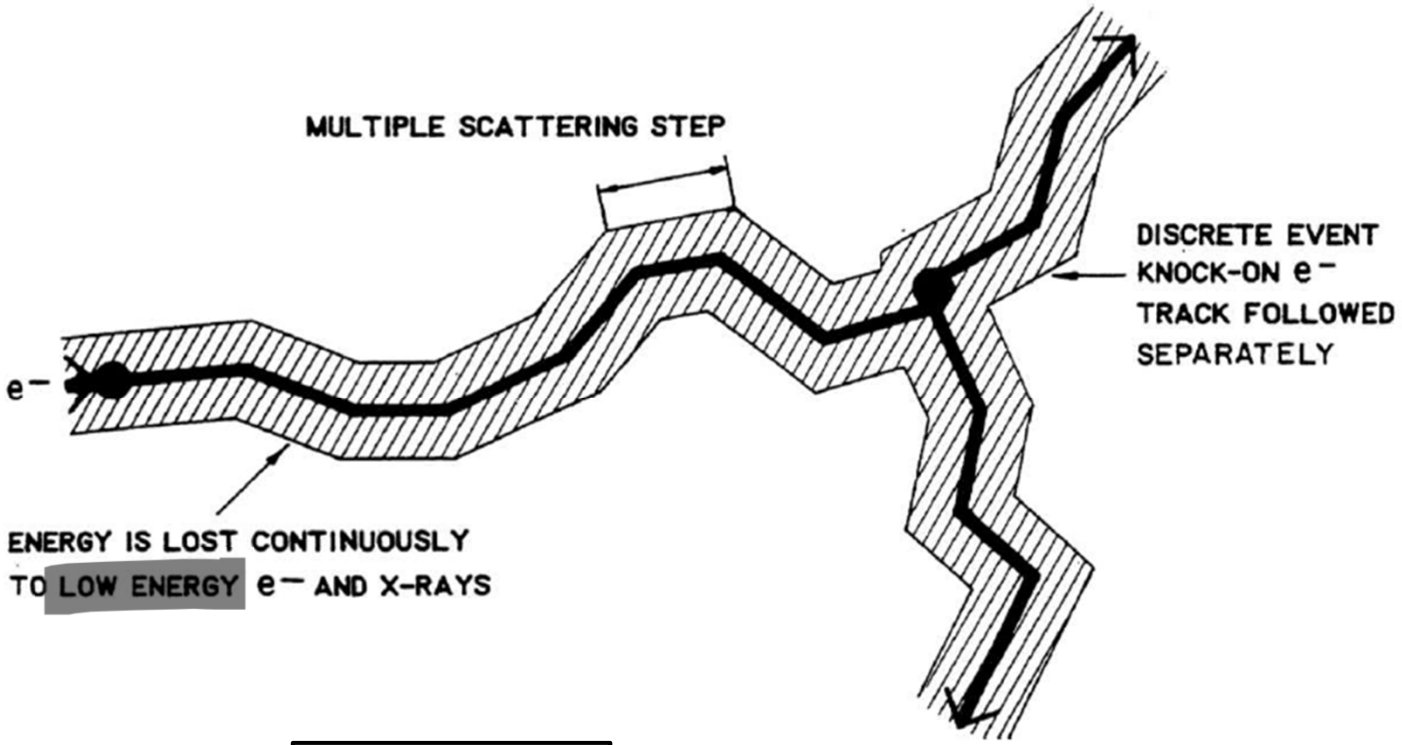


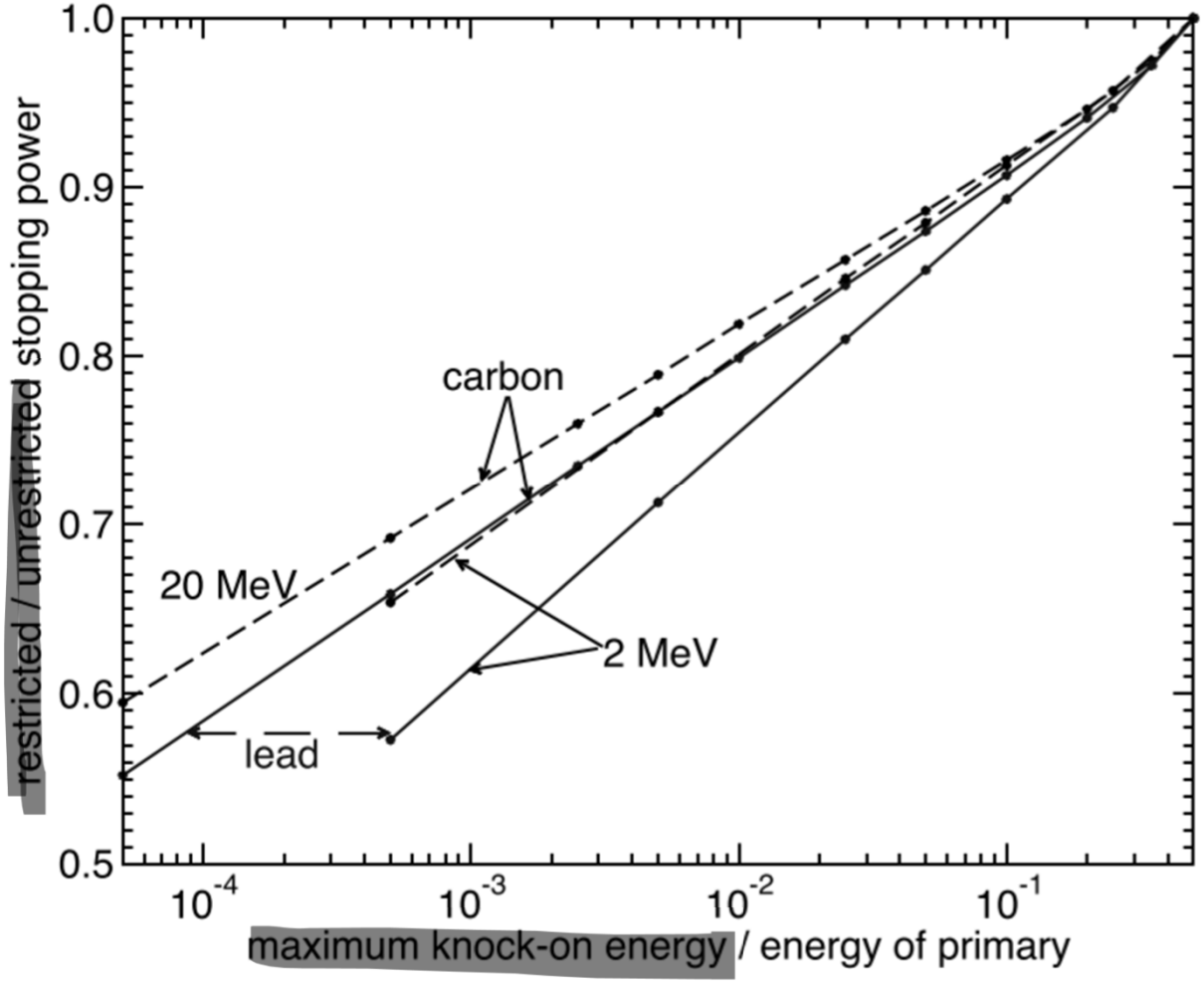
Figure 2: In a class II algorithm, the energy loss and deflection of the slowing electrons are broken into two components. The continuous energy loss is considered to occur along the straight-line path but in reality includes energy deposition throughout the shaded area by secondaries with energies less than the production thresholds AE and AP. Multiple scattering is taken into account by a deflection in each small step. Discrete interactions create knock-on electrons with energies above AE or bremsstrahlung photons with energies above AP.

