# Chapter 3 Mechanical properties of materials

loads

↓

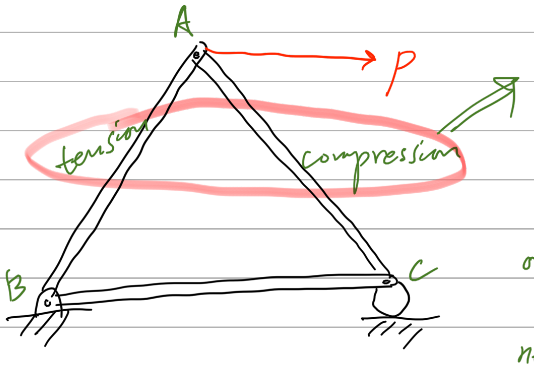
stresses

↓ ← material properties

strains

↓

displacements



$\sigma = \frac{f}{A}$

⇓

% elongation or contraction

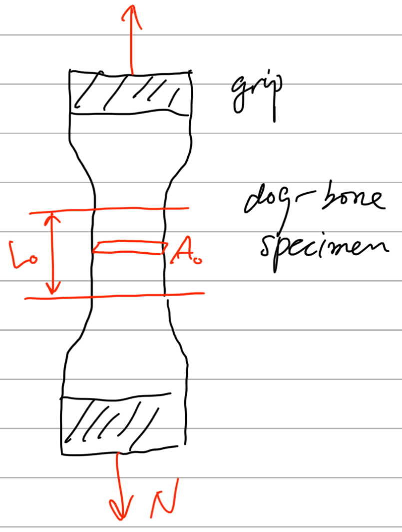
⇓

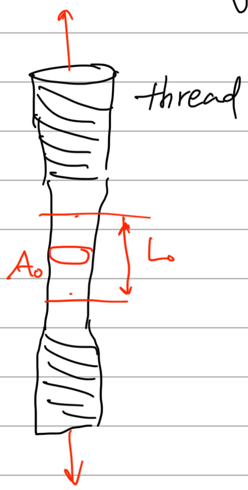
new positions of A, B and C

## Material property identification

- Coupon testing
  : standardized specimen testing (ASTM standard)
  American Society for Testing and Materials
- Computational simulations
  : useful for saving time and cost but cannot be trusted 100%

## Uniaxial tension test using load frame





Load (N)

↓

stress

$\sigma = \frac{N}{A_0}$

: nominal or engineering stress

($A_0$ : original cross-sectional area)

