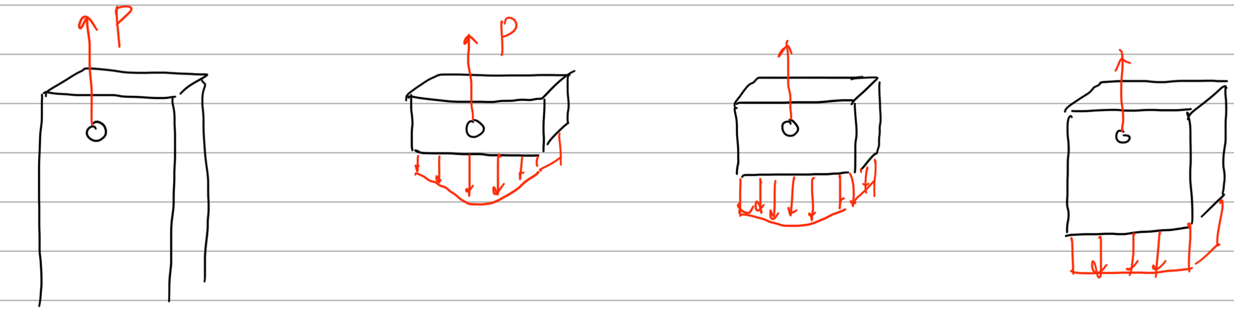
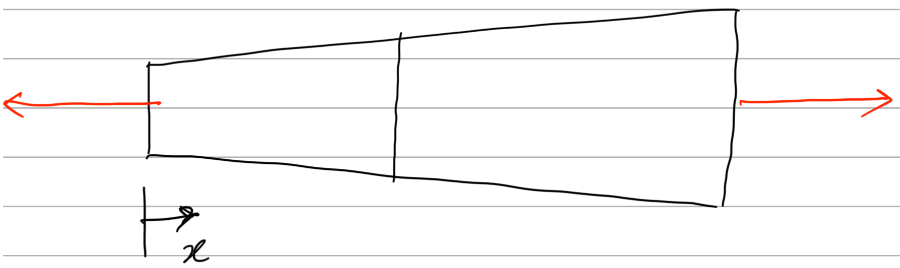
Chapter 4. Axial load



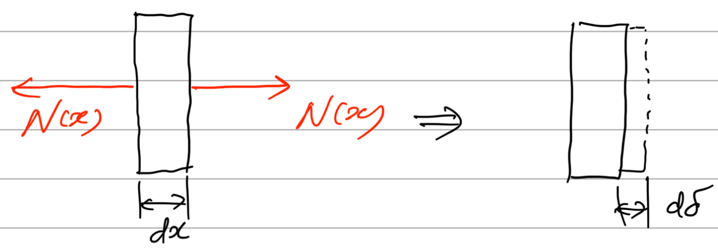
Localized stresses and deformation tend to become uniform away from the point loading

: Saint-Venant's principle



Load $(N(x))$

$\downarrow$



Stress

$$\sigma(x) = \frac{N(x)}{A(x)}$$

$\downarrow$

strain

$$\varepsilon(x) = \frac{\sigma(x)}{E(x)}$$

$(E(x) = E$ if the bar is made of a homogeneous material$)$

$\downarrow$

displacement

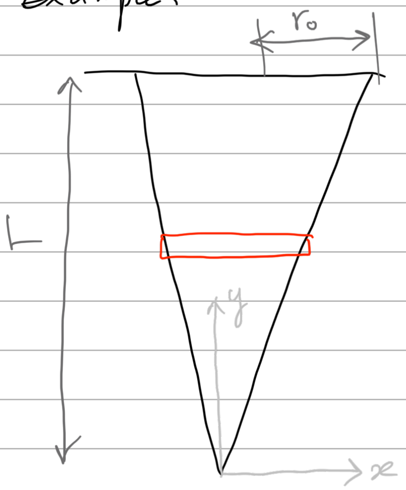
$$\frac{d\delta}{dx} = \varepsilon(x)$$

$$\delta = \int d\delta = \int \frac{\sigma(x)}{E(x)} \cdot dx = \int \frac{N(x)}{E(x) \cdot A(x)} \cdot dx$$

A, E (constant cross-section and Young's modulus)

$$\delta = \frac{NL}{EA} \qquad \left(\begin{array}{l} \delta > 0 \text{ for elongation} \\ \delta < 0 \text{ for contraction} \end{array}\right)$$

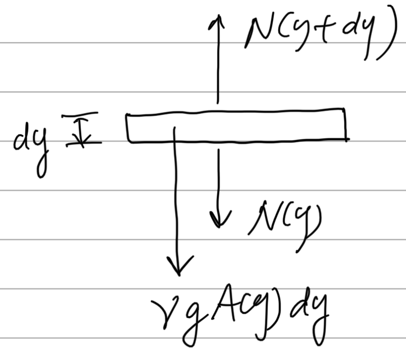
Example.