source: DWO Rogers and AF Bielajew "Monte Carlo techniques of electron and photon transport for radiation dosimetry" (1990)

Table 1:
DEFINITIONS OF ENERGY CUTOFFS AND THRESHOLDS

	Electrons or positrons	Photons	Meaning
Energy cutoffs	ECUT	PCUT	Particle histories are terminated for particles with kinetic energies below these values
Production thresholds	AE	AP	The production by electrons of secondary particles with kinetic energy greater than these values is modeled explicitly

source: DWO Rogers and AF Bielajew "Monte Carlo techniques of electron and photon transport for radiation dosimetry" (1990)



source: DWO Rogers and AF Bielajew "Monte Carlo techniques of electron and photon transport for radiation dosimetry" (1990)

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

* The number of subsets t_i (i = 1, 2, ···, n) also can be determined by selecting the n according to the Poisson distribution: $t_i = s/n$.

  - The probability distribution for a number n (≥ 0) of collisions in a path length s is approximated to Poisson distribution:

$$P(n) = \exp(-s/\lambda)\frac{(s/\lambda)^n}{n!}$$ with the mean $\langle n \rangle = s/\lambda$.

where $\lambda(W) = 1/N\sigma_{elas}(W)$ is the mean free path b/w elastic collisions and $\sigma_{elas}(W)$ is the elastic scattering cross section of the secondary electron of energy W.

Electron Transport Mechanics:

typical electron tracks

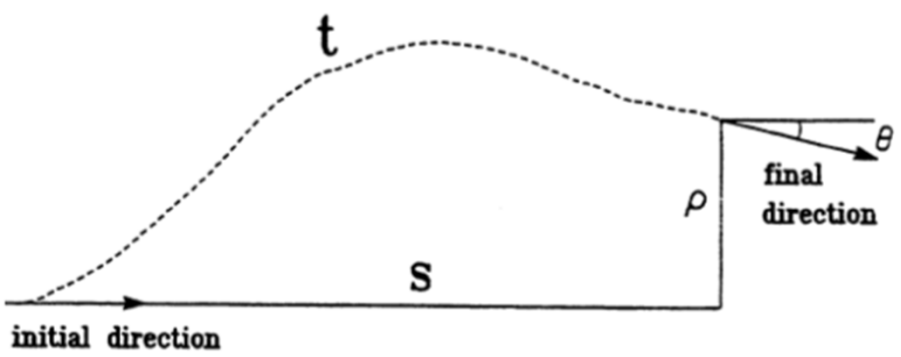
- ✓ An electron is being transported through a medium. Along the way energy is being lost "continuously" to sub-threshold knock-on electrons and bremsstrahlung.
- ✓ The track is broken up into small straight-line segments called multiple scattering substeps. In this case the length of these substeps was chosen so that the electron lost a selected fraction (ex. ESTEPE=0.01 in EGS4) of its subthreshold energy loss (W) during each substep.
- ✓ At the end of each of these substeps the multiple scattering angle is selected according to some theoretical distribution.
- ✓ Catastrophic events, each producing a single knock-on electron (or a hard bremsstrahlung), set other particles in motion. These particles are followed separately in the same fashion. The original particle, if it does not fall below the transport threshold (AE or AP), is also transported.

Electron Transport Mechanics:

