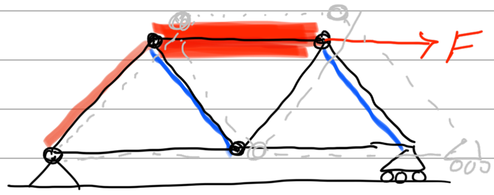
(Hibbler Chap. 1, Crandall, chap. 1)

Statics : analysis of force and moment (or torque) acting on physical systems

("study on "structures and applied loads")



What is force acting on each member?
→ Loads

Statics

What is the loading intensity?
→ Stresses

Relative elongation / contraction?
→ strains

Deformed shape?
→ Displacements

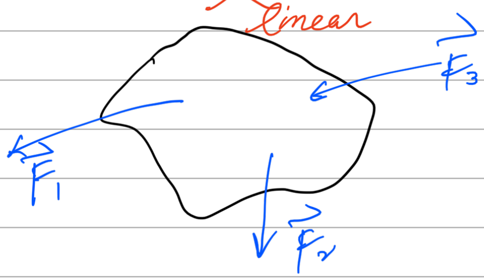
Mechanics of solids/structures

Equilibrium equations

$$\Sigma \vec{F} = m\vec{a} = 0$$

$\vec{a} = 0$

Statics : $\underbrace{\text{No acceleration}}_{\text{linear}}$ (i.e., at rest)



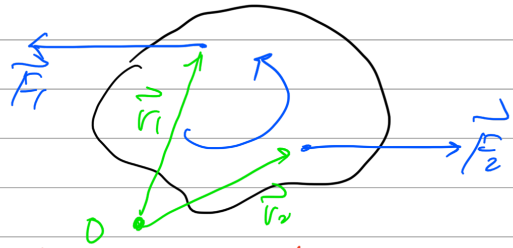
$$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = 0$$

$$\Sigma F_x = F_{1x} + F_{2x} + F_{3x} + \cdots = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

→ Force equilibrium



No rotational acceleration

With respect to arbitrary point O,

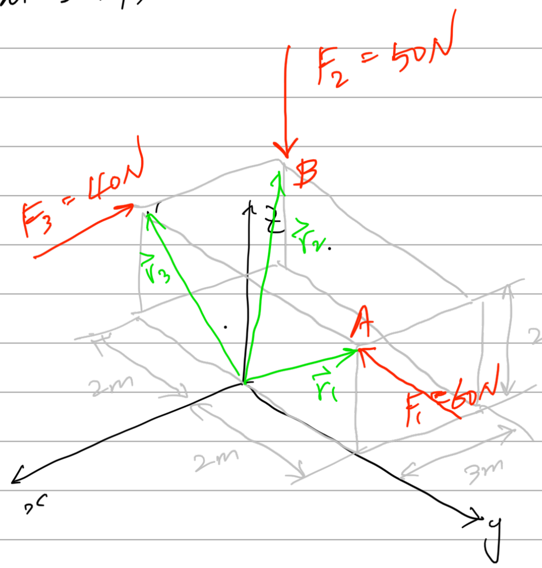
$$\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cdots$$

$$= \vec{M}_1 + \vec{M}_2 + \cdots = \Sigma \vec{M}_o = 0$$

→ Moment equilibrium

Example. Determine the resultant moment of the three forces about the x-, y-, and z-axis

(Hibbler statics 4.7)



$M_x = 2 \times 60 + 2 \times 50 = 220\,Nm$

$M_y = -2 \times 40 - 3 \times 50 = -230\,Nm$

$M_z = -2 \times 40 = -80\,Nm$

$$\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3$$

$$= [0\hat{i} + 2\hat{j} + 2\hat{k}] \times [0\hat{i} - 60\hat{j} + 0\hat{k}]$$
$$+ [-3\hat{i} - 2\hat{j} + 2\hat{k}] \times [0\hat{i} + 0\hat{j} - 50\hat{k}]$$
$$+ [0 - 2\hat{j} + 2\hat{k}] \times [-40\hat{i} + 0\hat{j} + 0\hat{k}]$$

$$= \begin{vmatrix} i & j & k \\ 0 & 2 & 2 \\ 0 & -60 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 2 \\ 0 & -60 \end{matrix} \Rightarrow +120\hat{i}$$

$$+ \begin{vmatrix} i & j & k \\ -3 & -2 & 2 \\ 0 & 0 & -50 \end{vmatrix} \begin{matrix} i & j \\ -3 & -2 \\ 0 & 0 \end{matrix} \Rightarrow 100i - 150j$$

$$+ \begin{vmatrix} i & j & k \\ 0 & -2 & 2 \\ -40 & 0 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & -2 \\ -40 & 0 \end{matrix} \Rightarrow -80j - 80\hat{k}$$

