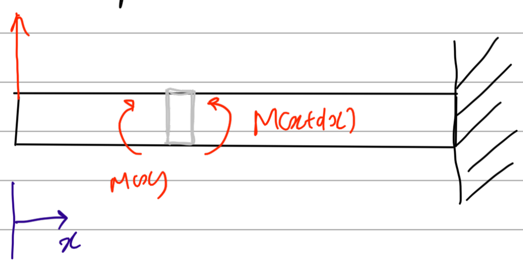
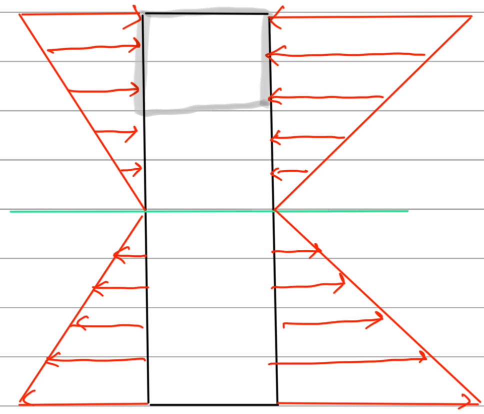
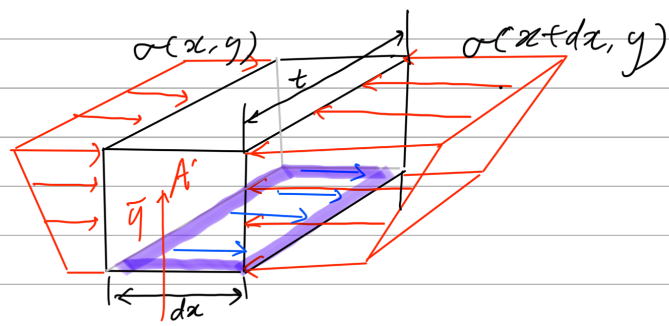
Chapter 7. Transverse Shear







$\Sigma F_x = 0 \qquad -\int_{A'} \sigma(x+dx, y)\, dA + \int_{A'} \sigma(x,y)\, dA + \tau\, t \cdot dx = 0$

$$\tau = \frac{1}{t} \cdot \int_{A'} \frac{\sigma(x+dx, y) - \sigma(x,y)}{dx}\, dA$$

$$= \frac{1}{t} \int_{A'} \frac{d\sigma}{dx}\, dA$$

$$\left(\sigma = + \frac{M}{I} y\right)$$

$$= \frac{1}{t} \int_{A'} \frac{dM}{dx} \cdot \frac{y}{I}\, dA$$

$$\left(\frac{dM}{dx} = V\right)$$

$$= \frac{V}{I \cdot t} \int_{A'} y\, dA$$

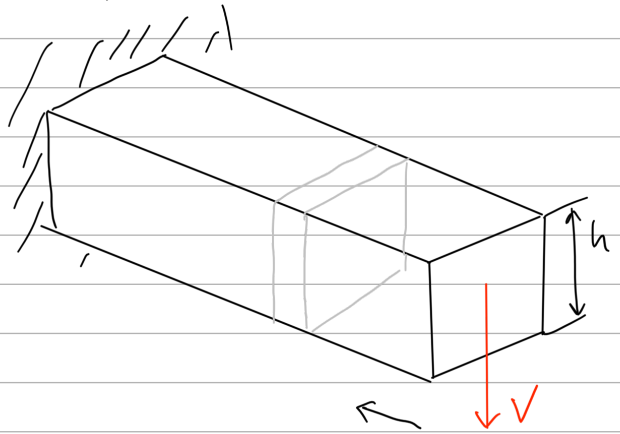
$Q = \int y\, dA$: First moment of the cross-sectional area