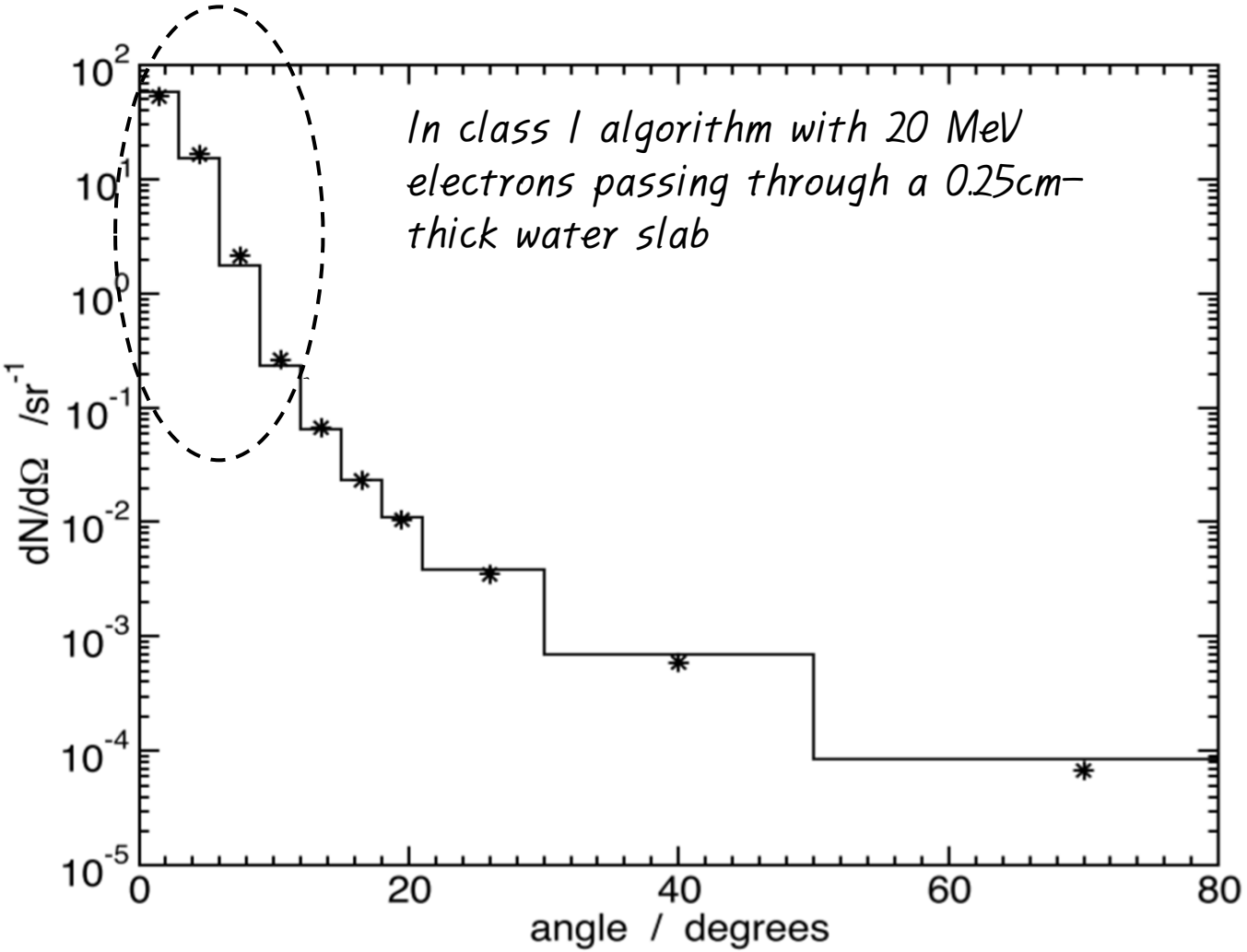
typical multiple scattering substeps

- ✓ A single electron substep is characterized by the length of total curved path-length to the end point of the substep, **t**. (This is a reasonable parameter to use because the number of atoms encountered along the way should be proportional to **t**.)
- ✓ At the end of all the substeps, the deflection from the initial direction, Θ, is sampled. Associated with the substeps is the average projected distance along the original direction of motion, **s**.
- ✓ The lateral deflection, ρ, the distance transported perpendicular to the original direction of motion, is often ignored by electron Monte Carlo codes. Such lateral deflections do occur as a result of multiple scattering.
- ✓ It is only the lateral deflection during the course of a subset which is ignored. One can guess that if the multiple scattering substeps are small enough, the electron track may be simulated more exactly.

typical multiple scattering substeps



$$\varsigma = \sum_{\tilde{n}} t_{\tilde{n}}$$



source: DWO Rogers and AF Bielajew "Monte Carlo techniques of electron and photon transport for radiation dosimetry" (1990)

Review

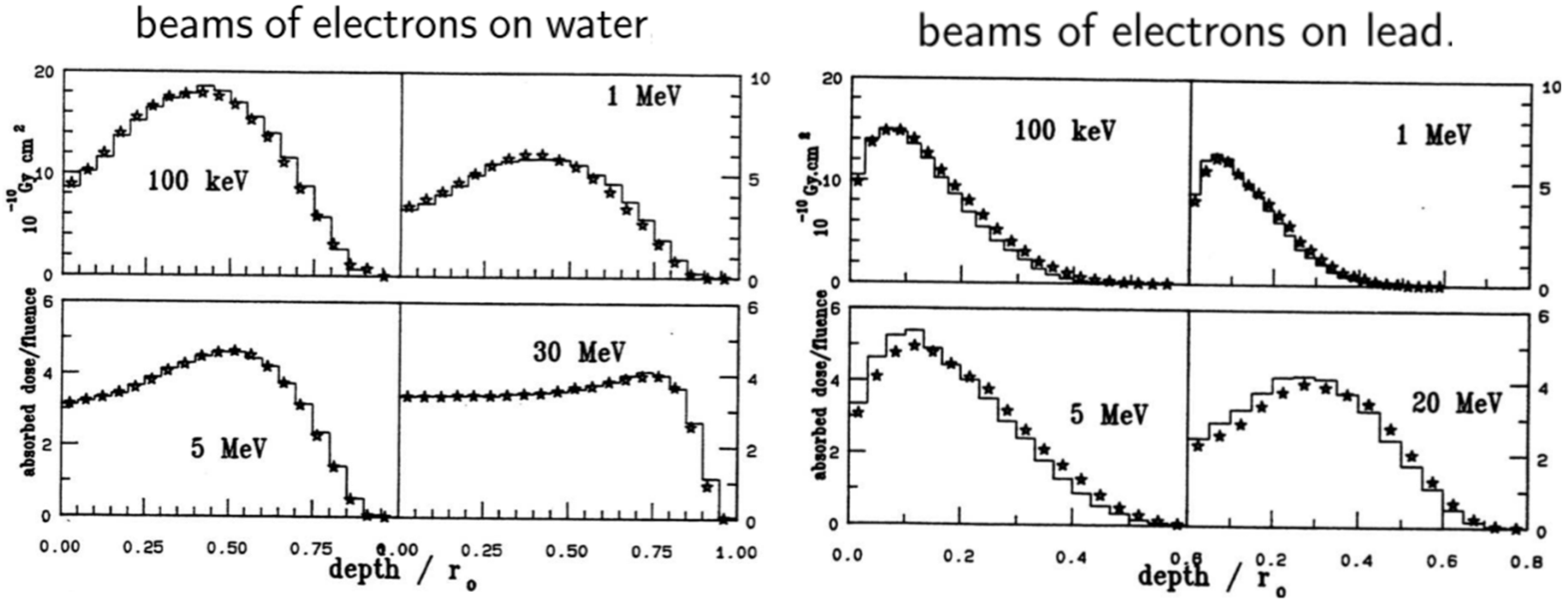
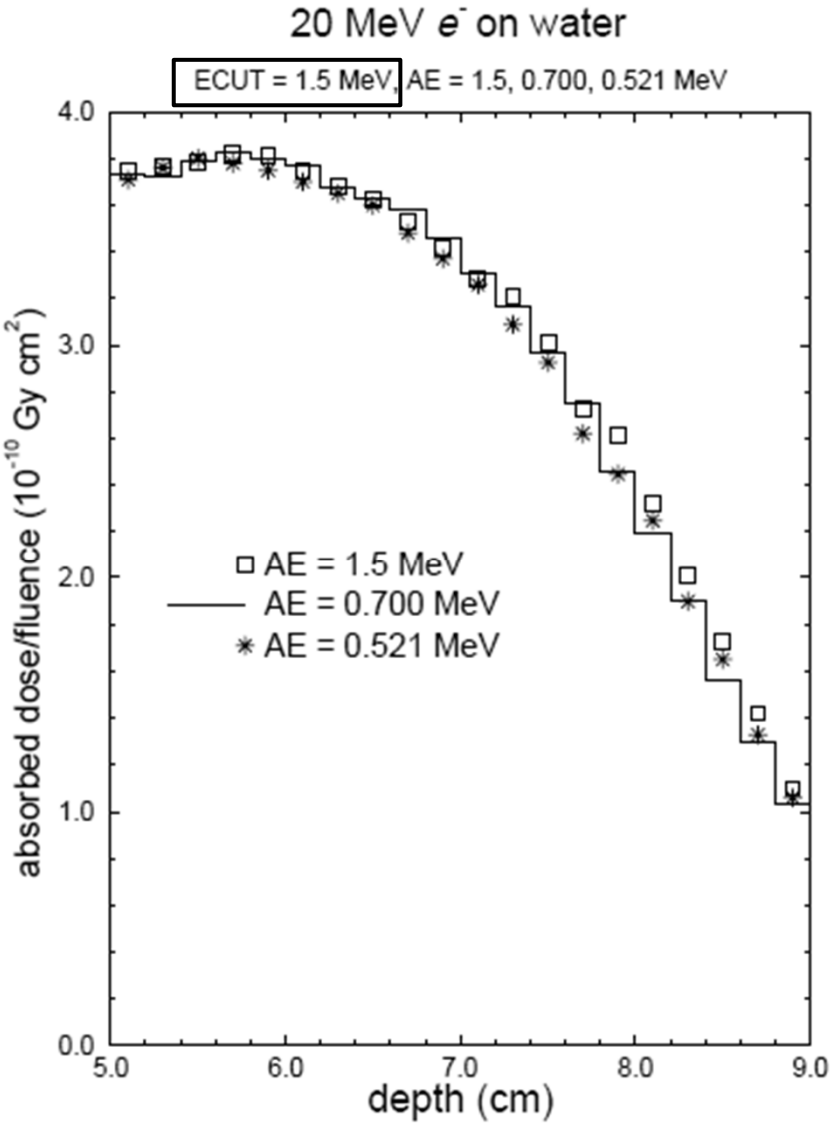


