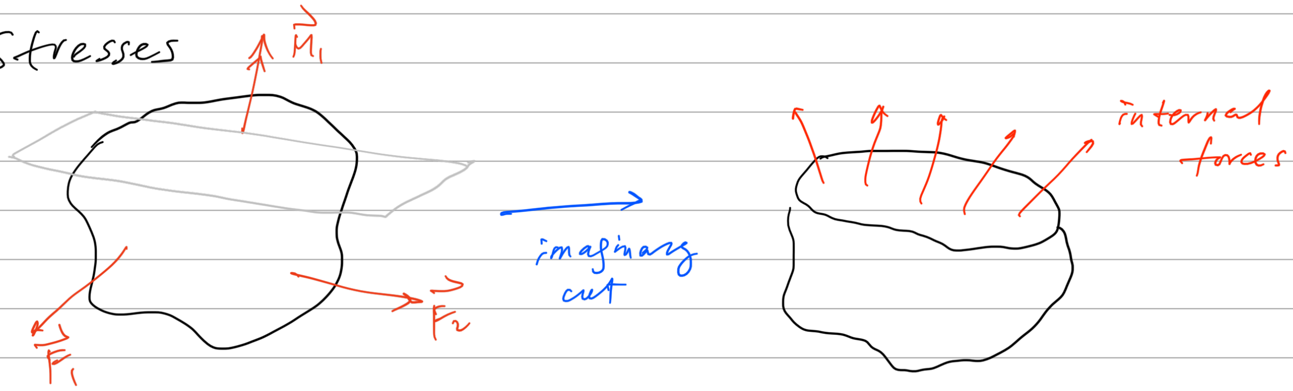
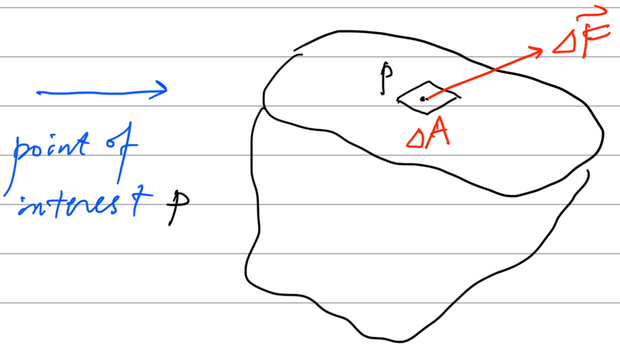
1.3. Stresses





Intensity of internal force

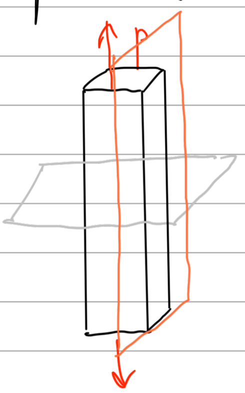
: Quotient of the force and area

⇒ stress

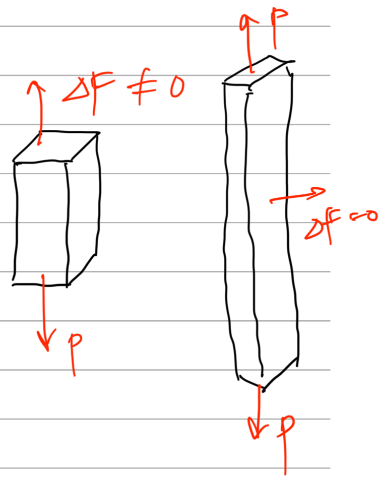
Challenge : This quotient depends on the cut surface direction

