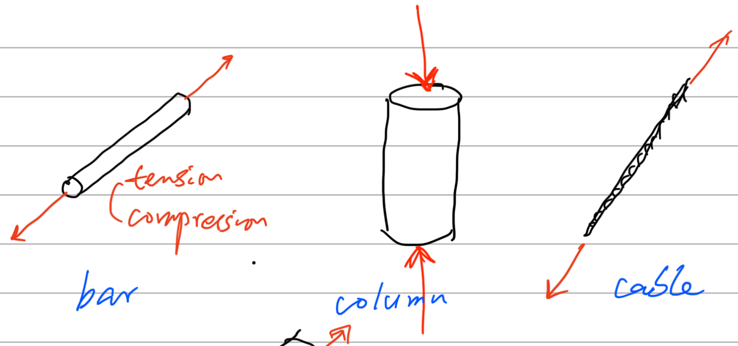
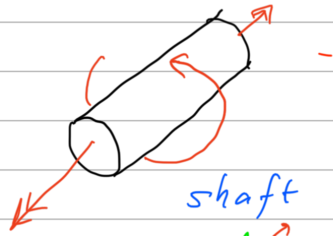
Structural components

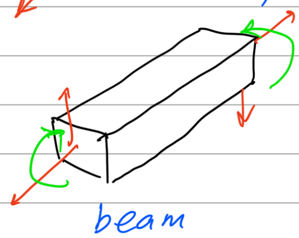
Axial loading :



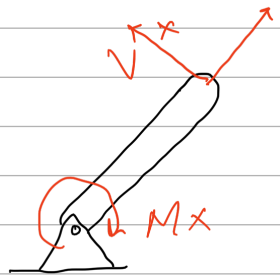
Torsional loading :



Bending loading

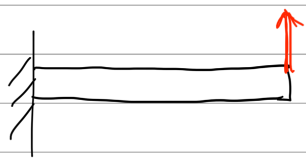


Axial loads
Shear loads
Bending loads

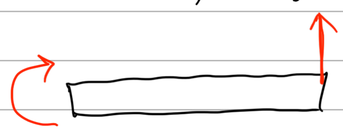


Pin joints cannot handle shear or bending loads.

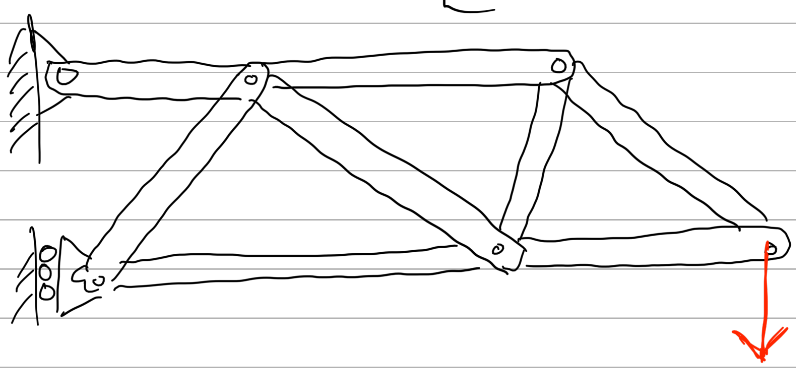
c.f.



Free-body diagram



Wall mount can handle torques



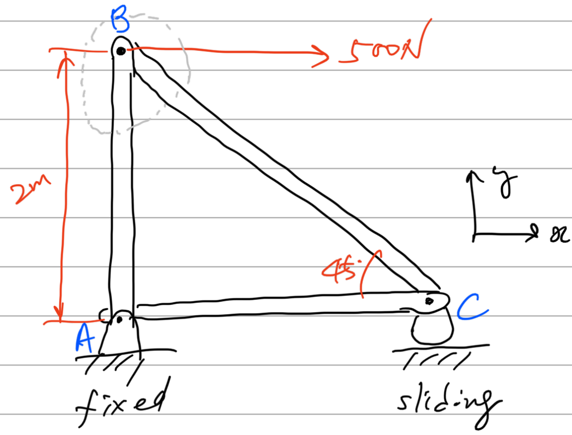
Combination of pins joints and bars can handle shear and bending loads