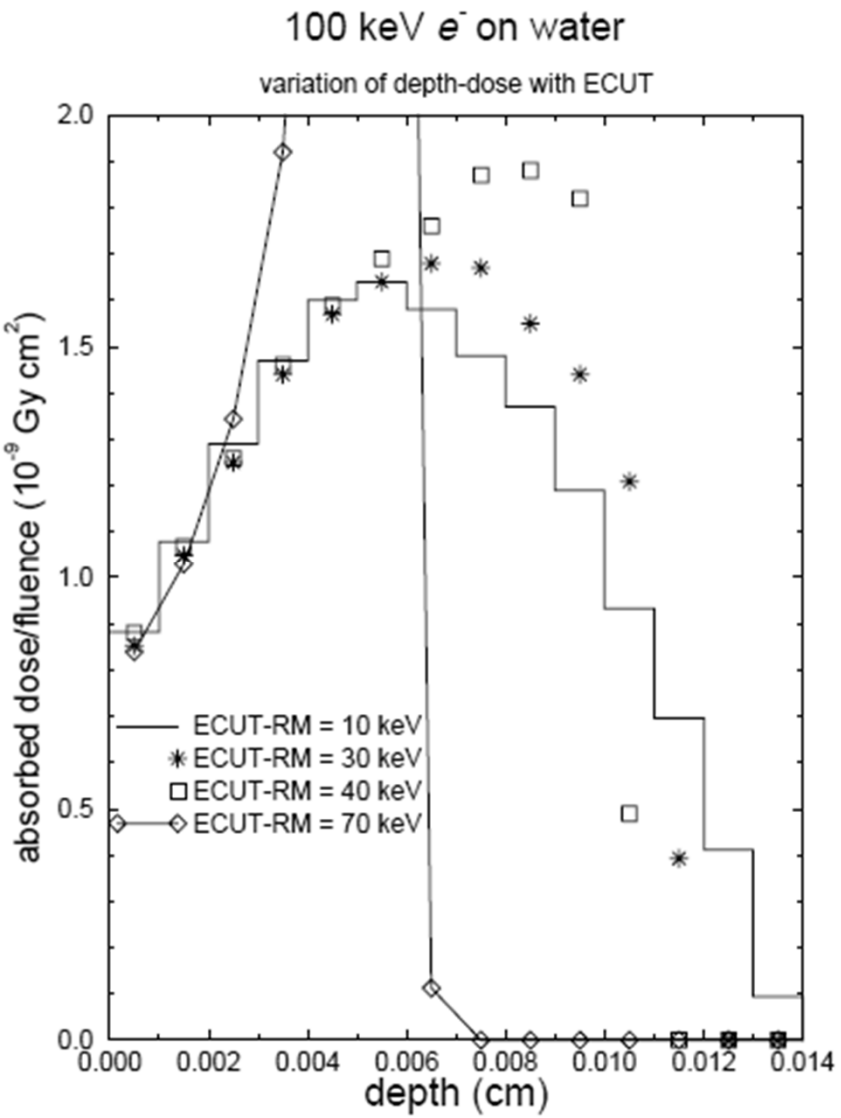
Figure 39: A comparison, at various energies, of the depth-dose curves for broad parallel beams of electrons on water, calculated using the Goudsmit-Saunderson multiple-scattering formalism in ETRAN (histogram) and the Moliere multiple-scattering formalism in EGS4 (stars). In both cases, the calculations were done in the CSDA approximation without secondary particles or energy-loss straggling. The depth axis is in terms of the CSDA range, r_0. The EGS4 calculations used the PRESTA algorithm (Bielajew and Rogers, 1986a, 1987).

source: DWO Rogers and AF Bielajew "Monte Carlo techniques of electron and photon transport for radiation dosimetry" (1990)

