Mechanics of Materials (Solid mechanics)

: A branch of mechanics that studies the internal effects of stress and strain in a solid body

Mechanics

: A branch of physical science that deals with energy and forces and their effect on bodies

(fluid mechanics, quantum mechanics, ...)

Simply put : A study on how materials deform under loading

Why do we learn mechanics of solids?

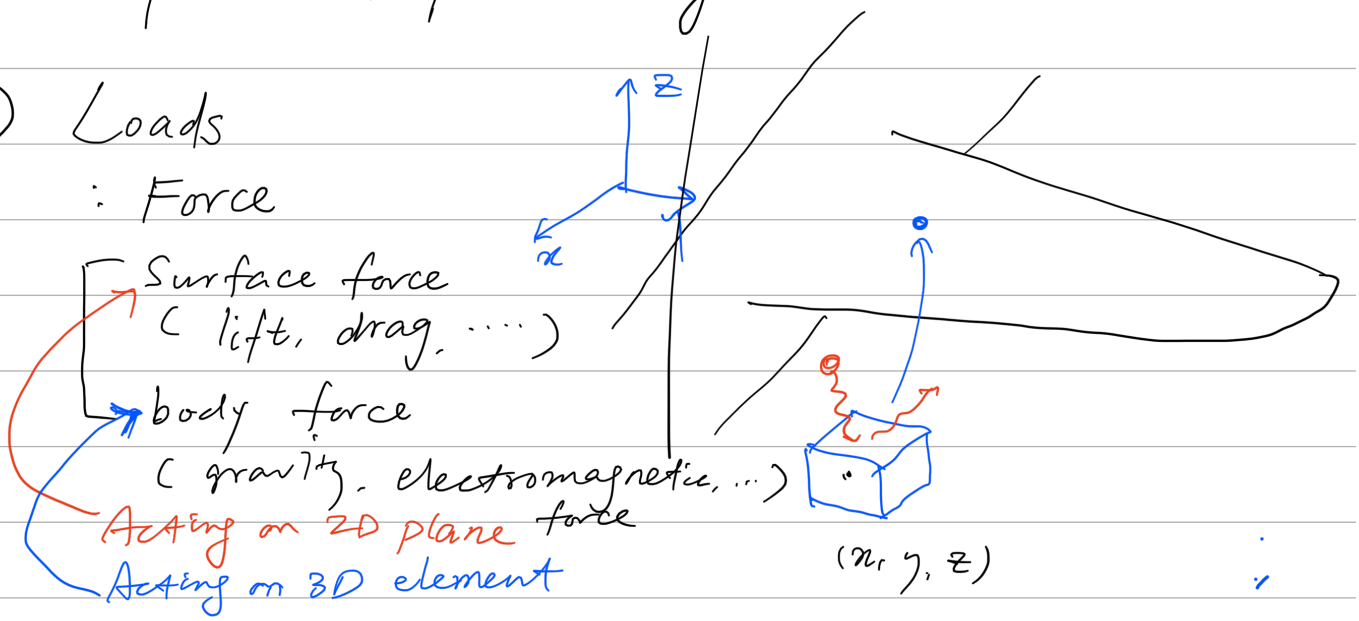
: To analyze solids under loading to ensure that they do not fail under predicted loading

<Example> Airplane wing

① Loads

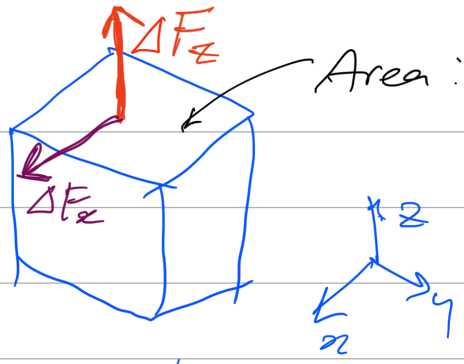
: Force

- Surface force (lift, drag, ...) — Acting on 2D plane
- body force (gravity, electromagnetic, ...) force — Acting on 3D element