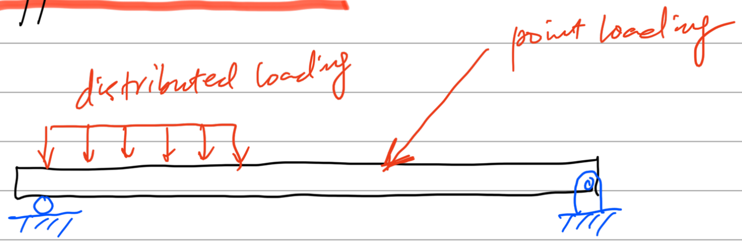
$$= 220\hat{i} - 230\hat{j} - 80\hat{k}$$

Question:

- Does a moment caused by multiple forces acting on a body depend on the choice of the pivoting point? Yes

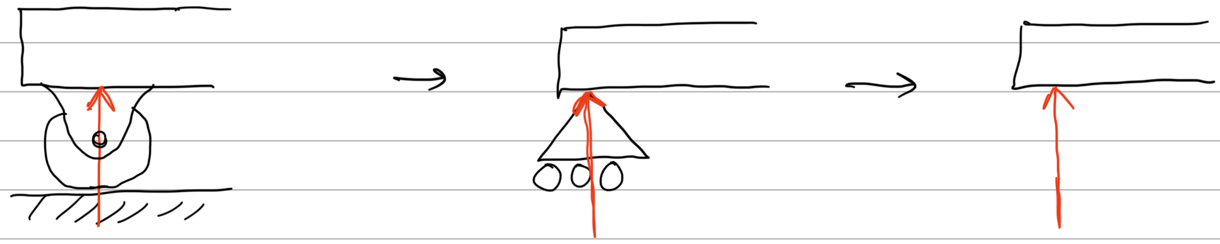
- For the establishment of the moment equilibrium equation, does it matter which pivoting point we pick?
  No, but for the sake of simplicity, we tend to have preference of some points over the others.

To account for forces, we often need to figure out support reactions.