Examples of Electron Transport: Parameter selection

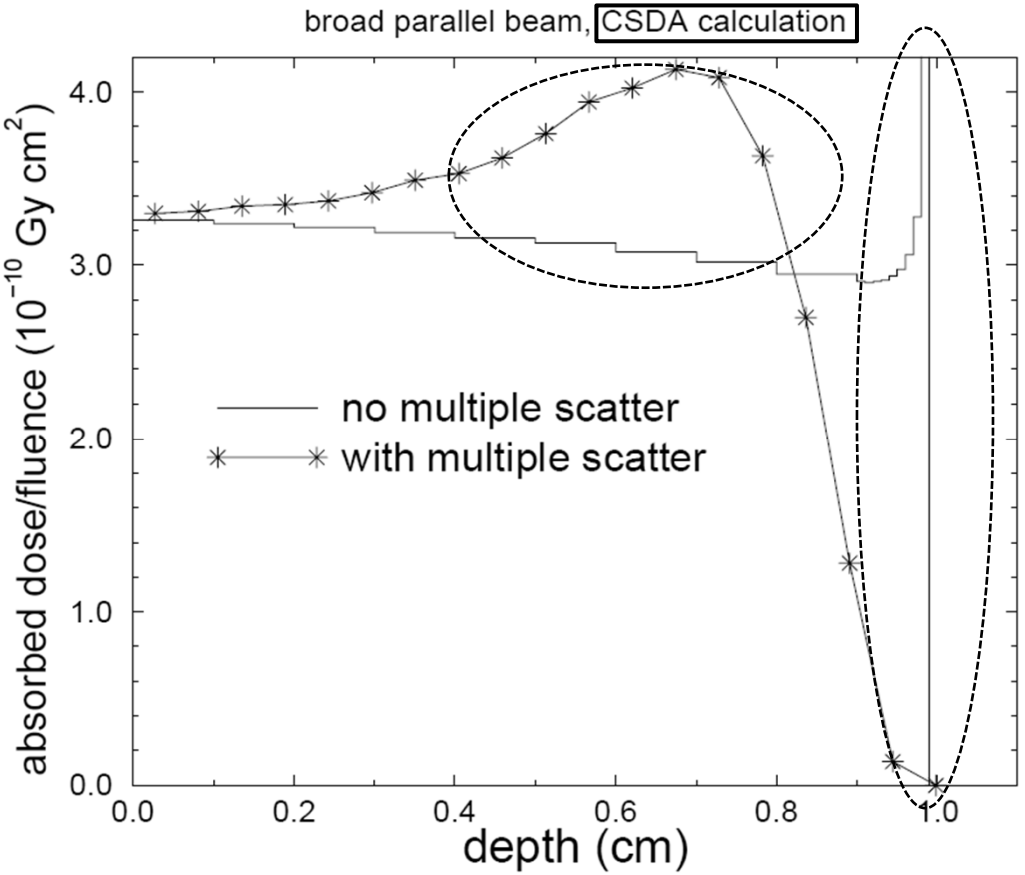


AE: secondary electron threshold

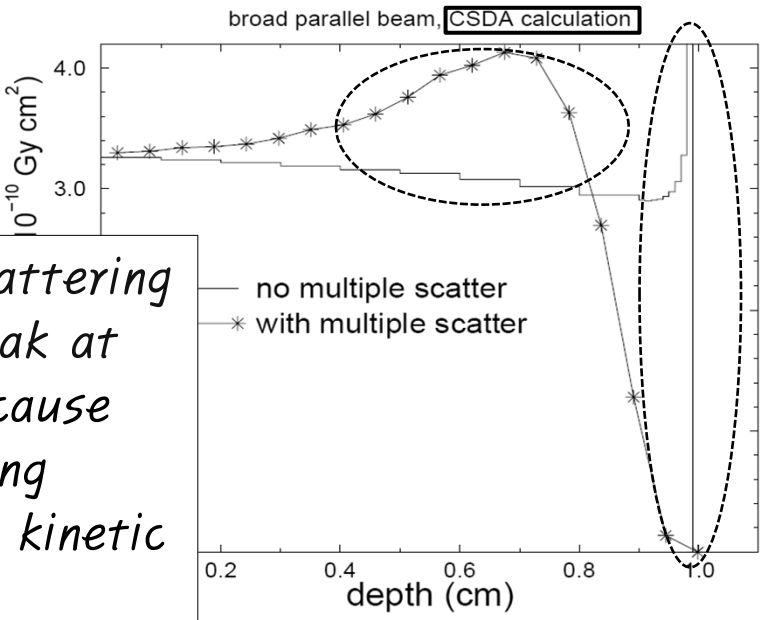


ECUT: electron transport cutoff

Examples of Electron Transport: w/ vs. w/o multiple scatterings

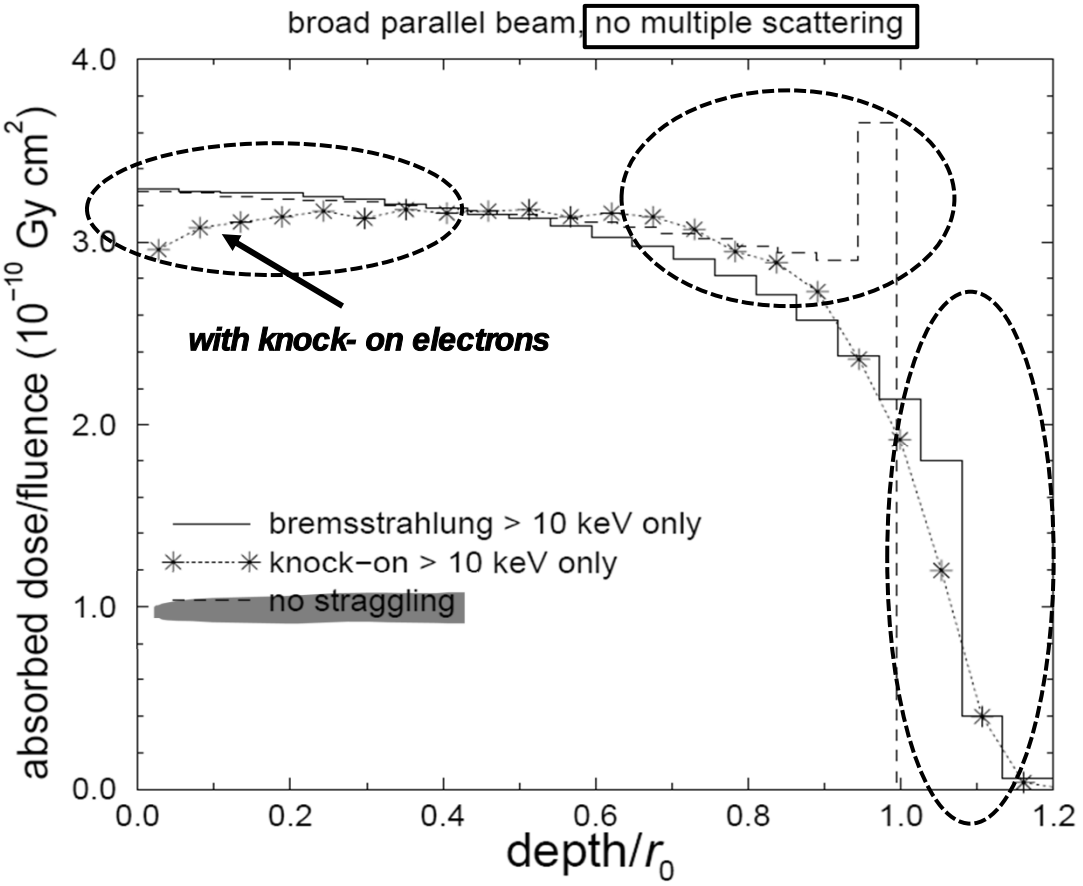


- Depth-dose curve for a broad parallel beam (BPB) of 20 MeV electrons incident on a water slab. The histogram represents a CSDA calculation in which multiple scattering has been turned off, and the stars show a CSDA calculation which includes multiple scattering.



- ✓ For the dashed histogram, no multiple scattering is modeled and hence there is a large peak at the end of the range of the particles because they all reach the same depth before being terminating and depositing their residual kinetic energy (189 keV in this case).
- ✓ The curve with the stars includes the effect of multiple scattering. This leads to a lateral spreading of the electrons which shortens the depth of penetration of most electrons and increases the dose at shallower depths because the fluence has increased.
- ✓ In this case, the depth-straggling is entirely caused by the lateral scattering since every electron has traveled the same distance.