→ Truss structure

: Widely used due to the advantages, such as

- lightweight
- easy to assemble
- adjustable under varying loading conditions

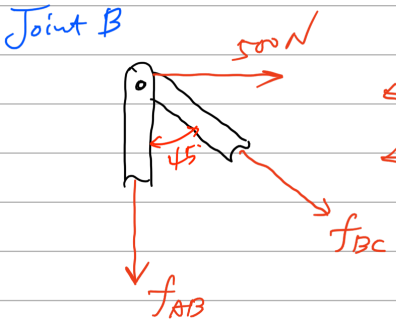
Analysis on joints (method of joints)



[Loading condition
geometry
~~material properties~~
(deformation is not of interest in statics)

Key assumption: Bar elements in the truss structure can only support axial forces

Joint B

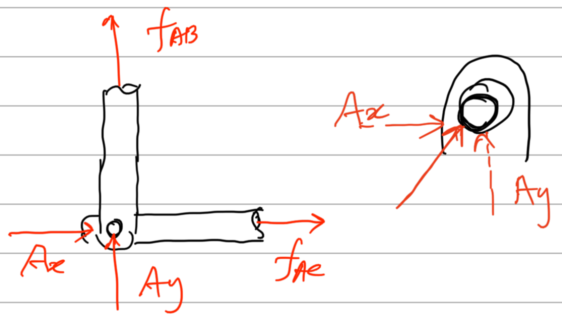


$\Sigma F_x = 0$: $f_{BC} \sin 45^\circ + 500 = 0$

$\Sigma F_y = 0$: $-f_{AB} - f_{BC} \cos 45^\circ = 0$

(Assume all bar elements are under tension)

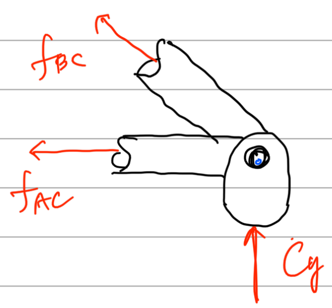
Joint A



$\Sigma F_x = 0$: $f_{AC} + A_x = 0$

$\Sigma F_y = 0$: $f_{AB} + A_y = 0$

Joint C



$\Sigma F_x = 0$: $-f_{AC} - f_{AB} \cos 45^\circ = 0$

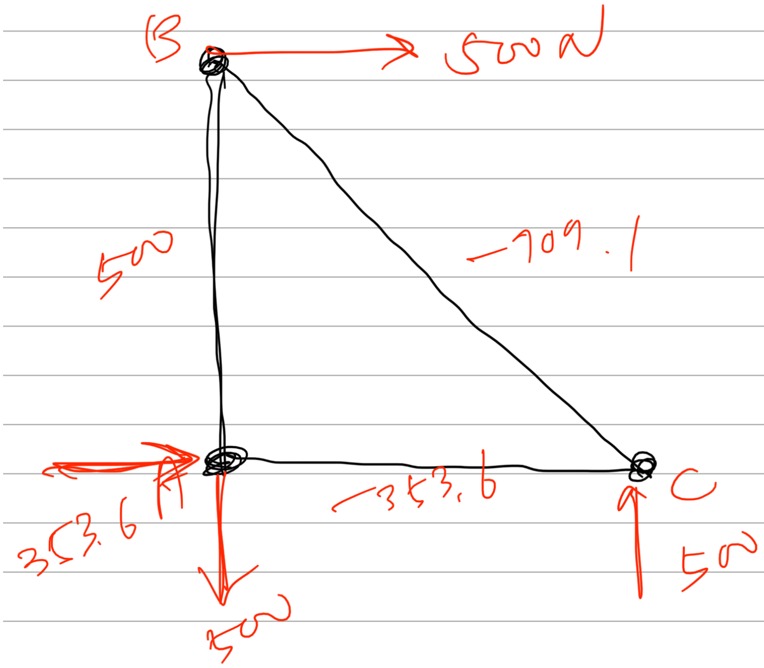
$\Sigma F_y = 0$: $f_{BC} \sin 45^\circ + C_y = 0$

$$\begin{bmatrix} 0 & 0 & \sqrt{2}/2 & 0 & 0 & 0 \\ -1 & 0 & -\sqrt{2}/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ -\sqrt{2}/2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2}/2 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} f_{AB} \\ f_{AC} \\ f_{BC} \\ A_x \\ A_y \\ C_y \end{Bmatrix} = \begin{Bmatrix} -500 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