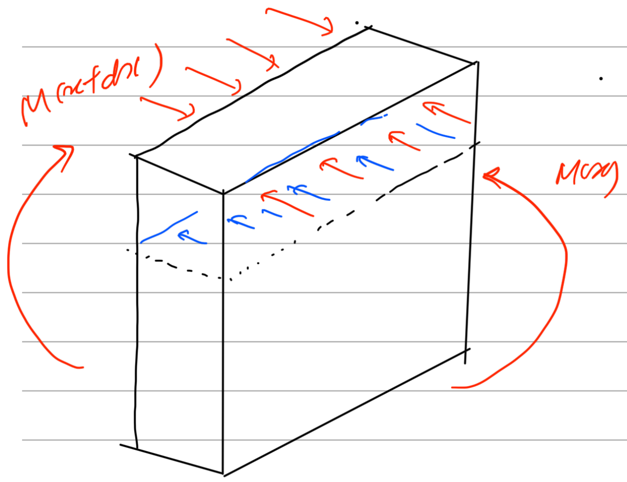
$= \bar{y} \cdot A'$

distance from the neutral axis to the centroid of A'

Example 7.2.

Shear stress distribution





$$-\int \sigma(x+dx,y)\,dA + \int \sigma(x,y)\,dA + \tau b\,dx = 0$$

$$\tau = \frac{1}{b}\cdot\int \frac{d\sigma}{dx}\,dA$$

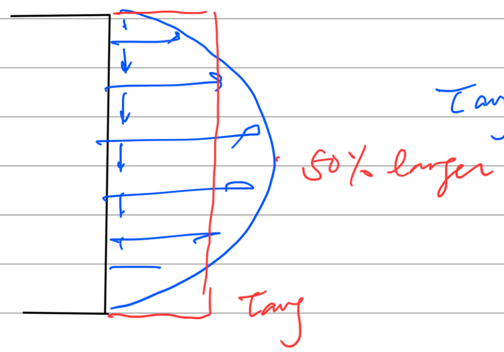
$$\left(\sigma = \frac{M}{I}y, \quad \frac{d\sigma}{dx} = \frac{y}{I}\frac{dM}{dx} = \frac{Vy}{I}\right)$$

$$= \frac{V}{I\cdot b}\int y\,dA$$

$$Q = \int_y^{h/2} y\,b\,dy = \frac{b}{2}\left(\frac{h^2}{4}-y^2\right)$$

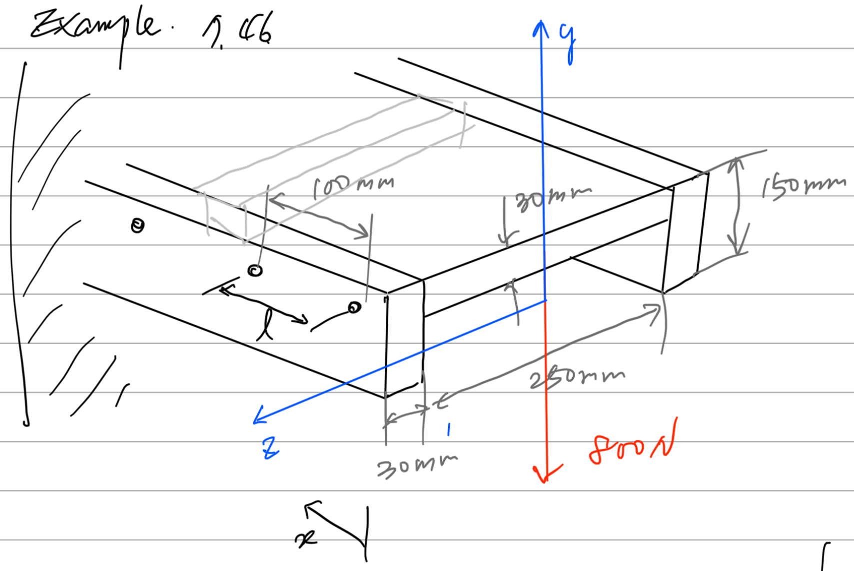
$$\therefore \tau = \frac{V}{\frac{bh^3}{12}\cdot\cancel{b}}\cdot\frac{\cancel{b}}{2}\left(\frac{h^2}{4}-y^2\right)$$