↓

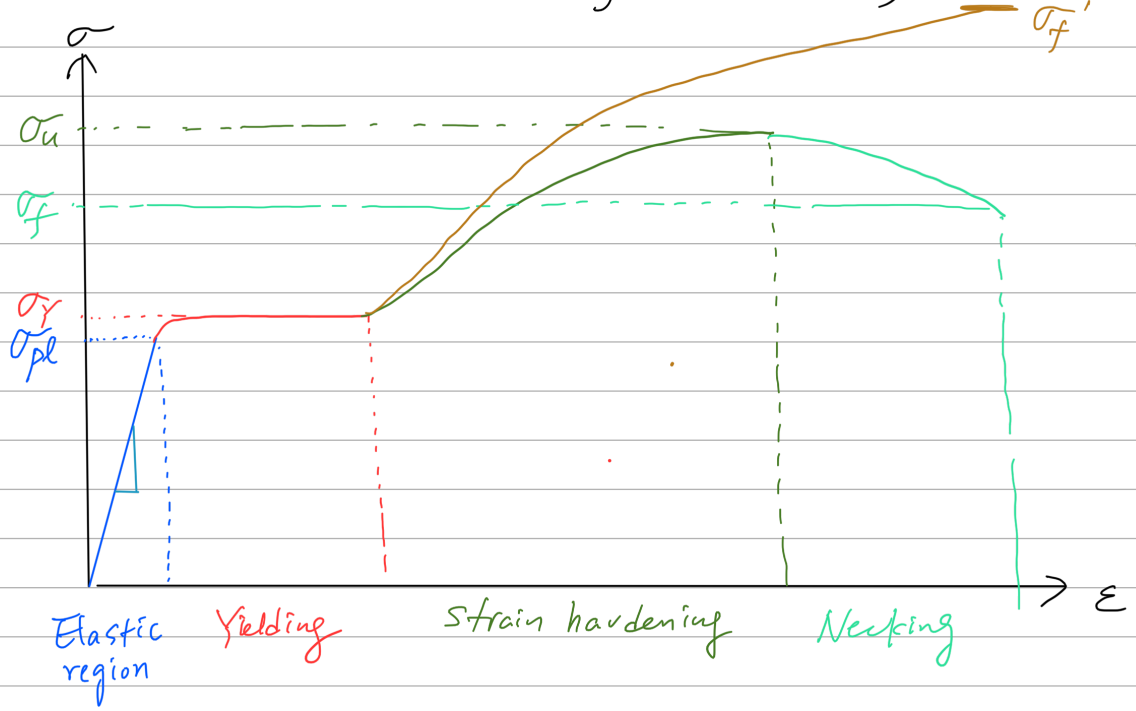
strain

$\varepsilon = \frac{\delta}{L_0}$

How to measure strain

- extensometer (contact) to get $\delta$
- strain gage (contact) to get $\varepsilon$ directly by measuring electrical resistance
- digital image correlation (non-contact)

## 3.2. Stress–strain diagram (curve)



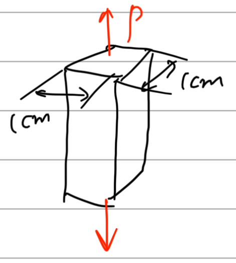
$\sigma_{pl}$ : proportional limit

$E$ : (slope) Elastic modulus, Young's modulus

$\sigma = E\varepsilon$ : Hooke's law

unit : Pa

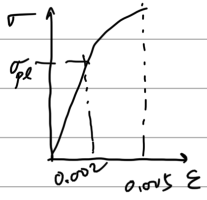
Aluminum : 70 GPa, Stainless steel : 200 GPa