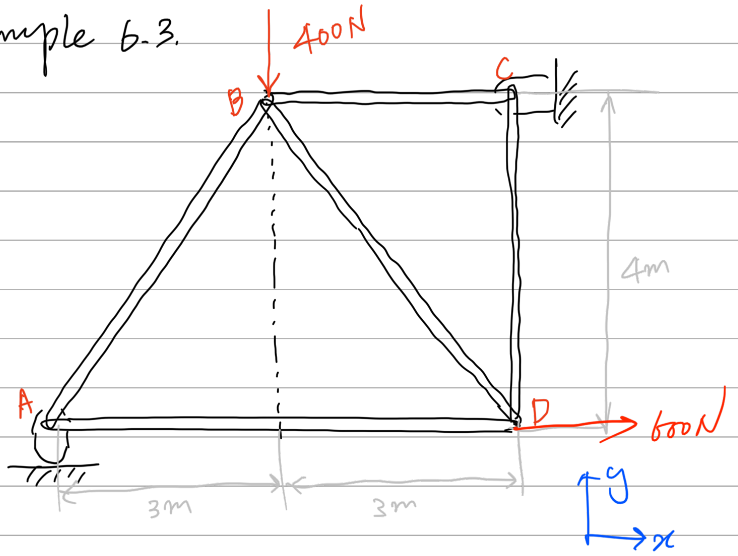
$$A\vec{x} = \vec{y} \qquad \vec{x} = A^{-1}\vec{y}$$

(A \ y in Matlab)

$$\begin{Bmatrix} f_{AB} \\ f_{AC} \\ f_{BC} \\ A_x \\ A_y \\ C_y \end{Bmatrix} = \begin{Bmatrix} 500 \\ -353.6 \\ -709.1 \\ 353.6 \\ -500 \\ 500 \end{Bmatrix}$$

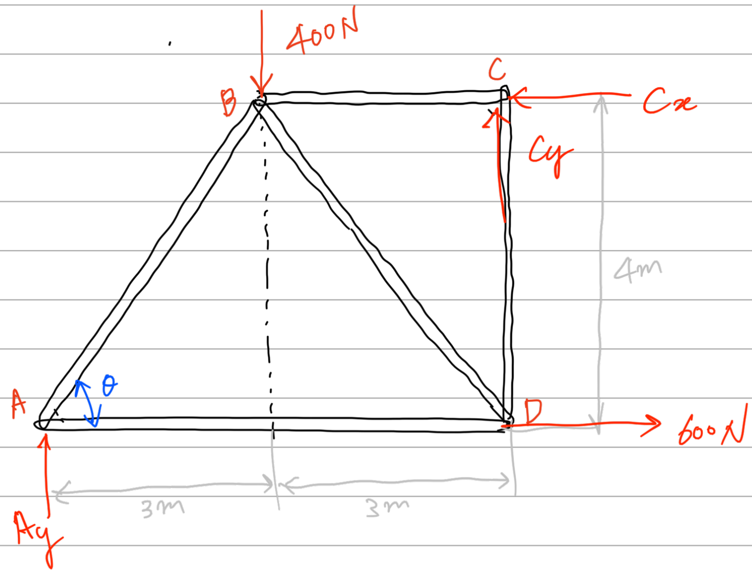


Example 6.3.



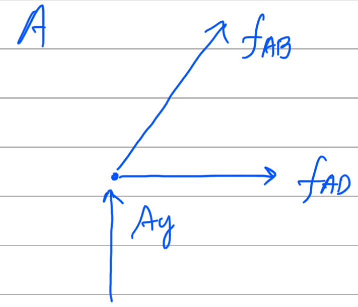
Determine the force in each member of the truss.