Cone with specific weight γ ($W = \gamma \cdot V$)

Q. Tip displacement due to gravity.

- Geometry
- Material property
- Boundary condition & loading condition



1) Load

$$A(y) = \pi\, r(y)^2 = \pi\left(\frac{L}{r_0}\cdot y\right)^2$$

$$\Sigma F_y = 0$$

$$N(y+dy) - N(y) - \gamma \cdot \pi\left(\frac{L}{r_0}y\right)^2 \cdot dy = 0$$

$$\frac{N(y+dy) - N(y)}{dy} = \gamma \cdot \pi\left(\frac{L}{r_0}y\right)^2 = \frac{dN_y}{dy}$$

$$\therefore N_y = \int \gamma\, \pi\left(\frac{L}{r_0}y\right)^2 dy = \pi\gamma\, \frac{L^2}{r_0^2}\cdot\frac{y^3}{3} + C$$

$$N_y(y=0) = C = 0$$

2) Stress

$$\sigma(y) = \frac{N(y)}{A(y)} = \frac{\cancel{\pi}\gamma\, \cancel{\frac{L^2}{r_0^2}}\, \frac{y^{\cancel{3}}}{3}}{\cancel{\pi}\, \cancel{\frac{L^2}{r_0^2}}\cdot \cancel{y^2}} = \frac{\gamma}{3} y$$

3) strain

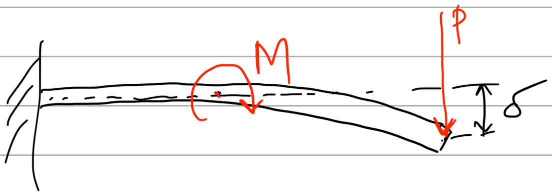
$$\varepsilon(y) = \frac{\sigma(y)}{E} = \frac{\gamma}{3E}\,y$$

4) displacement

$$\frac{d\delta}{dy} = \varepsilon \rightarrow \delta = \int d\delta = \int_0^L \varepsilon\, dy$$

$$= \frac{\gamma}{3E} \cdot \left.\frac{y^2}{2}\right|_0^L = \frac{\gamma L^2}{6E}$$

4.3. Principle of superposition



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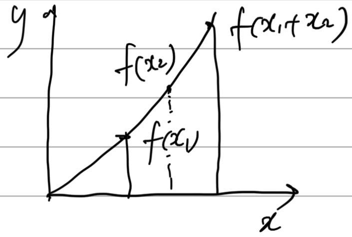
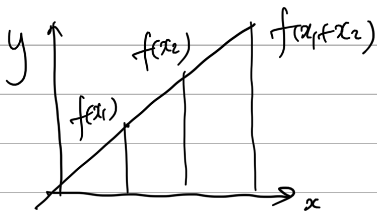
Q: $\delta = \delta_1 + \delta_2$?

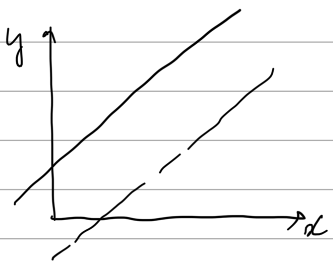
Yes, only when the structure behaves linearly under loading



$$f(x_1) + f(x_2) \neq f(x_1 + x_2)$$



$$f(x_1) + f(x_2) = f(x_1 + x_2)$$

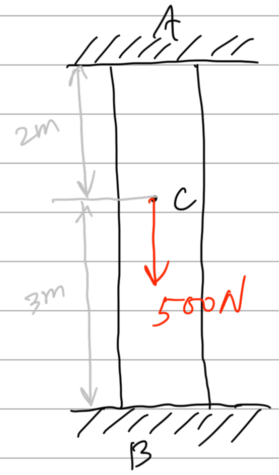


Q: superposition principle works?