$$\frac{P}{(0.01)^2} = 140 \times 10^6 \text{ Pa}$$

$$\therefore P = 140 \times 10^2 \text{ N}$$
$$= 1400 \text{ kgf}$$

$$\sigma_{pl} = 70 \times 10^9 \times 0.002 = 140 \times 10^6 \text{ Pa}$$



$\sigma_Y$ : yield stress

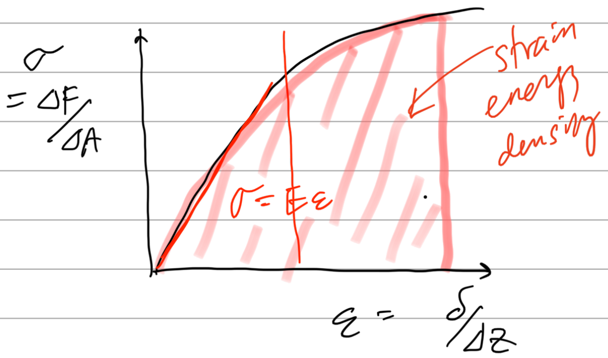
$\sigma_u$ : ultimate stress

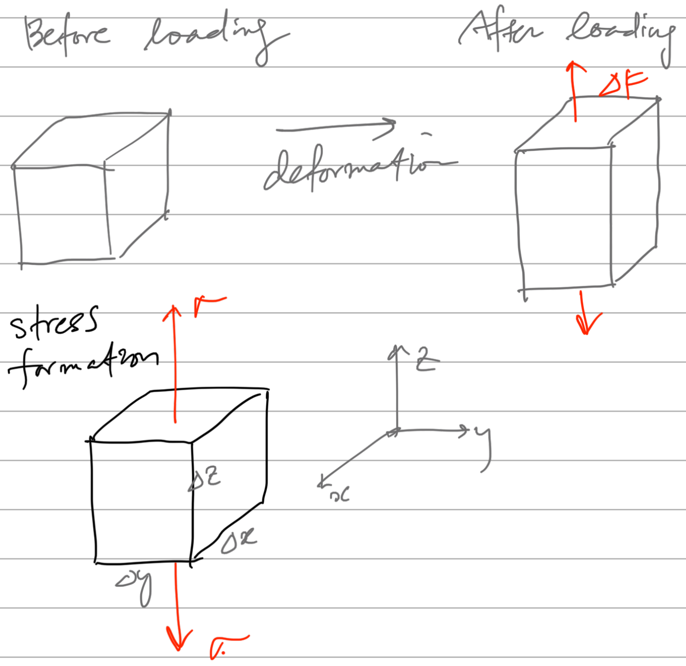
$\sigma_f$ : fracture stress

$\sigma_f'$ : true fracture stress

Visual aids : Load frame
Non-contact strain measurements
Stress-strain movie clip
Stress-strain diagram examples

# 3.4. Strain energy





Work done by an external load

$$\Delta W = \int \Delta F \, d\delta$$

$$= \int \sigma \Delta A \cdot \Delta z \cdot d\varepsilon$$

$$= \underline{\Delta V} \cdot \int \sigma \, d\varepsilon$$

ΔxΔyΔz : volume

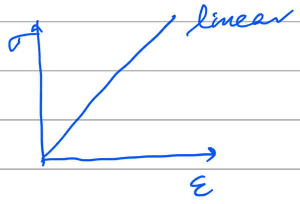
$$= \Delta U$$ : strain energy

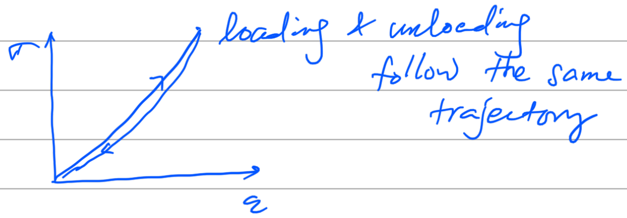
Strain energy density

$$u = \frac{\Delta U}{\Delta V} = \underline{\int \sigma \, d\varepsilon}$$

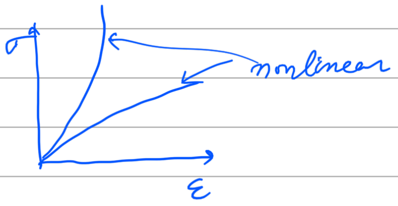
area under the stress-strain curve

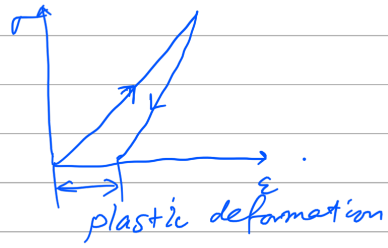
In the linear elastic regime





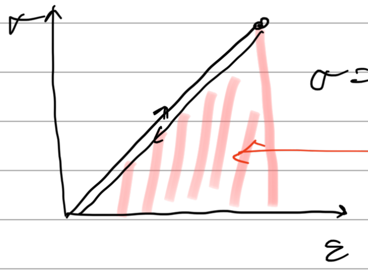
loading & unloading follow the same trajectory





plastic deformation

linear elastic

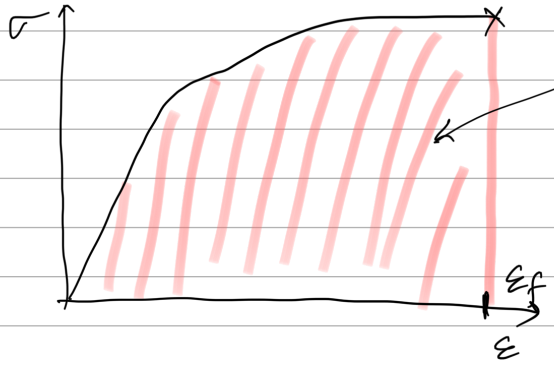


$\sigma = E\varepsilon$ : Hooke's law

$$u = \int \sigma \, d\varepsilon = \int E\varepsilon \, d\varepsilon = \frac{1}{2}E\varepsilon^2$$