$$= \frac{3V}{2bh}\left[1-\left(\frac{y}{2h}\right)^2\right]$$

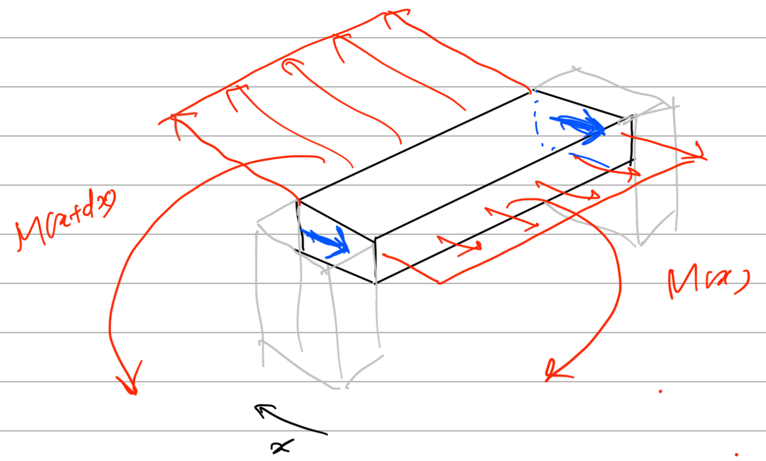


$$\tau_{avg} = \frac{V}{A} = \frac{V}{bh}$$

Example. 7.46



Nail diameter : 2 mm
Average shear stress developed in the nail?



$$\int \sigma(x+dx)\cdot dA - \int \sigma(x)\,dA - 2\tau\cdot t\cdot dx = 0$$

$$\rightarrow \tau = \frac{1}{2t}\int \frac{\sigma(x+dx)-\sigma(x)}{dx}\,dA$$

$$= \frac{1}{2t}\cdot\int \frac{d\sigma}{dx}\cdot dA$$

$$\left(\sigma = \frac{M}{I}y,\quad \frac{dM}{dx} = V\right)$$

$$= \frac{V}{2tI}\cdot\int y\,dA$$

$$\bar{y}\cdot(2\times 30\times 150 + 30\times 250) = 75\times(30\times 150\times 2) + (150-15)\times(30\times 250)$$

$$\left(\text{centroid}\quad \bar{y} = \frac{1}{A}\int y\,dA\right)$$

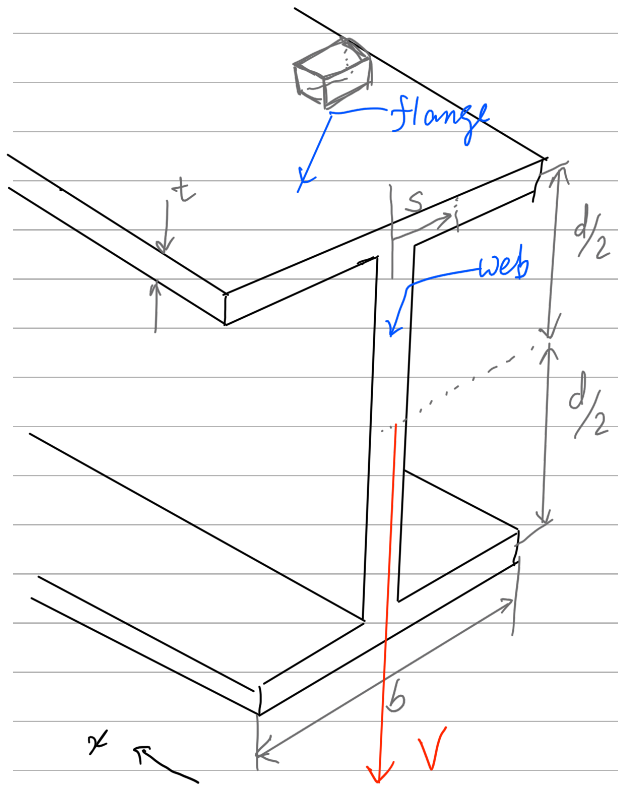
$$= 102.3\ \text{mm}$$

$$I = \frac{1}{12}\times 0.250\times 0.03^3 + 0.250\times 0.03\times(0.135-0.1023)^2$$
$$+ 2\left[\frac{1}{12}\times 0.03\times 0.150^3 + 0.03\times 0.150\times(0.1023-0.075)^2\right]$$
$$= 32.165\times 10^{-6}\ m^4$$