FBD



Equilibrium at joints

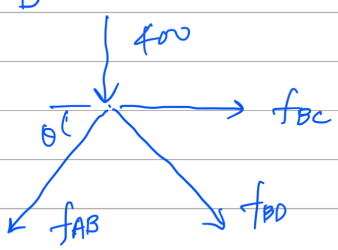
Joint A



$\Sigma F_x = 0: \quad f_{AB}\cos\theta + f_{AD} = 0$

$\Sigma F_y = 0: \quad f_{AB}\sin\theta + A_y = 0$

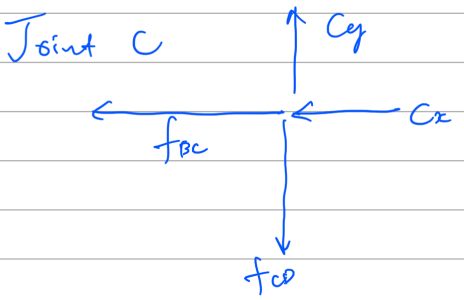
Joint B



$\Sigma F_x = 0: \quad -f_{AB}\cos\theta + f_{BC} + f_{BD}\cos\theta = 0$

$\Sigma F_y = 0: \quad -f_{AB}\sin\theta - f_{BD}\sin\theta - 400 = 0$

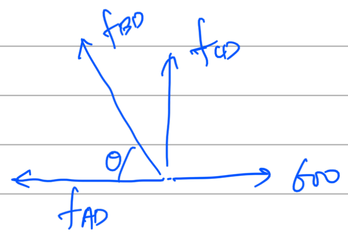
Joint C



$\Sigma F_x = 0: \; -f_{BC} - C_x = 0$

$\Sigma F_y = 0: \; -f_{CD} + C_y = 0$

Joint D



$\Sigma F_x = 0: \; -f_{AD} - f_{BD}\cos\theta + 600 = 0$

$\Sigma F_y = 0: \; f_{BD}\sin\theta + f_{CD} = 0$

4 joints × 2 equilibrium = 8 equations

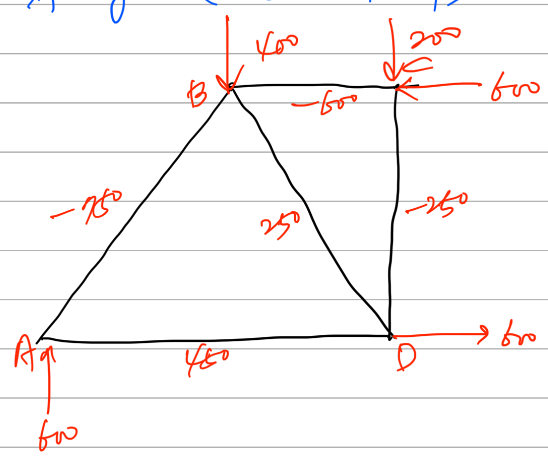
5 bars + 3 reaction forces = 8 unknowns

$$\left[\begin{array}{cccc:cccc} 3/5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4/5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -3/5 & 0 & 1 & 3/5 & 0 & 0 & 0 & 0 \\ -4/5 & 0 & 0 & -4/5 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -3/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4/5 & 1 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} f_{AB} \\ f_{AD} \\ f_{BC} \\ f_{BD} \\ f_{CD} \\ A_y \\ C_x \\ C_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 400 \\ 0 \\ 0 \\ -600 \\ 0 \end{Bmatrix}$$

$(\cos\theta = 3/5, \; \sin\theta = 4/5)$

$A\vec{x} = \vec{y}$, $\quad A^{-1}y \quad (\vec{x} = A^{-1}\cdot\vec{y})$.