$$= \frac{\sigma^2}{2E} \qquad \left(\varepsilon = \frac{\sigma}{E}\right)$$

$$= \frac{1}{2}\sigma\varepsilon$$



modulus of toughness

$$u_t = \int_0^{\varepsilon_f} \sigma \, d\varepsilon$$

$u_t$ ↑ : ductile

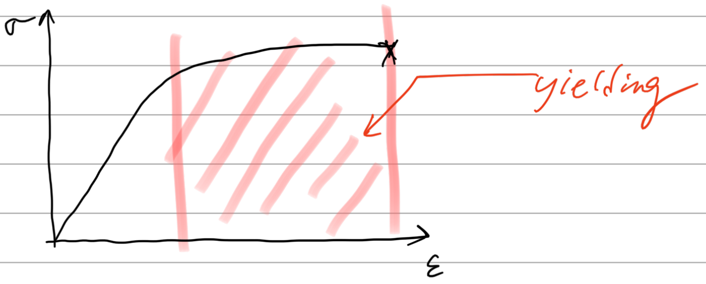
$u_t$ ↓ : brittle

### 3.3. Ductile and brittle material

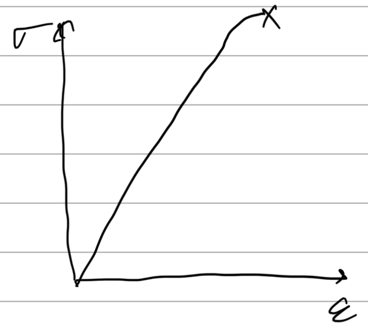
- Ductile material : subject to large strain before fracture
  - · percent elongation $= \dfrac{L_f - L_0}{L_0}$

    (mild steel : 38%)
  - · percent reduction of area $= \dfrac{A_0 - A_f}{A_0}$

    (mild steel : 60%)



- Brittle material : little or no yielding before fracture

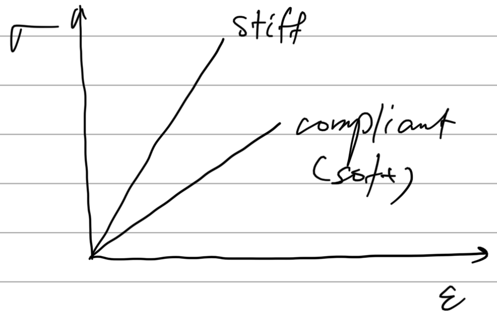


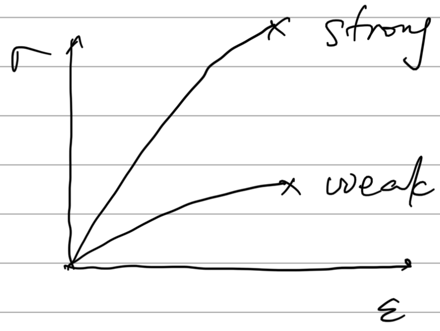
- cast iron
- concrete
- glass

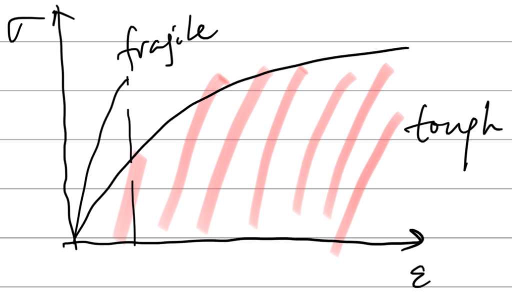
Q. Brittle or ductile for structural engineering?