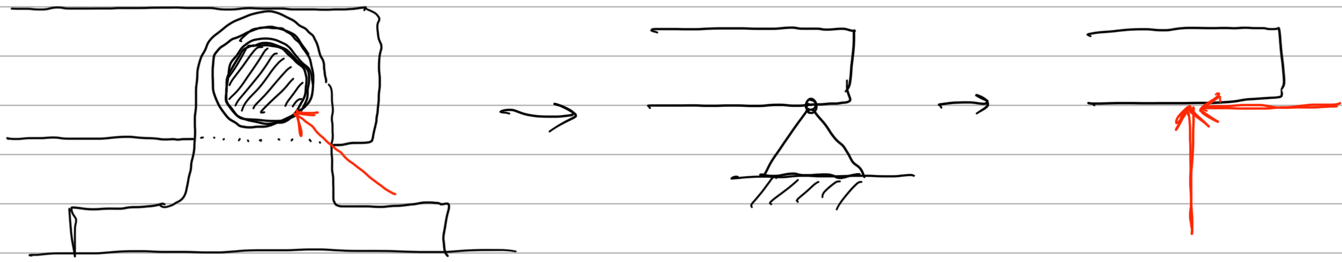


Support reactions (i.e., reaction forces) : prevents translations and rotations

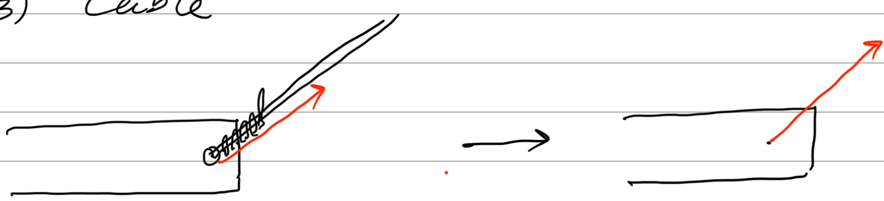
1) Roller joint



2) Pin joint

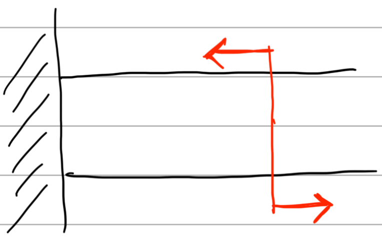


3) Cable



along the direction of the cable

4) Fixed support

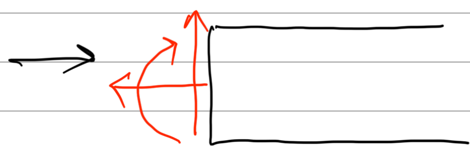


can handle

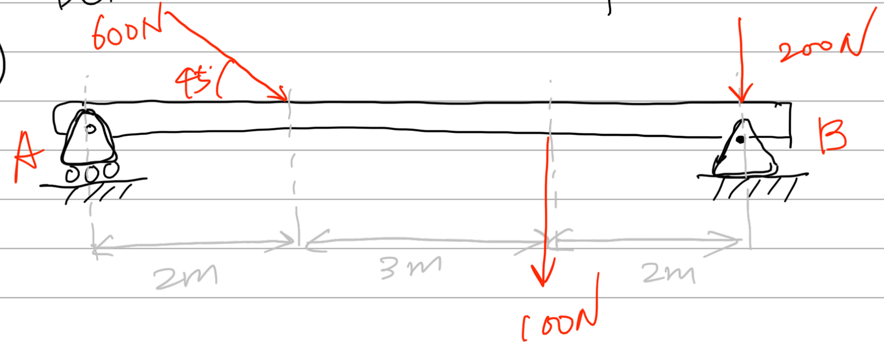
axial loads

shear loads

torsional loads

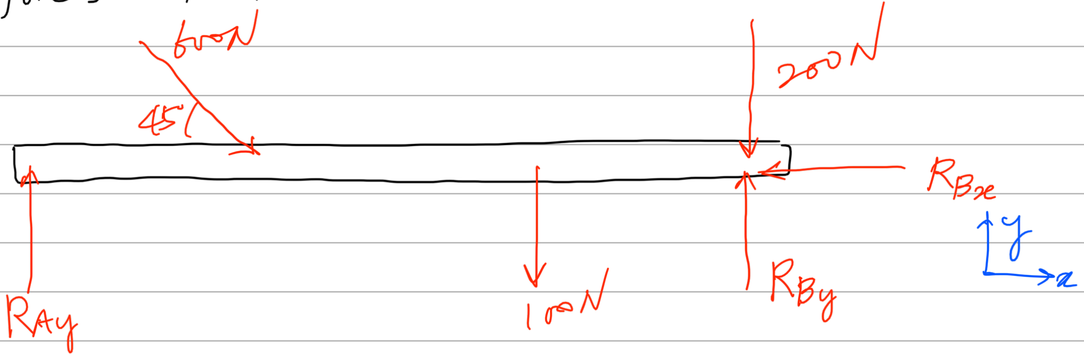


Example. Determine the reaction forces
(Hibbler statics 5.5)



Free-body diagram

: A drawing that shows the structure with all forces that act on it



Force equilibrium

$\sum F_x = 0$: $600\cos 45^\circ - R_{Bx} = 0$

$\sum F_y = 0$: $R_{Ay} - 600\sin 45^\circ - 100 - 200 + R_{By} = 0$

Moment equilibrium

$\sum M_B = 0$: $100 \times 2 + 600\sin 45^\circ \times 5 - R_{Ay} \times 7 = 0$

(↺+) (Assume the thickness of the beam is negligible)