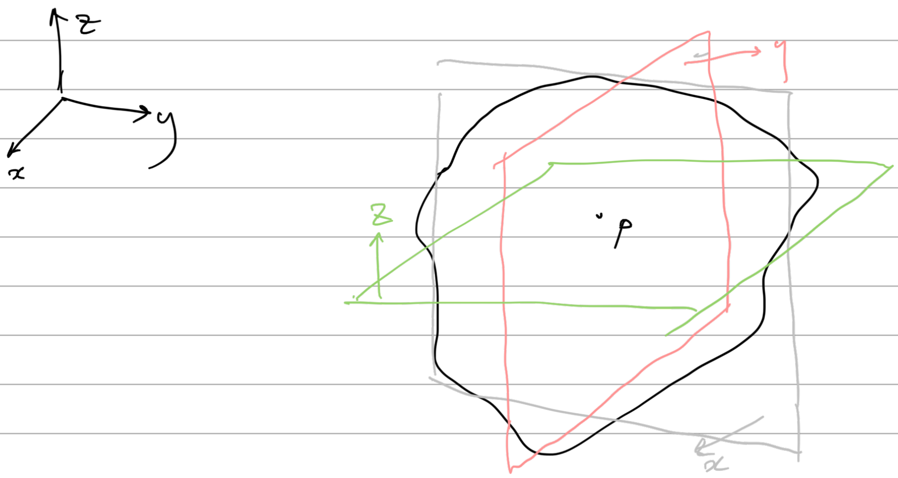
Solution : set a coordinate of your choice, then cut in three perpendicular directions

Example: Uniaxial tension test

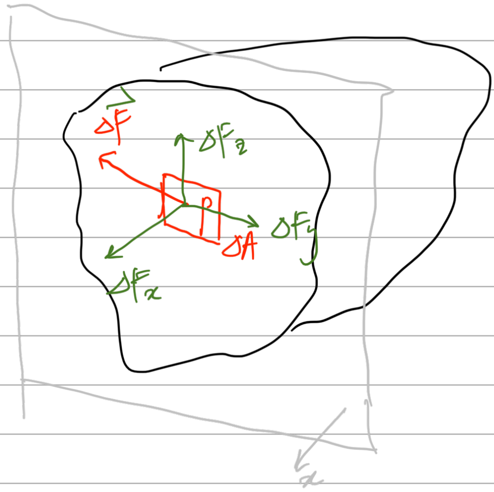


- Horizontal cut
  : Non-zero stress created by the application of P
- Vertical cut
  : ideally zero stress created.





① x − normal direction



Normal stress

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

if $\sigma_x > 0$: tensile stress

$\sigma_x < 0$: compressive stress

Shear stress

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

② y − normal direction

Normal stress

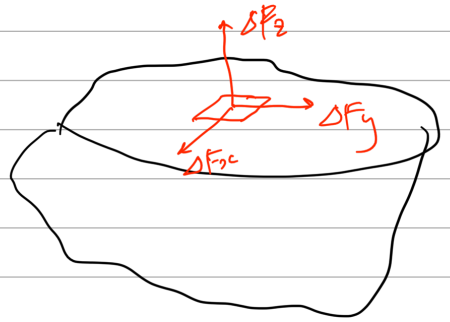
$$\sigma_y = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

Shear stress

$$\tau_{yx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{yz} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

③ z − normal direction



Normal stress

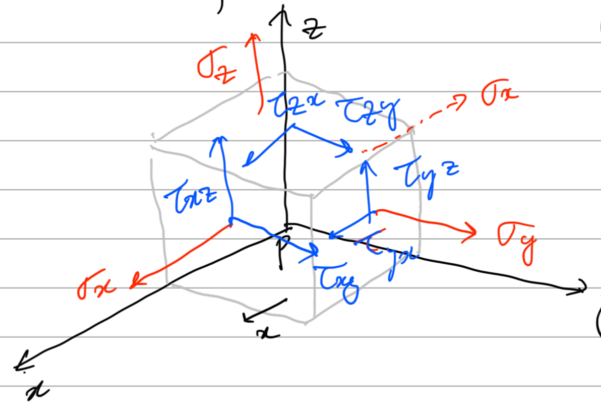
$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

shear stresses

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

State of stress at point P.



Based on action & reaction, σ (or τ) should be the same whether you see the front surface or rear surface (positive)

(positive surface : areal normal vector pointing to the positive coordinate direction.)