Examples of Electron Transport: w/o multiple scatterings



- ✓ Depth-dose curves for a BPB of 20 MeV electrons incident on a water slab, but with multiple scattering turned off. The dashed histogram calculation models no straggling.
- ✓ Note the difference caused by the different bin size. The solid histogram includes energy-loss straggling due to the creation of bremsstrahlung photons with an energy above 10 keV. The curve denoted by the stars includes only that energy-loss straggling induced by the creation of knock-on electrons with an energy above 10 keV.

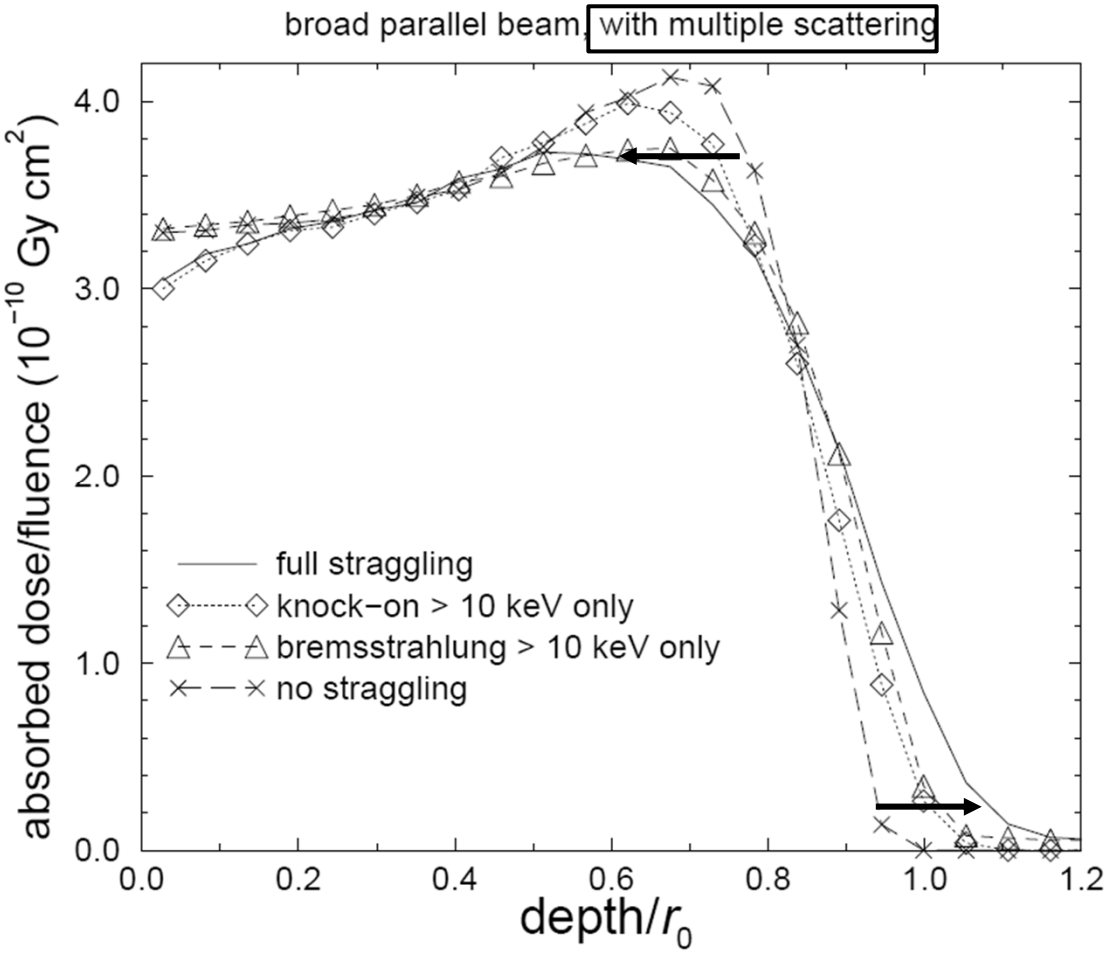
- ✓ Three depth-dose curves calculated with all multiple scattering turned off – i.e., the electrons travel in straight lines (except for some minor deflections when secondary electrons are created).
- ✓ In the cases including energy-loss straggling, a depth straggling is introduced because the actual distance traveled by the electrons varies, depending on how much energy they give up to secondaries. Two features are worth noting.

  1. Firstly, the energy-loss straggling induced by the creation of bremsstrahlung photons plays a significant role despite the fact that far fewer secondary photons are produced than electrons. They do, however, have a larger mean energy.

  2. Secondly, the inclusion of secondary electron transport in the calculation leads to a dose buildup region near the surface.

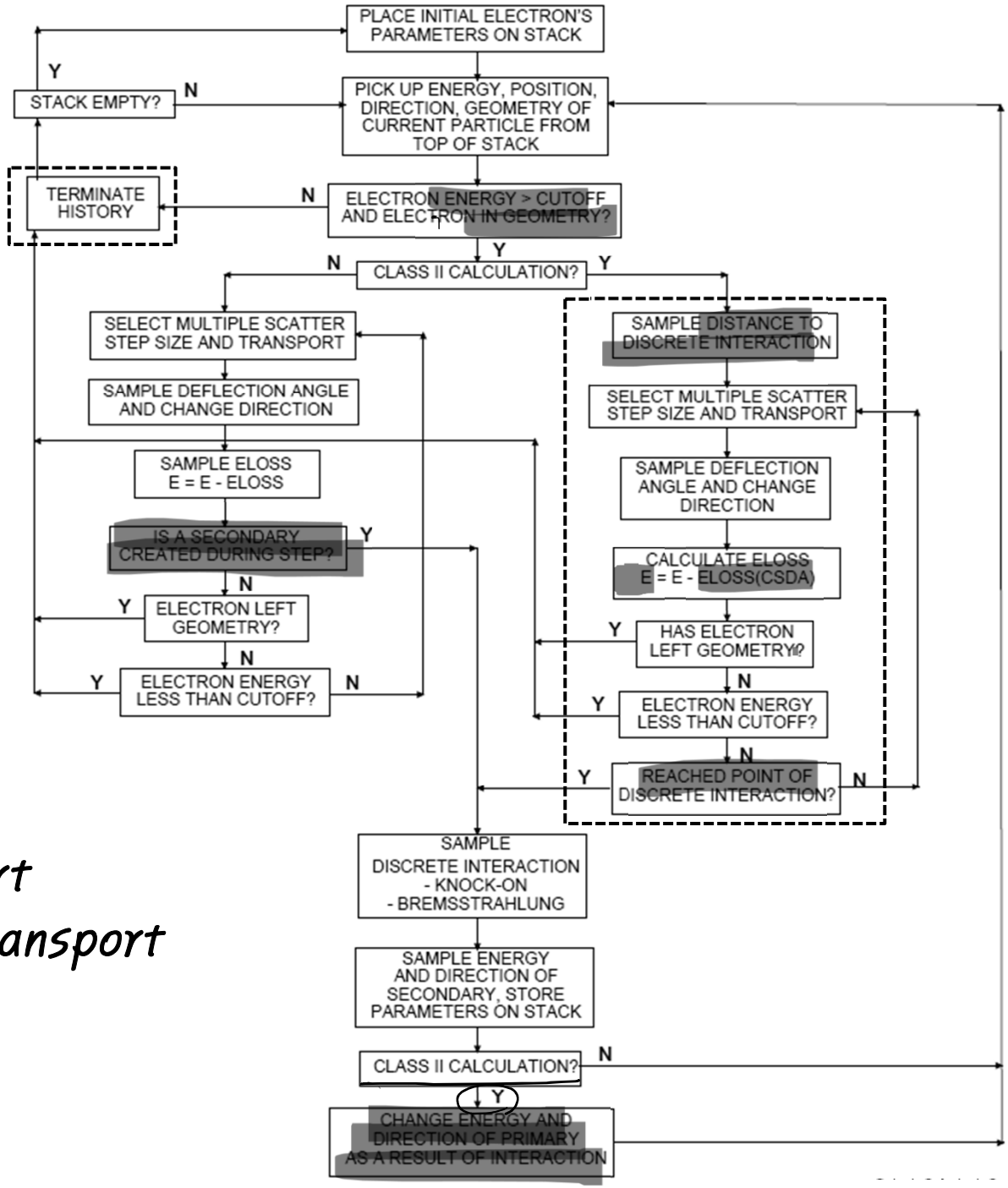
- ✓ The extremes of no energy-loss straggling and the full simulation are shown to bracket the results in which energy-loss straggling from either the creation of bremsstrahlung or knock-on electrons is included. The bremsstrahlung straggling has more of an effect, especially near the peak of the depth-dose curve.