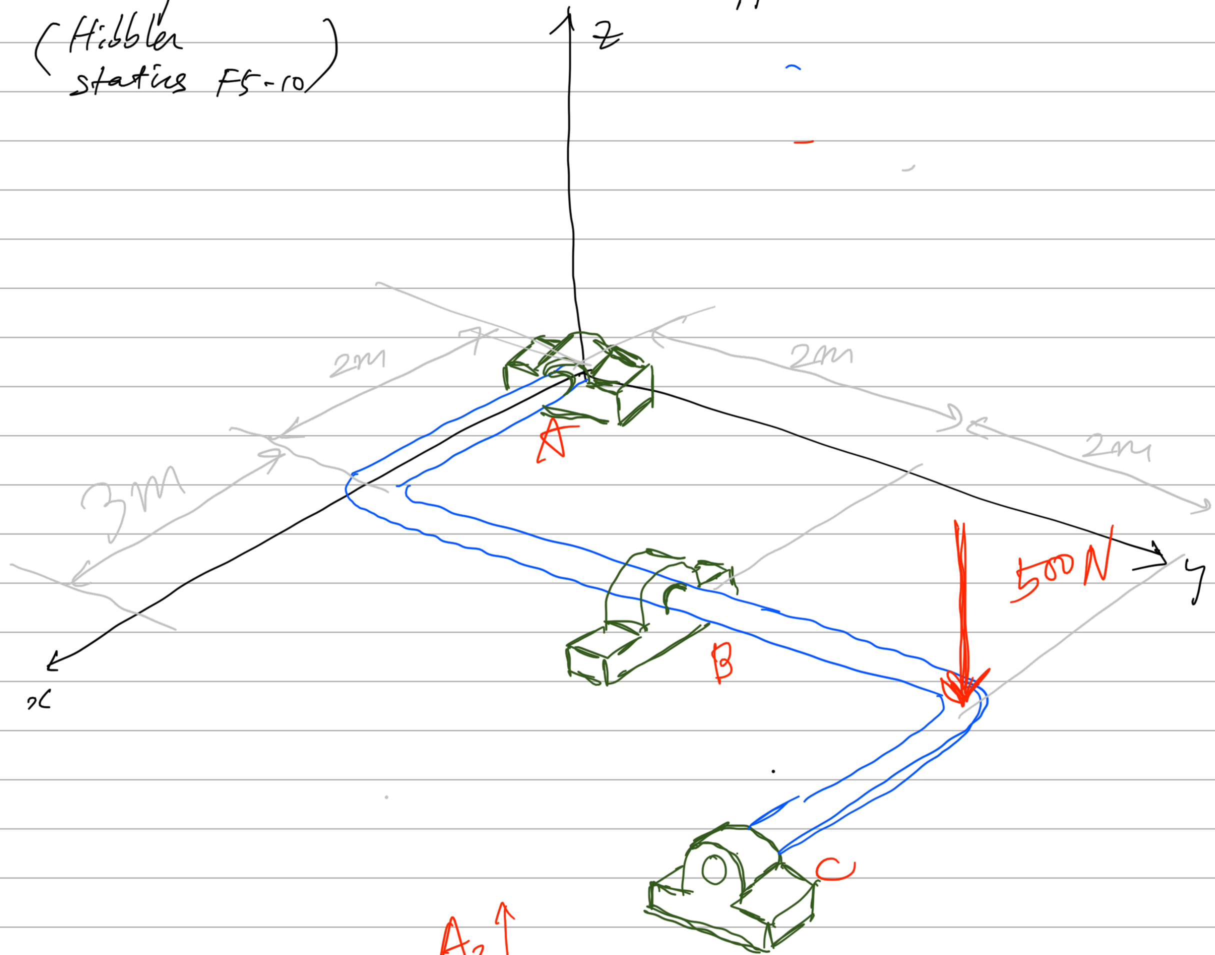
$\therefore R_{Ay} = \dfrac{200 + 1500\sqrt{2}}{7}$ N

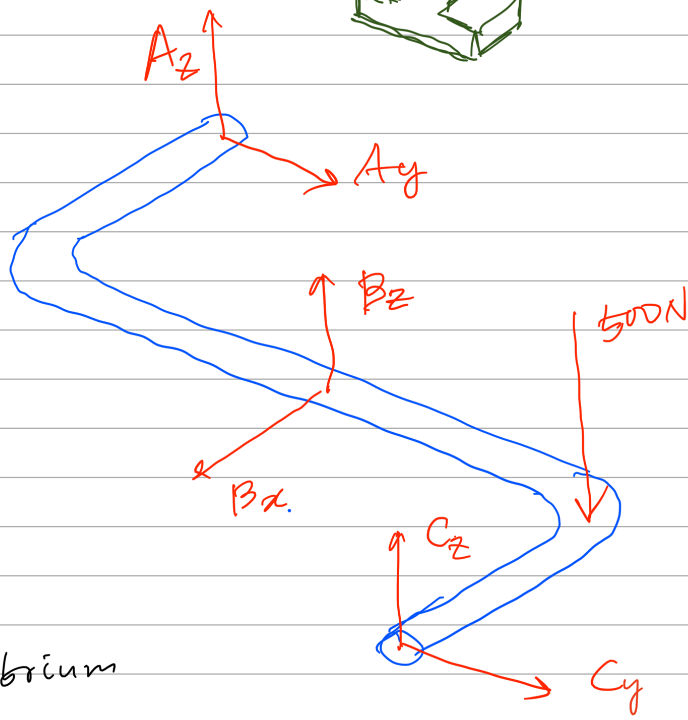
$R_{Bx} = 300\sqrt{2}$ N

$R_{By} = 300 + 300\sqrt{2} - \left(\dfrac{200 + 1500\sqrt{2}}{7}\right)$

$= \dfrac{1900 - 600\sqrt{2}}{7}$ N

Example : Determine support reactions
(Hibbler statics F5-10)