- more energy absorption before failure
- sign of failure due to plasticity

Q. stiffness, strength, and toughness

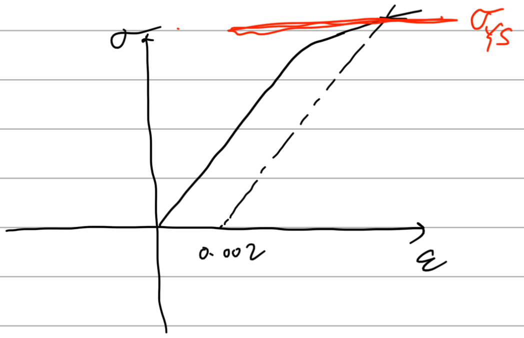




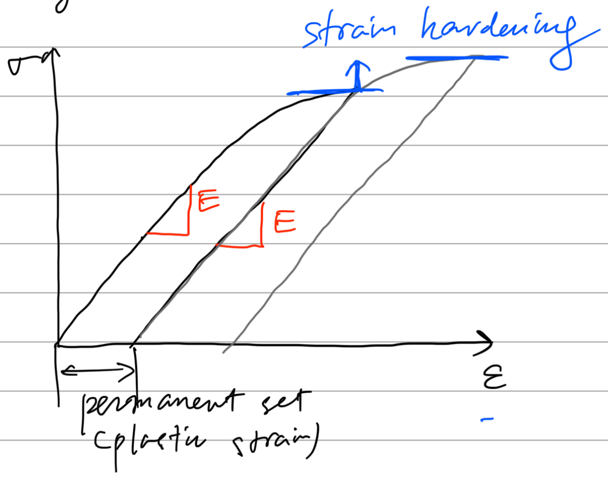


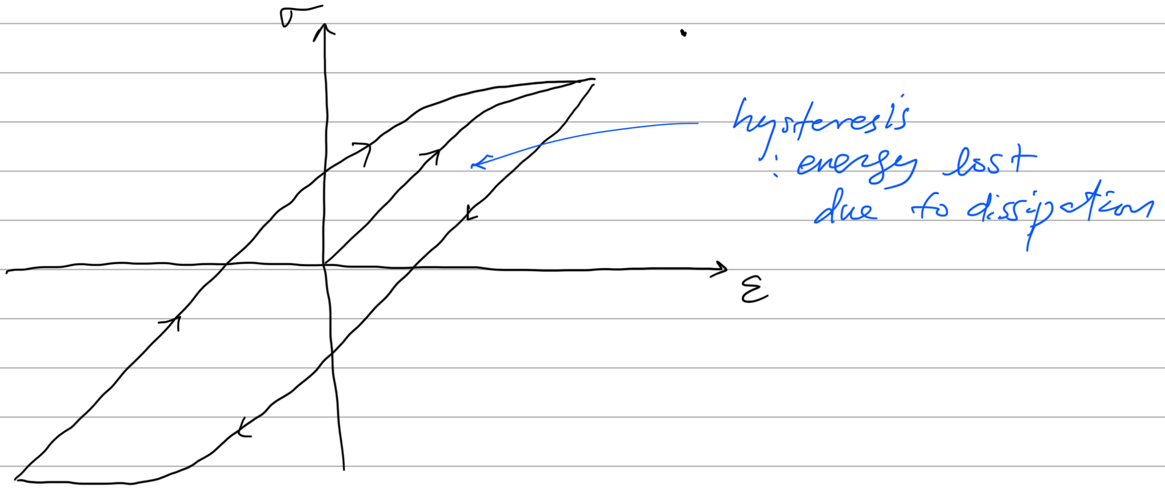
glass : stiff but not strong or tough

Q When yield stress cannot be clearly identified.