Examples of Electron Transport: w/ multiple scatterings



- ✓ BPB of 20 MeV electrons on water with multiple scattering included in all cases and various amounts of energy-loss straggling included by turning on the creation of secondary photons and electrons above a 10 keV threshold.

Flow Chart for Electron Transport



KE/EHK

Electron Transport Logic (EGS4)

- ✓ EGS4 is a "class II" algorithm which samples interactions discretely and correlates the energy loss to secondary particles with an equal loss in the energy of the primary electron (or positron).

  Class II (CSDA + multiple scattering)
  vs. Class I (all discrete interaction modes)

- ✓ There is a close similarity between this flow chart and the photon transport flow chart. The essential differences are the nature of the particle interactions as well as the additional continuous energy-loss mechanism and multiple scattering. Positrons are treated by the same subroutine in EGS4 although it is not shown in the flow chart.

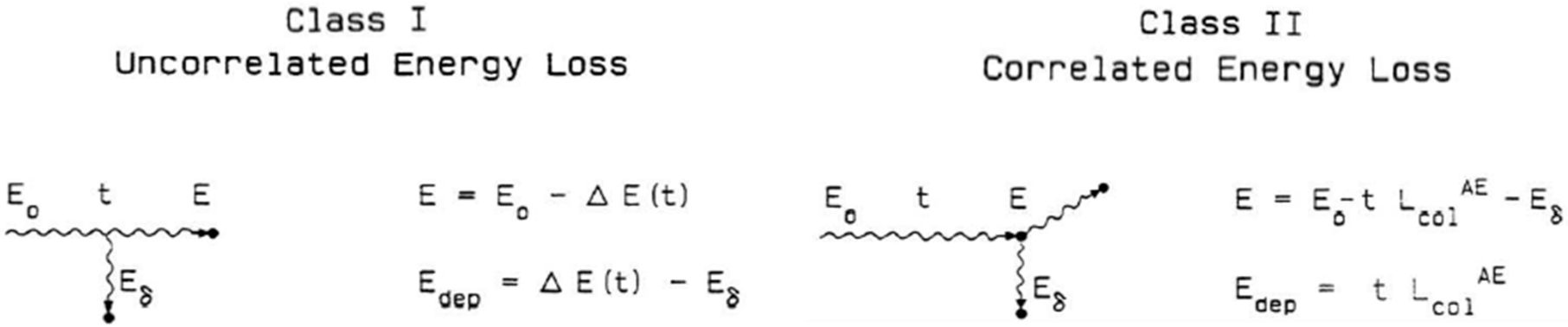
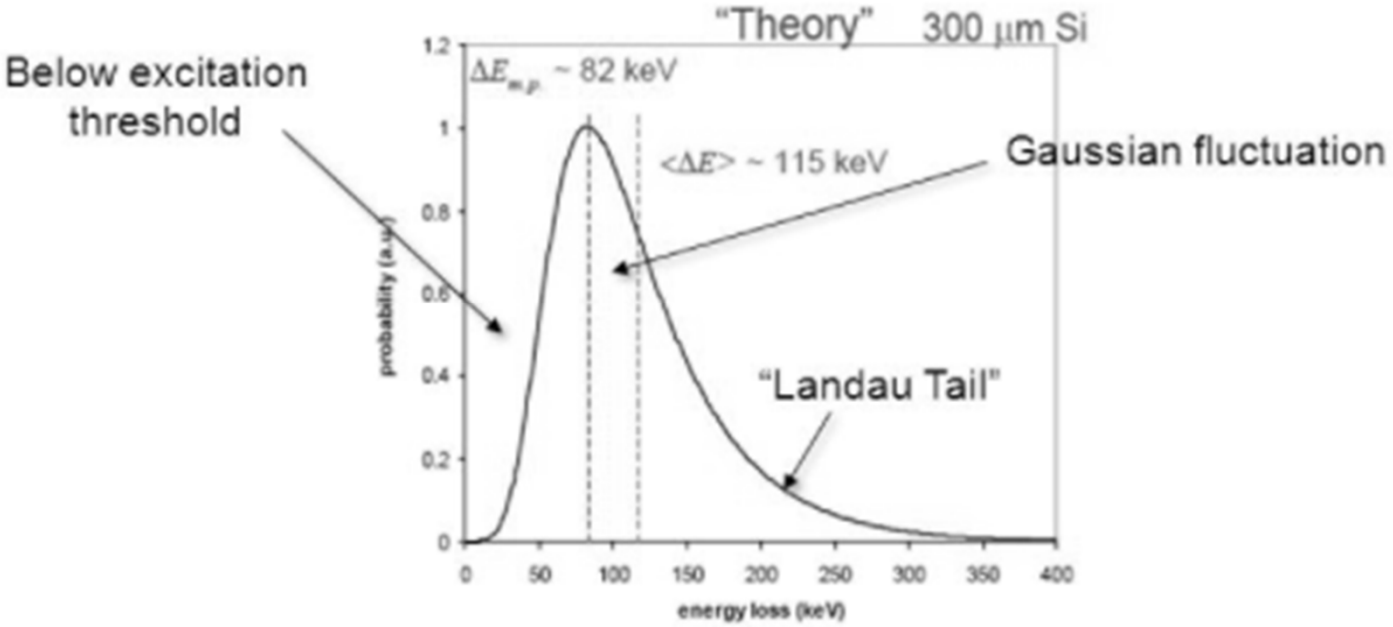


Figure 6: Distinctions between the uncorrelated energy-loss mechanisms used in class I algorithms and the correlated energy-loss mechanisms used in class II algorithms. For simplicity, only discrete interactions creating knock-on electrons are considered. $\Delta E(t)$ is the energy loss sampled from an energy-loss straggling distribution and L is the restricted collision stopping power for secondaries below energy AE.