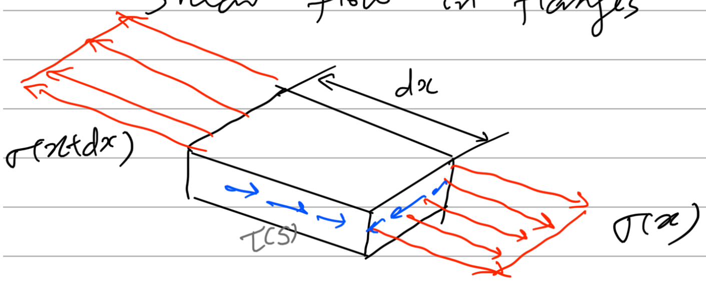
$$Q = \int y\,dA = (0.150-0.1023)\cdot(0.250\times 0.030) = 0.245\times 10^{-3}\ m^3$$

$$\tau\cdot t\cdot l = \tau_n\cdot\pi\left(\frac{d}{2}\right)^2 \rightarrow \tau_n = \frac{\frac{VQ}{2I}\cdot l}{\pi\left(\frac{d}{2}\right)^2} = 97.2\ \text{MPa}$$

7.4. Shear flow in thin-walled members



– Shear flow in flanges



$$\int \sigma(x+dx)\,dA - \int \sigma(x)\,dA$$
$$- \tau(s)\cdot t\,dx = 0$$

$$\tau(s) = \frac{1}{t}\int \frac{d\sigma}{dx}\,dA$$

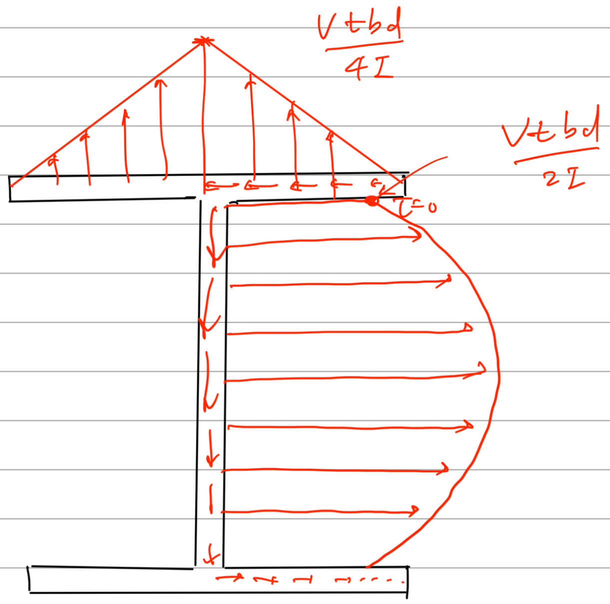
$$\left(\sigma = \frac{M}{I}\cdot y, \quad \frac{dM}{dx} = V \right)$$

$$= \frac{VQ}{tI}$$

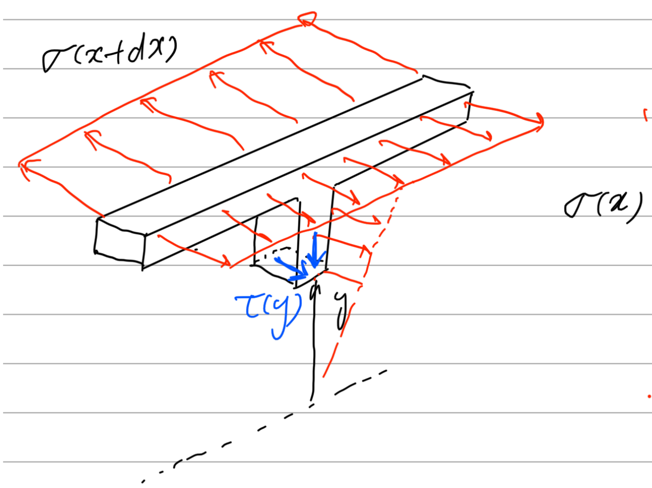
$$Q = \frac{d}{2}\cdot\left(\frac{b}{2} - s\right)\cdot t$$