② Stress

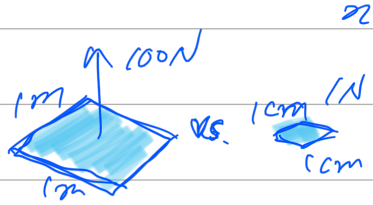
: intensity of loads



- Normal stress

$$\sigma_z\Big|_{(x,y,z)} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

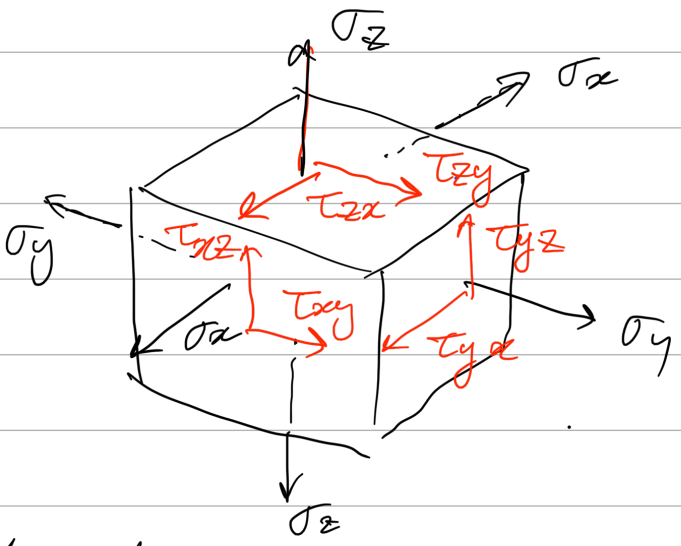
normal force



[$\sigma_z > 0$: tensile stress
$\sigma_z < 0$: compressive stress

- Shear stress

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$



σ_x, σ_y, σ_z

$\tau_{zx} = \tau_{xz}$

$\tau_{zy} = \tau_{yz}$

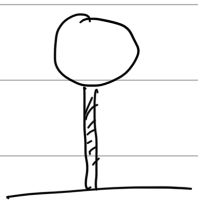
$\tau_{xy} = \tau_{yx}$

∴

→ At any point of solids, you need six stress components to fully describe the stress condition

→ Stress is important since it often serves as a criterion for material failure.

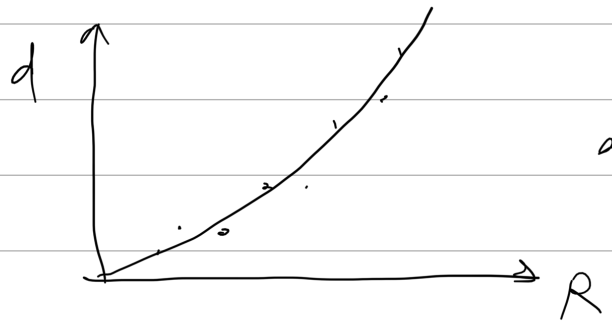
<Example> Elephant vs. bird



$W \propto \rho \cdot R^3$ (similar between animals: body density)

$\sigma \propto \frac{W}{d^2} \propto \frac{R^3}{d^2} = \text{constant} = 1/c$ (similar bone materials)

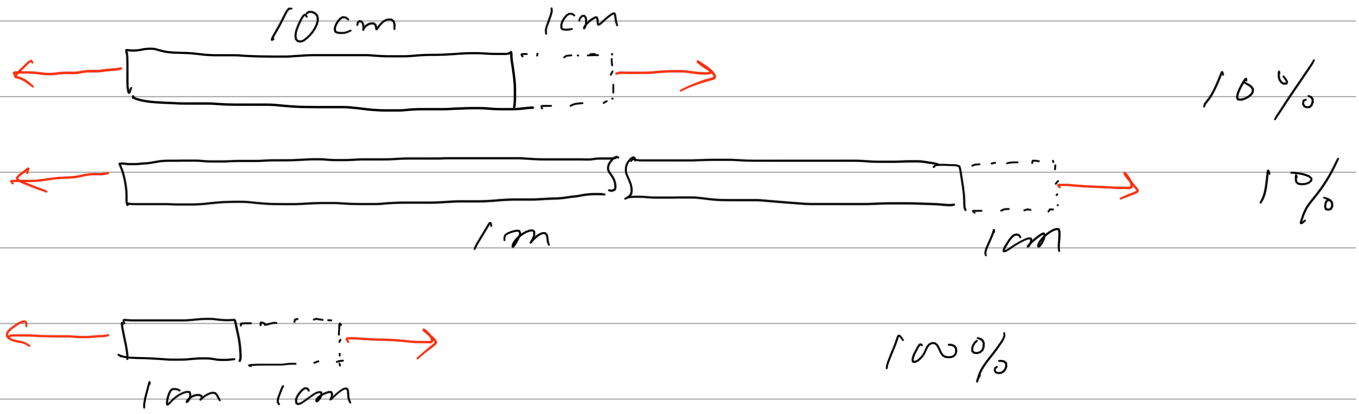
$d^2 = c \cdot R^3 \rightarrow d \propto R^{3/2}$



As a body size increases, a bone diameter needs to increase more drastically.