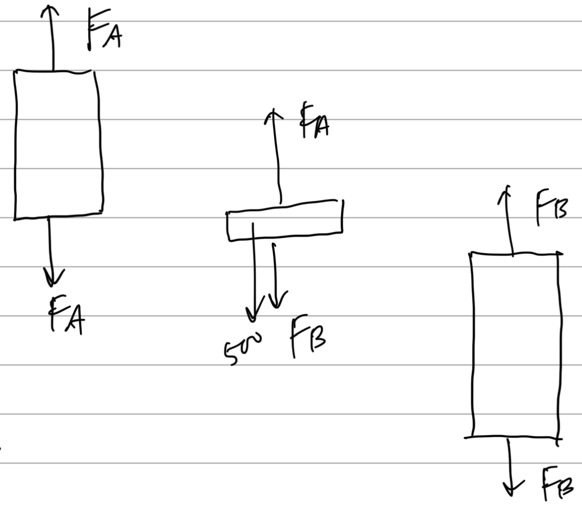
4.4. Statically determinate vs. indeterminate

If loads can be determined by using conventional statics approach, the structure is statically determinate.

If not, statically indeterminate.



A: cross-sectional area



$$F_A - F_B - 500 = 0 \quad (1)$$

2 unknowns and 1 equation → Need one more equation

$$\delta_{AC} = \frac{F_A}{EA} \cdot \overline{AC}$$

$$\delta_{BC} = \frac{F_B}{E \cdot A} \cdot \overline{BC}$$

$$\delta_{AC} + \delta_{BC} = \frac{1}{EA} \cdot (2F_A + 3F_B) = 0 \quad (2)$$

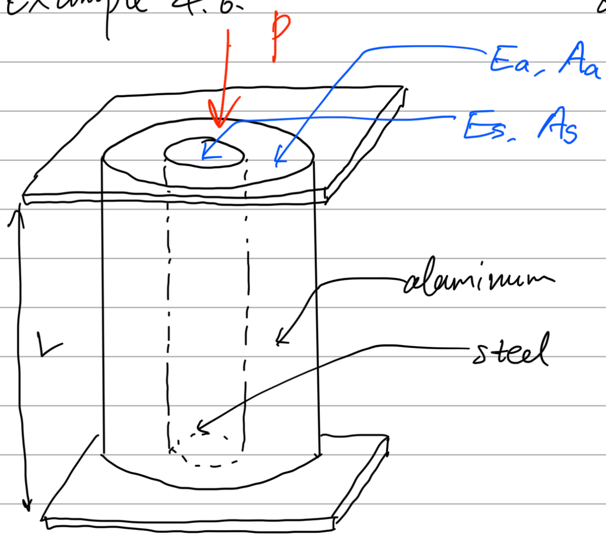
From eqs. (1) & (2)

$$F_A - \left(-\frac{2}{3}F_A\right) - 500 = \frac{5}{3}F_A - 500 = 0$$

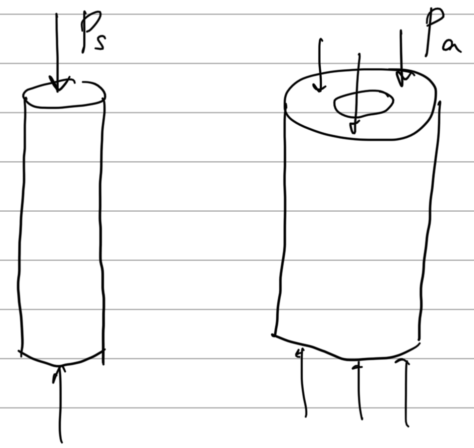
$\therefore F_A = 300$, $F_B = -200$

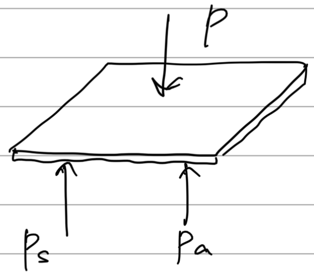
(tension) (compression)

Example 4.6.



stresses taken by aluminum & steel





$P = P_a + P_s$ —(1)

→ statically indeterminate

stresses

$\sigma_s = \dfrac{P_s}{A_s}, \qquad \sigma_a = \dfrac{P_a}{A_a}$

strains

$\varepsilon_s = \dfrac{P_s}{E_s \cdot A_s}, \qquad \varepsilon_a = \dfrac{P_a}{E_a A_a}$