Force equilibrium

$\sum F_x = 0$: $B_x = 0$

$\sum F_y = 0$: $A_y + C_y = 0$ ~ (4)

$\sum F_z = 0$: $A_z + B_z + C_z - 500 = 0$ ~ (5)

Moment equilibrium (with respect to B)

$\Sigma M_x = 0$: $-2A_z - 500 \times 2 + 2C_z = 0$ — (1)

$\Sigma M_y = 0$: $2A_z - 3C_z = 0$ — (2)

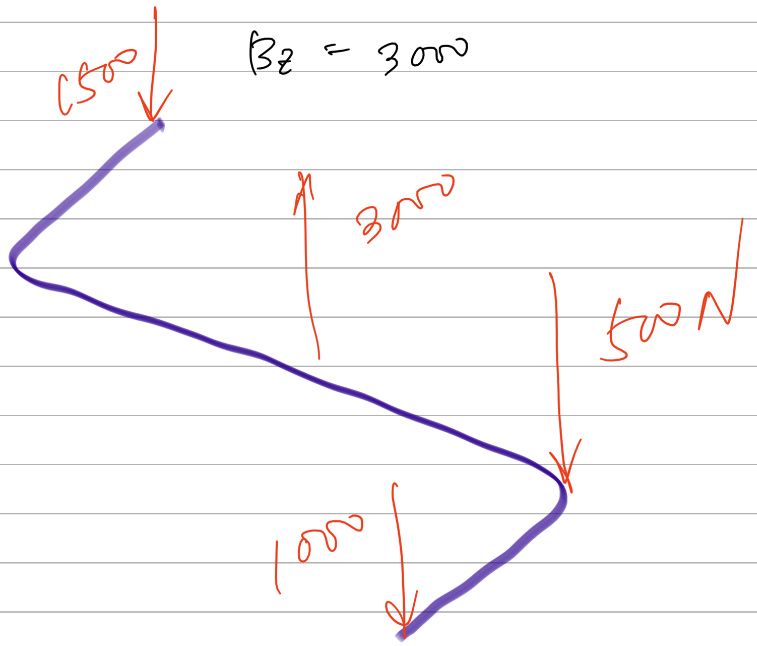
$\Sigma M_z = 0$: $3C_y - 2A_y = 0$ — (3)

From (1) & (2) : $C_z = -1000$, $A_z = -1500$

From (3) & (4) : $A_y = C_y = 0$

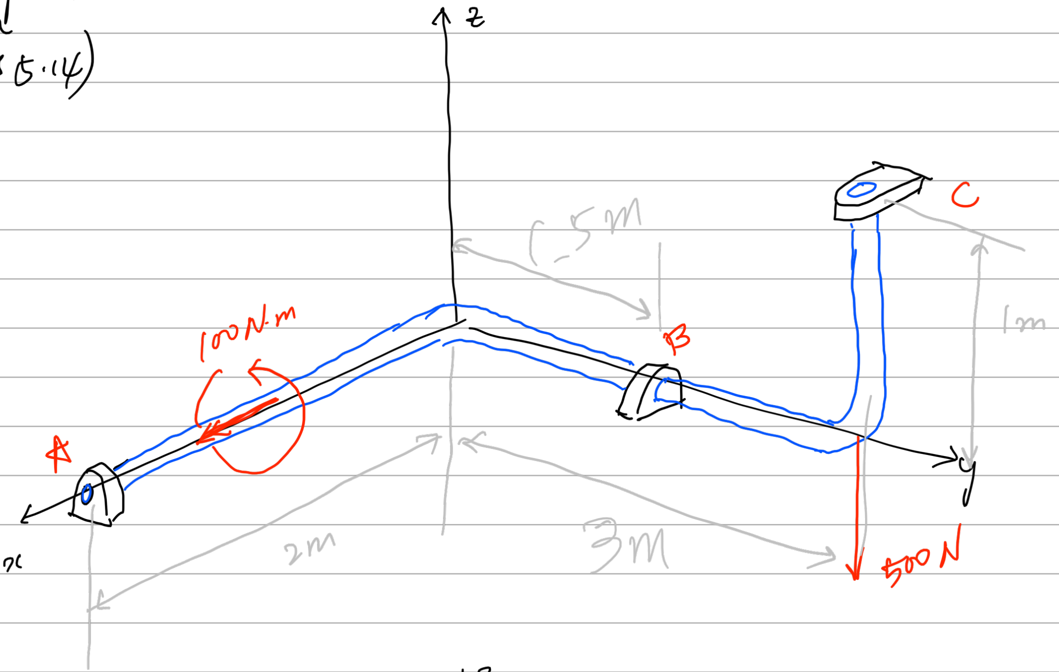
From (5) : $-1500 + B_z - 1000 - 500 = 0$