Three normal stresses :

$\sigma_x \quad \sigma_y \quad \sigma_z$

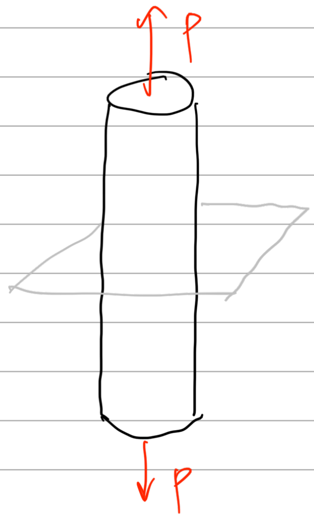
three shear stresses

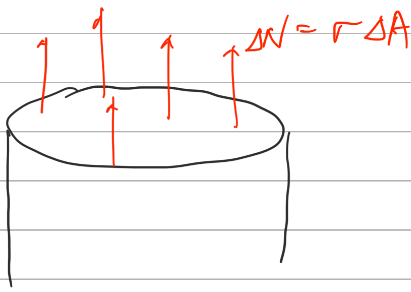
$\tau_{xy} = \tau_{yx}, \quad \tau_{xz} = \tau_{zx}, \quad \tau_{yz} = \tau_{zy}$

Units : $1\ N/m^2 = 1\ Pa$

(same unit as pressure)

1.4. Average normal stress in an axially loaded bar





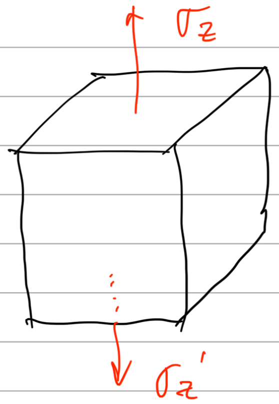
$$\int dN = \int \sigma\, dA$$

$$N = \sigma \cdot A$$

Average normal stress

$$\sigma = \frac{N}{A} = \frac{P}{A}$$

internal normal force

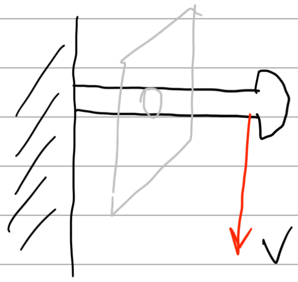


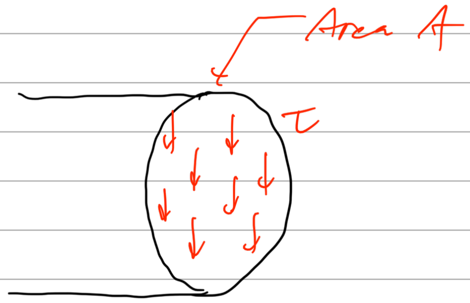
Based on equilibrium :