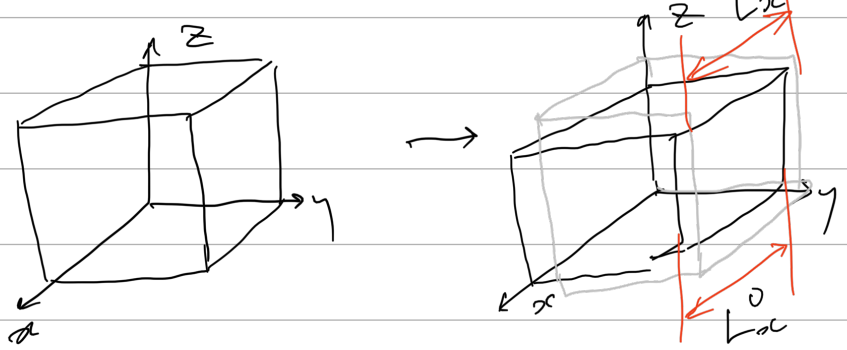
③ Strain

: Severity of deformation



Normal strain

$$\varepsilon = \frac{L - L_x^0}{L_x^0}$$

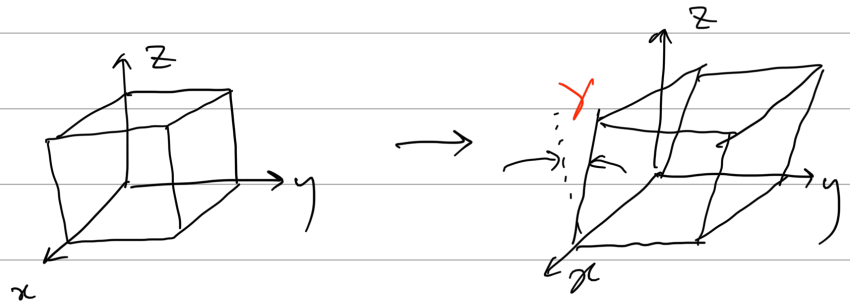


$$\varepsilon_x = \frac{L_x - L_x^0}{L_x^0}$$

ε_y

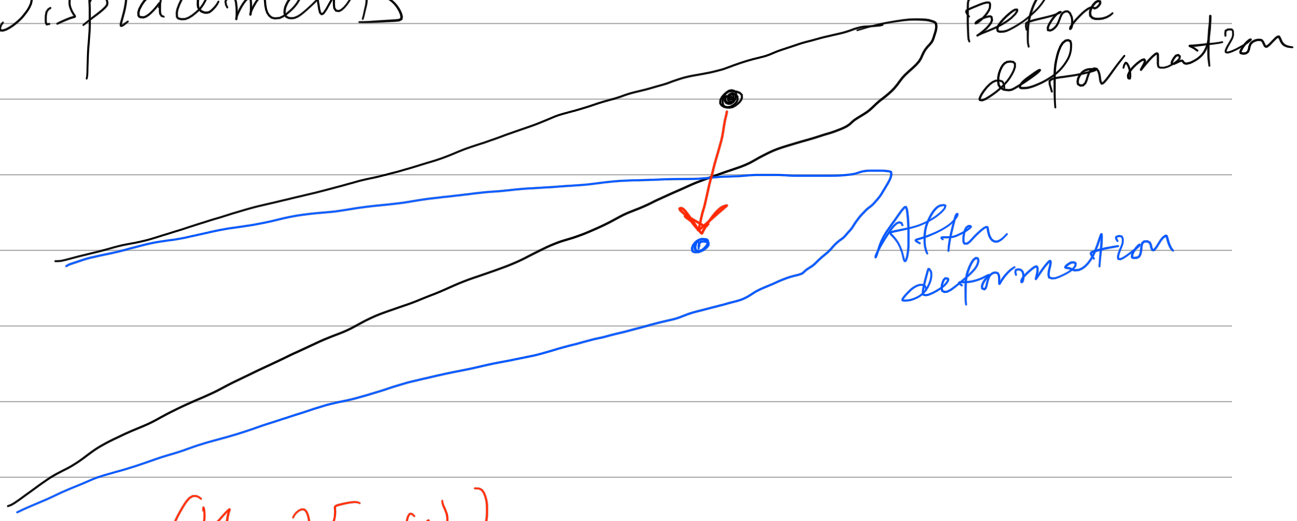
ε_z

Shear strain



γ_{yz} . γ_{xz}, γ_{xy}

④ Displacements



(u. v. w)

x-directional displacement

y- " "

z- " "

Total number of unknowns

- 6 stresses
- 6 strains
- 3 displacements

} 15 unknowns

Total number of equations

Loads

↓ ← 3 equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$)

stresses

↓ ← (Material property needed) 6 constitutive equations

strains

↓ ← 6 strain – displacement equations

displacements

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15 equations.

To solve solid mechanics problems, what should you know a priori?

- Loading condition
- Geometry
- Material properties

⇒ Learning solid mechanics will enable you to analyze, design, and manufacture advanced materials and structures!