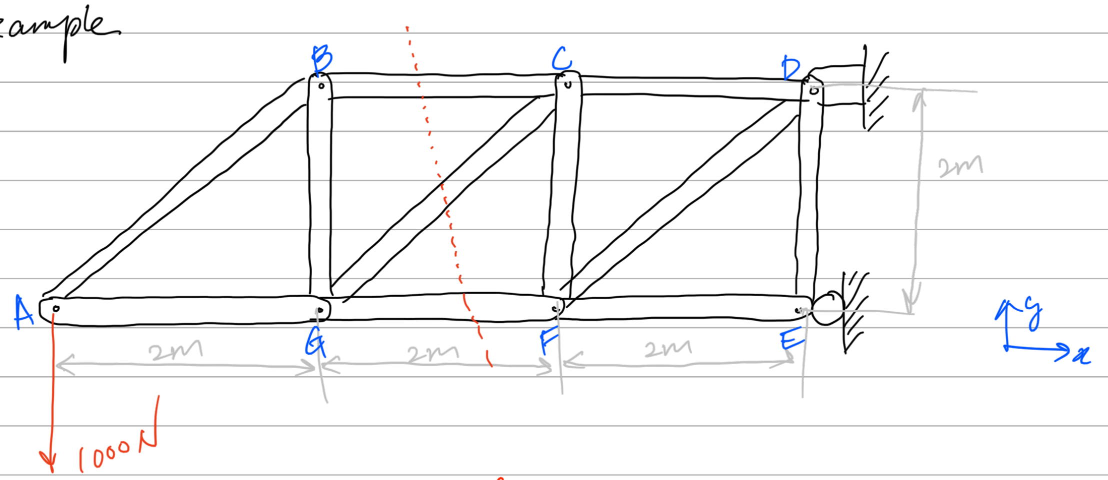
$$\begin{Bmatrix} f_{AB} \\ f_{AD} \\ f_{BC} \\ f_{BD} \\ f_{CD} \\ A_y \\ C_x \\ C_y \end{Bmatrix} = \begin{Bmatrix} -750 \\ 450 \\ -600 \\ 250 \\ -250 \\ 600 \\ 600 \\ -200 \end{Bmatrix}$$



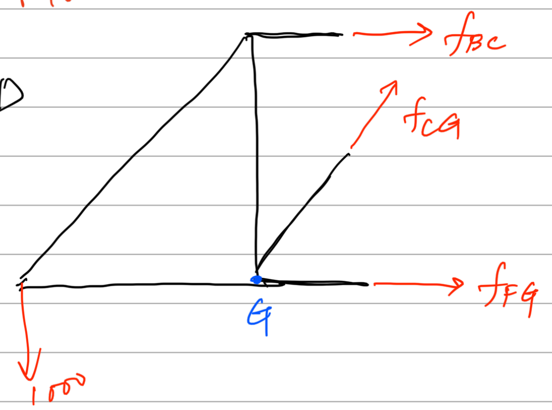
Analysis on sections (the method of sections)

: Based on the principle that if the truss is in equilibrium, then any section of the truss is also in equilibrium.

Example



FBD



Force equilibrium $\quad \Sigma F_x = 0 : \ f_{BC} + \frac{\sqrt{2}}{2} f_{CG} + f_{FG} = 0$

$\Sigma F_y = 0 : \frac{\sqrt{2}}{2} f_{CG} - 1000 = 0$

Moment equilibrium $\quad \Sigma M_G = 0 : \ 1000 \times 2 - f_{BC} \times 2 = 0$

$(\circlearrowleft +)$

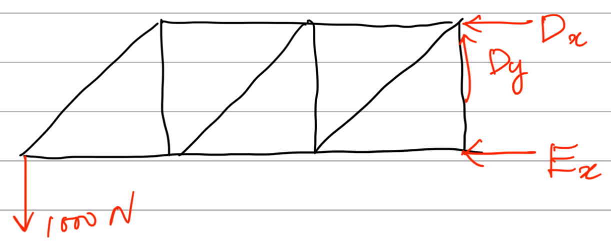
$\rightarrow f_{BC} = 1000 \ N$

$f_{CG} = 1000\sqrt{2} \ N$

$f_{FG} = -f_{BC} - \frac{\sqrt{2}}{2} f_{CG}$

$= -1000 - 1000 = -2000 \ N$

Similarly reaction forces can be easily figured out :



$\Sigma F_x = 0 : \quad -D_x - E_x = 0$

$\Sigma F_y = 0 : \quad -1000 + D_y = 0$

$\Sigma M = 0 : \quad 6 \times 1000 - 2 \times E_x = 0$

(w.r.t. D)

$\therefore \quad E_x = 3000 \text{ N}, \quad D_x = -3000 \text{ N}, \quad D_y = 1000 \text{ N}$

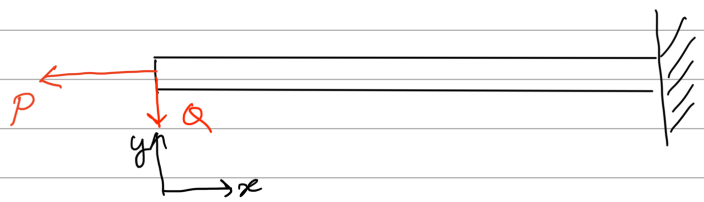
Internal forces

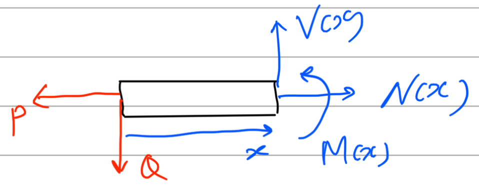
: loads acting within the structure

→ can be obtained by using the method of sections

Example.