$B_z = 3000$

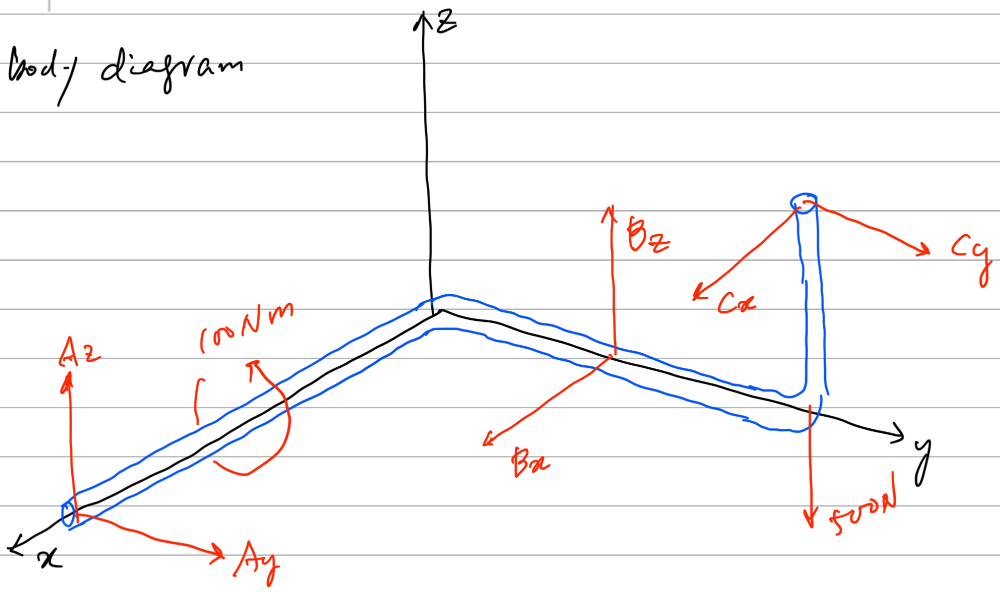


Example.
(Hibbler statics 5.14)

Calculate reaction forces



Free-body diagram



Force equilibrium

$\Sigma F_x = 0 \quad : \quad B_x + C_x = 0$

$\Sigma F_y = 0 \quad : \quad \cancel{A_y} + \cancel{C_y} = 0$

$\Sigma F_z = 0 \quad : \quad A_z + B_z - 500 = 0$

Moment equilibrium

$\Sigma M_x = 0 \quad : \quad 100 + 1.5 \times B_z - 1 \times \cancel{C_y} - 3 \times 500 = 0$

$\Sigma M_y = 0 \quad : \quad 1 \times C_x - 2 \times A_z = 0$

$\Sigma M_z = 0 \quad : \quad -3 \times C_x - 1.5 \times B_z + 2 \times \cancel{A_y} = 0$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1.5 & 0 & -1 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 2 & 0 & -1.5 & 0 & -3 & 0 \end{bmatrix} \begin{Bmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500 \\ 1400 \\ 0 \\ 0 \end{Bmatrix}$$

$\rightarrow$ Rank $= 5$

If joint A is a roller joint : $A_y = 0$

$C_y = 0$

$B_z = 900$

$A_z = -400$

$C_x = -800$

$B_x = 1600$

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1.5 & 0 & -1 \\ -2 & 0 & 0 & 1 & 0 \\ 0 & -1.5 & 0 & -3 & 0 \end{bmatrix} \begin{Bmatrix} A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500 \\ 1400 \\ 0 \\ 0 \end{Bmatrix}$$