$$\sigma_z \cdot dA = \sigma_z' dA$$

$$\therefore \sigma_z = \sigma_z'$$

1.5. Average shear stress





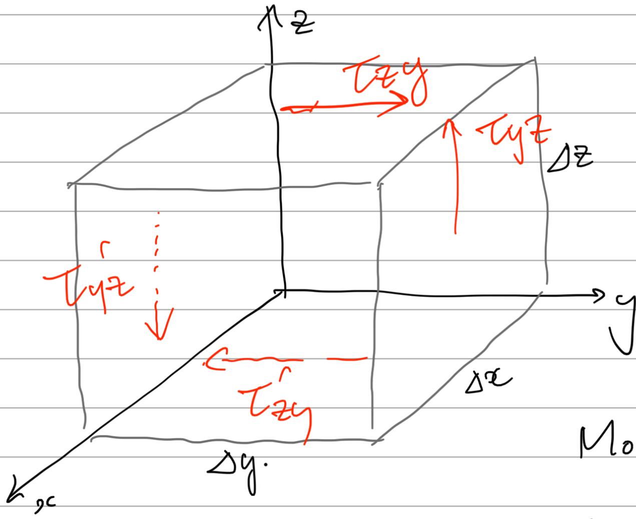
$$\int dV = \int \tau \, dA$$

$$V = \tau A$$

$$\tau = \frac{V}{A}$$

V: resultant internal shear force

τ: average shear stress



Force Equilibrium

$$\Sigma F_y = 0$$

$$\tau_{zy} \cdot \Delta x \Delta y - \tau'_{zy} \Delta x \Delta y = 0$$

$$\therefore \tau_{zy} = \tau'_{zy}$$

Moment equilibrium w.r.t. x-axis

$$\Sigma M_x = 0$$
$(\circlearrowleft +)$

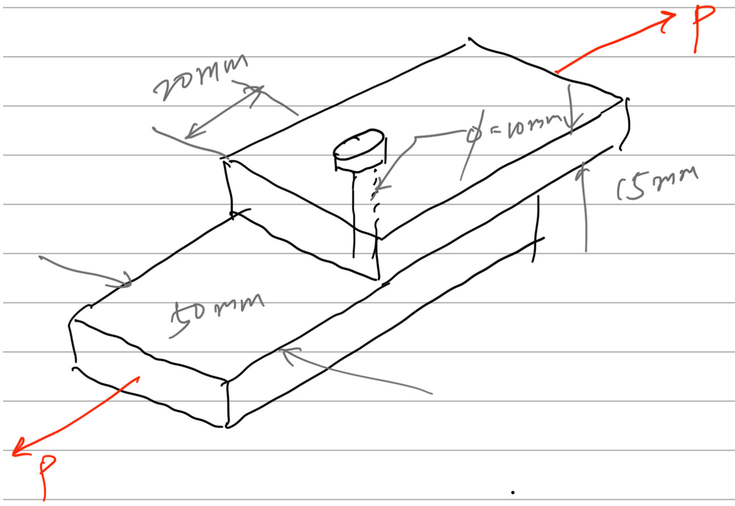
$$\tau_{zy} \cdot \Delta x \Delta y \cdot \Delta z - \tau_{yz} \cdot \Delta x \Delta z \cdot \Delta y = 0$$

$$\therefore \tau_{zy} = \tau_{yz}$$

1.6. Allowable stress design

$$\text{Factor of safety (F.S.)} = \frac{F_{fail}}{F_{allow}}$$

Example 1.12. Determine the largest load P

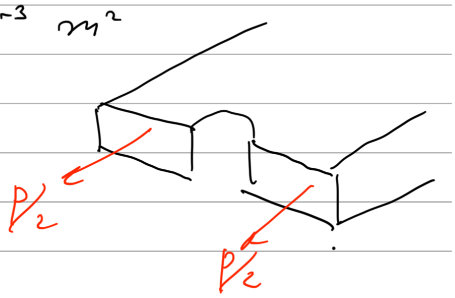


Allowable shear stress of the bolt
: 80 MPa
Allowable tensile stress of the plate
: 50 MPa
Allowable bearing stress
: 80 MPa
Allowable shear stress
: 30 MPa

1) plate tension

$$50 \times 10^6 \ N/m^2 = \frac{P/2}{15 \times 10^{-3} \times 20 \times 10^{-3} \ m^2}$$

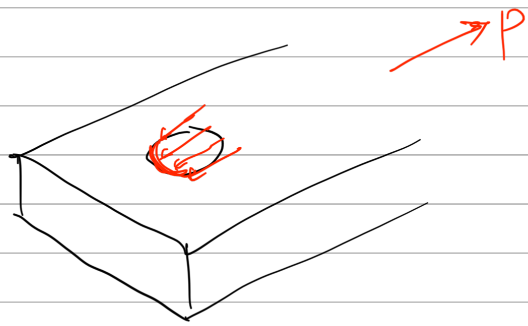
$$P = 30 \ kN$$



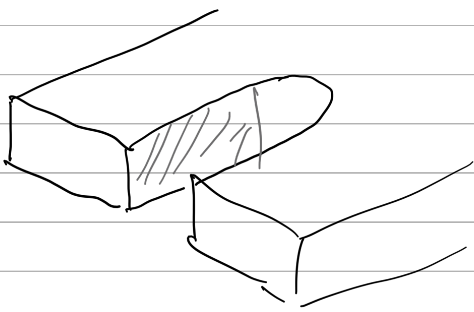
2) plate bearing

$$80 \times 10^6 \ N/m^2 = \frac{P}{15 \times 10^{-3} \times 10 \times 10^{-3} \ m^2}$$

$$P = 12 \ kN$$



3) plate shear



$$30 \times 10^6 \ N/m^2 = \frac{P/2}{20 \times 10^{-3} \times 15 \times 10^{-3} \ m^2}$$

$$P = 18 \ kN$$

4) Bolt shear ✓

$80 \times 10^6 \ N/m^2$

$$= \frac{P}{\pi \times (5 \times 10^{-3})^2 \ m^2}$$

$P = 6.28 \ kN$ ✓