Axial loads
Shear loads
Bending moment ?





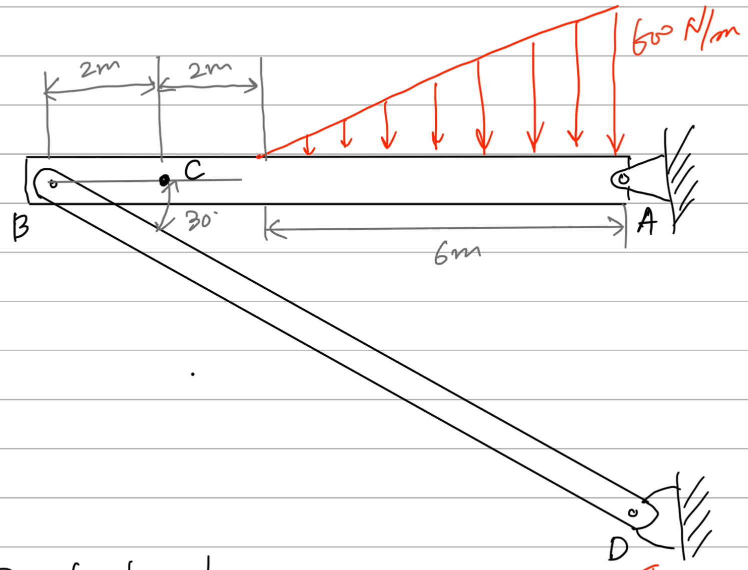
$V(x) = Q$

$N(x) = P$

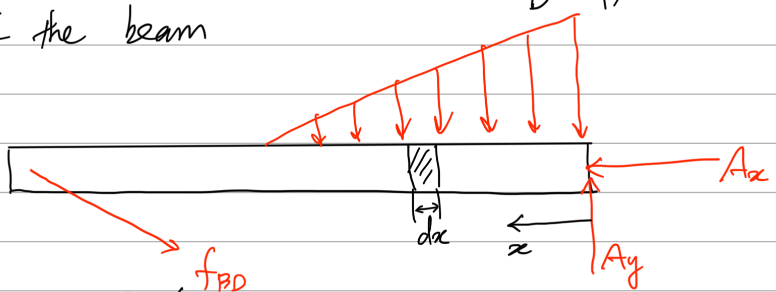
$M(x) + Q \cdot x = 0$

$M(x) = -Qx$

Example : Determine the resultant internal loadings acting at point C on the beam



FBD of the beam



$w = 100 \cdot (6 - x)$

$df = w \cdot dx$

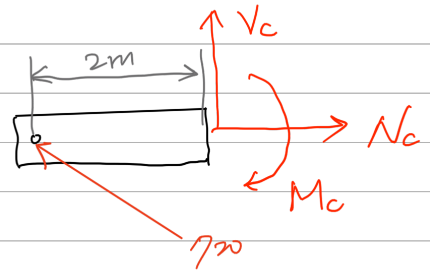
$$M_A = \int dM_A = \int_0^{x=6} x \cdot df$$

$$= \int_0^6 x \cdot 100(6-x)\,dx = 100\left(3x^2 - \frac{x^3}{3}\right)\Big|_0^6 = 100(108 - 72)$$

$$= 3600 \text{ Nm}$$

$\Sigma M = 0$: $360 + 10 \cdot f_{BD} \cdot \sin 30^\circ = 360 + 5 f_{BD} = 0$
$(\circlearrowleft +)$

$\therefore f_{BD} = -720\ N$



$N_c - 720 \cos 30^\circ = 0$

$V_c + 720 \sin 30^\circ = 0$

$M_c + 720 \sin 30^\circ \times 2 = 0$

$\therefore N_c = 360\sqrt{2}\ N$
$V_c = -360\ N$
$M_c = -720\ N \cdot m$