Displacements

$\delta_s = \varepsilon_s \cdot L = \dfrac{P_s \cdot L}{E_s \cdot A_s}, \qquad \delta_a = \varepsilon_a L = \dfrac{P_a L}{E_a A_a}$

$\delta_s = \delta_a \rightarrow \dfrac{P_s}{E_s A_s} = \dfrac{P_a}{E_a A_a}$ —(2)

From eqns (1) & (2)

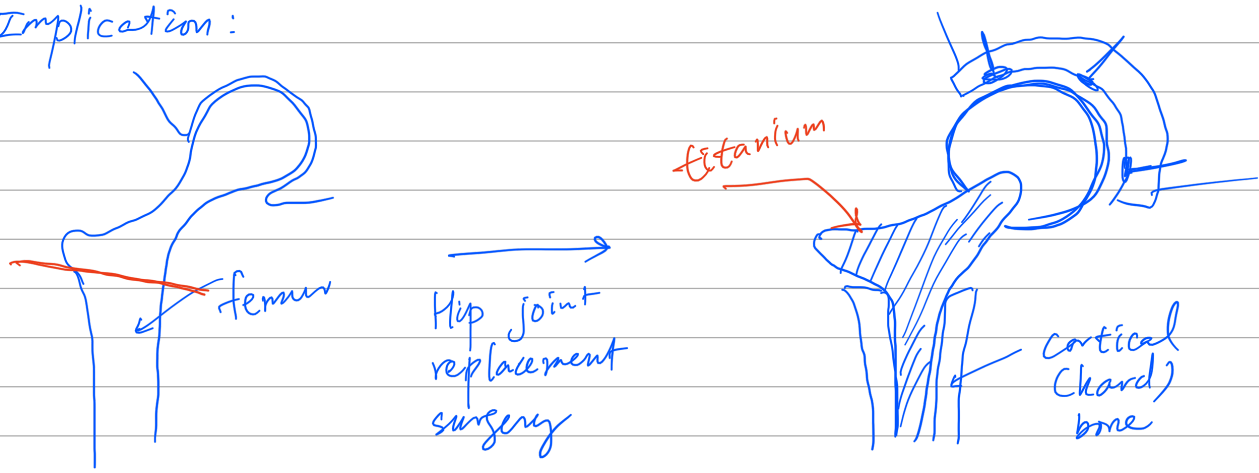
$P = P_a + \dfrac{E_s A_s}{E_a A_a} \cdot P_a \qquad \therefore P_a = \dfrac{E_a A_a}{E_a A_a + E_s A_s} P$

$P_s = \dfrac{E_s A_s}{E_a A_a + E_s A_s} P$

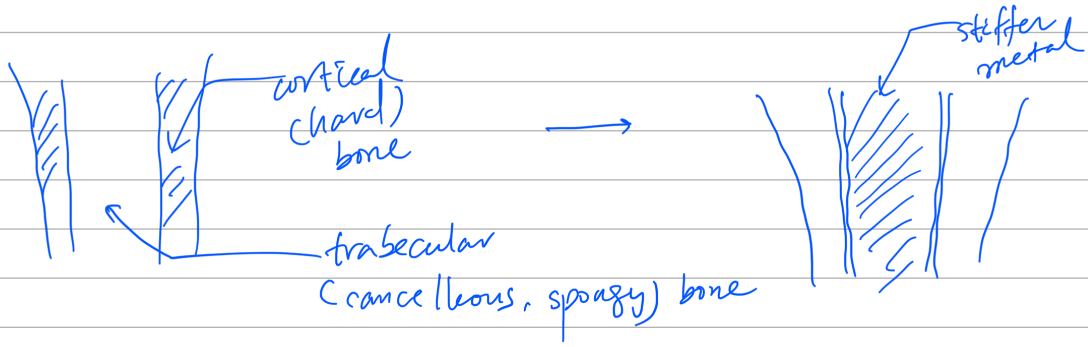
stress ratio $\dfrac{\sigma_s}{\sigma_a} = \dfrac{P_s / A_s}{P_a / A_a} = \dfrac{E_s}{E_a}$

→ stiffer materials take more stresses.