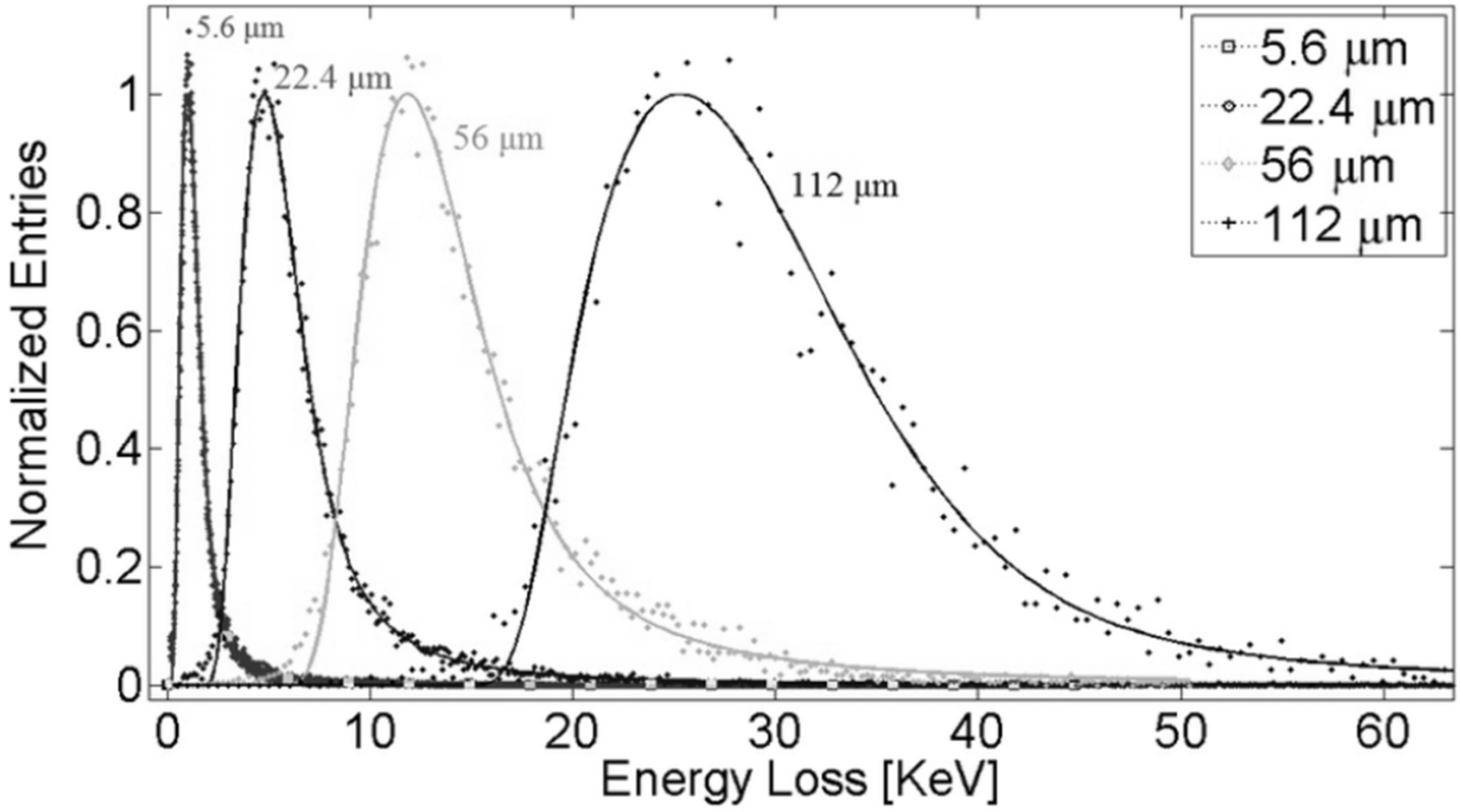
- ✓ *distingushed by how the energy (and direction) of primary electron is related to the energy lost in individual interactions*

Energy loss straggling

- Bethe Bloch formula gives **average** energy loss
- Fluctuations about the mean value are significant and non-gaussian
  - Collisions with small energy transfers are more probable
  - ⇒ Most probable energy loss shifted to lower values



Energy loss straggling varying with the thickness of shield



Energy loss distributions with fits for 12GeV protons passing through several silicon thicknesses.

Electron Transport Logic (cont.)

- ✓ Imagine that an electron's parameters (energy, direction, etc.) are on top of the particle stack. (STACK is an array containing the phase-space parameters of particles awaiting transport.)
- ✓ The electron transport routine picks up these parameters and first asks if the energy of this particle is greater than the transport cutoff energy, called ECUT. If it is not, the electron is discarded. ("Discard" means that the scoring routines are informed that an electron is about to be taken off the transport stack.)
- ✓ If there is no electron on the top of the stack, control is given to the photon transport routine. Otherwise, the next electron in the stack is picked up and transported.

Electron Transport Logic (cont.)

- ✓ If the original electron's energy was great enough to be transported, the distance to the next catastrophic interaction point is determined, exactly as in the photon case.
- ✓ The multiple scattering step-size t is then selected and the particle transported, taking into account the constraints of the geometry.
- ✓ After the transport, the multiple scattering angle is selected and the electron's direction adjusted. The continuous energy loss is then deducted.
- ✓ If the electron, as a result of its transport, has left the geometry defining the problem, it is discarded. Otherwise, its energy is tested to see if it has fallen below the cutoff as a result of its transport.

Electron Transport Logic (cont.)

- ✓ If the electron has not yet reached the point of interaction, a new multiple scattering step is effected. This innermost loop undergoes the heaviest use in most calculations because often many multiple scattering steps occur between points of interaction.
- ✓ If the distance to a discrete interaction has been reached, then the type of interaction is chosen. Secondary particles resulting from the interaction are placed on the stack as dictated by the differential cross sections, lower energies on top to prevent stack overflows.
- ✓ The energy and direction of the original electron are adjusted and the process starts all over again.