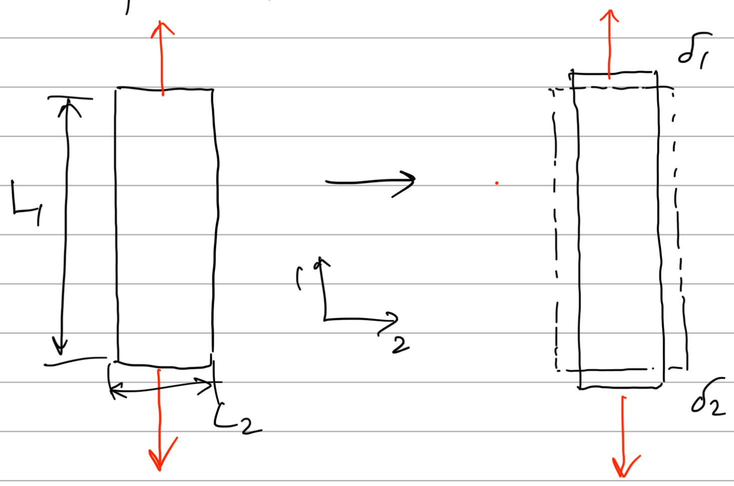


# Strain hardening





## 3.5. Poisson's ratio



$$\varepsilon_1 = \frac{\delta_1}{L_1}$$

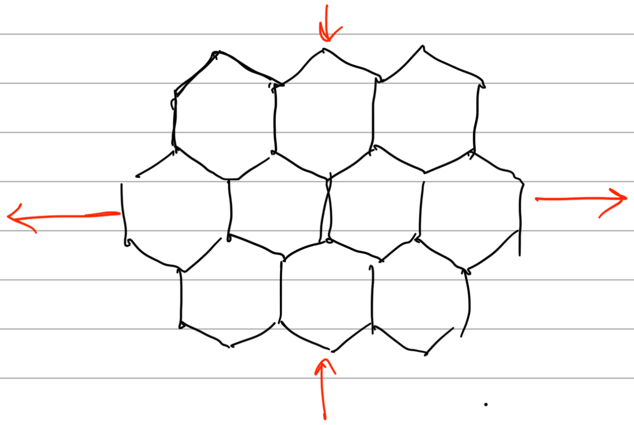
$$\varepsilon_2 = \frac{\delta_2}{L_2}$$

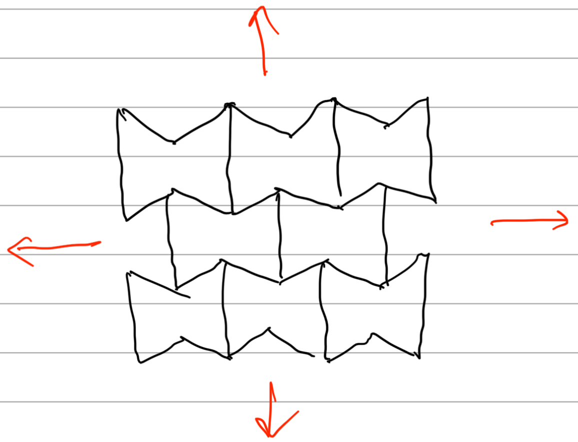
$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$ : Poisson's ratio

Typically $0 \lesssim \nu \lesssim 0.5$ ($\simeq 0.3$ for most materials)

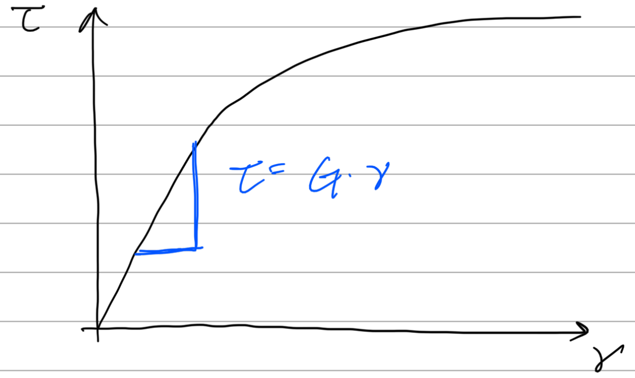
## Negative poisson's ratio materials
(i.e. Auxetic materials)

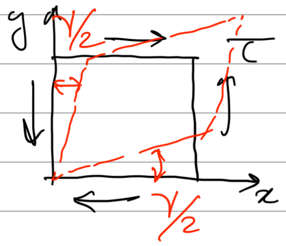
e.g. bone
architected material





## 3.6. Shear stress– strain diagram

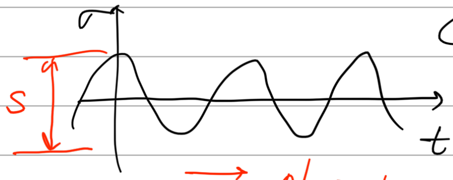




G : shear modulus (of elasticity)

$G = \frac{E}{2(1+\nu)}$ for isotropic material (linear elastic)

## 3.7. Fatigue



Cyclic loading

→ N cycles until failure