Implication:



original bone



Wolff's law : bone adapts to loads

e.g., astronauts

tennis player

senior citizen

→ Bone resorption, often requiring re-surgery after THA (total hip arthroplasty)

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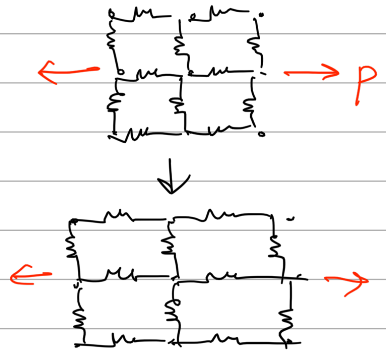
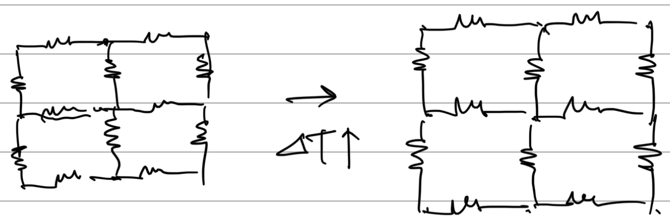
Handout: Wolff's law

4.6. Thermal Stress

strain
- by mechanical loading
- by thermal loading (i.e., temperature change)





$$\varepsilon_T = \alpha \Delta T$$

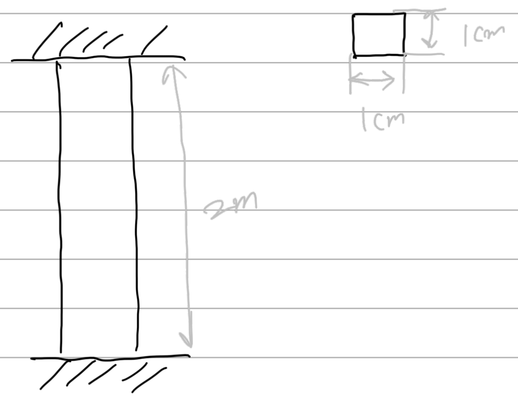
α : CTE (coefficient of thermal expansion) [1/K, 1/°C]

CTE of aluminum 24×10^{-6} /°C

e.g.) 100°C ↑ → $\varepsilon_T = 24 \times 10^{-4} = 0.24\%$ (1m → 2.4mm)

$$\delta_T = \varepsilon_T \cdot L = \alpha \Delta T \cdot L$$

Example 4.10.



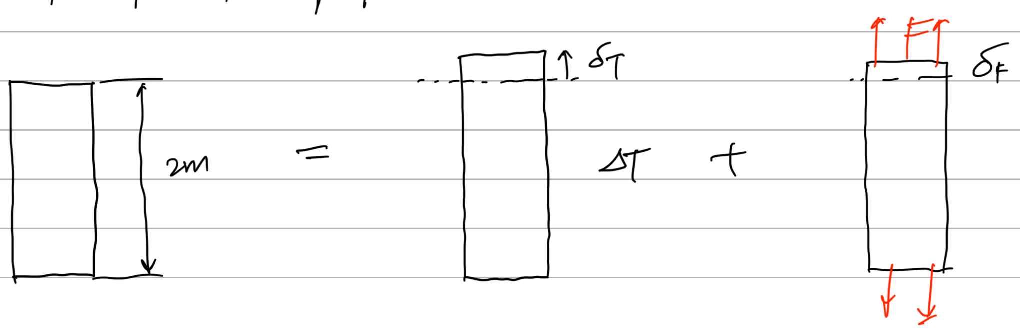
Bar constrained by a fixed wall

$\Delta T = 100°C$

Thermal stresses developed in the bar.

$\alpha = 20 \times 10^{-6}/°C$

Using principle of superposition :



$\delta_T = \varepsilon_T L = \alpha \Delta T L$
$= 20 \times 10^{-6} \times 100 \times 2 = 4 \times 10^{-3} = 4mm$

$\delta_F = \varepsilon \cdot L = \frac{F}{EA} L$

$\delta_T + \delta_F = \alpha \Delta T L + \frac{FL}{EA} = 0$

$\therefore F = -\alpha \Delta T EA$
$= -20 \times 10^{-6} \times 100 \times 70 \times 10^9 \times (10^{-2})^2$
$= -1.4 \times 10^4 N \quad (\simeq -1400 \text{ kgf})$

$\sigma_T = \frac{F}{A} = \frac{-1.4 \times 10^4}{10^{-4}} = -1.4 \times 10^8 \text{ Pa}$
$= -1400 \text{ MPa}$

(yield stress of aluminum in tension : ~300 MPa)

Handout : temperature effect