$$\therefore \tau(s) = \frac{Vd}{2I}\left(\frac{b}{2} - s\right)$$

Shear flow $q(s) = \tau(s)\cdot t = \frac{Vtd}{2I}\left(\frac{b}{2} - s\right)$



Shear flow in web



$$\int \sigma(x+dx)\,dA - \int \sigma(x)\,dA$$
$$- \tau(y)\cdot t \cdot dx = 0$$

$$\tau(y) = \frac{1}{t}\int \frac{d\sigma}{dx}\,dA$$
$$= \frac{VQ}{tI}$$

$$Q = \int y\,dA$$
$$= bt\cdot\frac{d}{2} + \int_y^{d/2} y\cdot t\,dy$$
$$= \frac{b\cdot t\cdot d}{2} + t\left[\frac{y^2}{2}\right]_y^{d/2}$$
$$= \frac{btd}{2} + \frac{t}{2}\left[\frac{d^2}{4} - y^2\right]$$

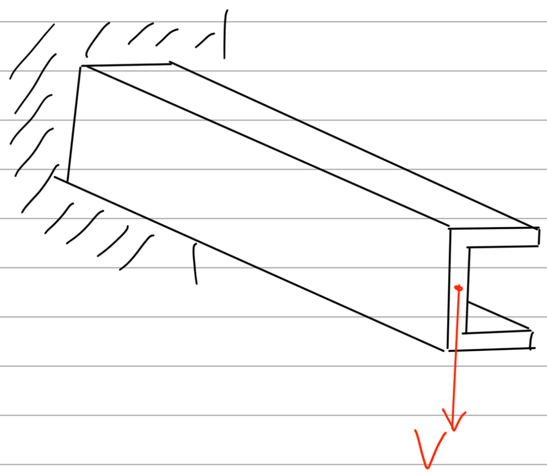
$$\tau(y) = \frac{V}{I}\left[\frac{bd}{2} + \frac{1}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

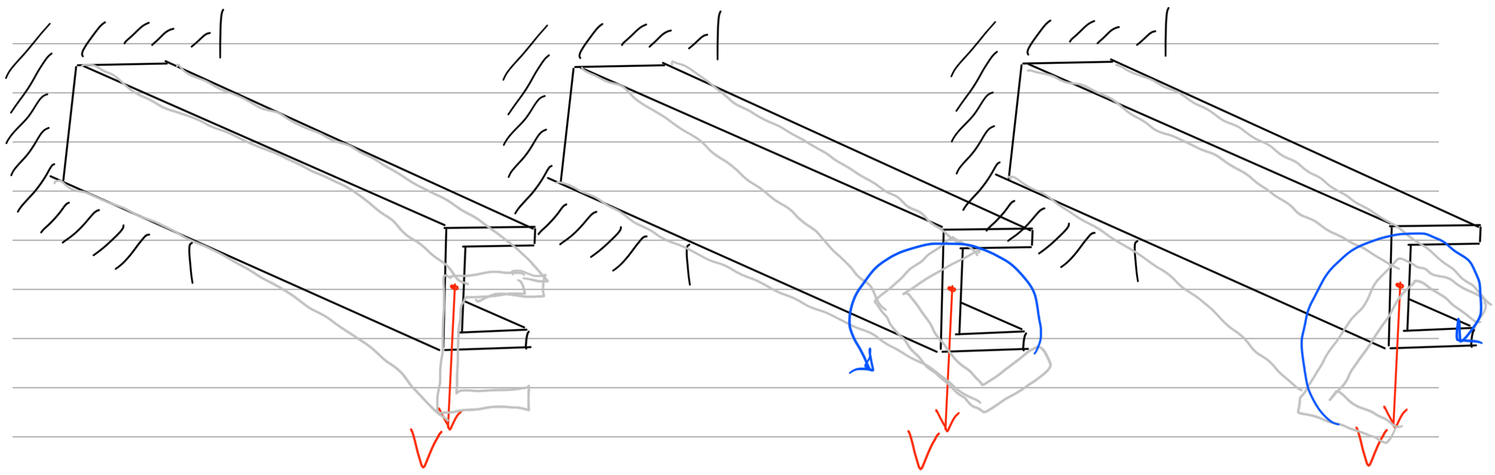
$$q(y) = \frac{Vt}{I}\left[\frac{bd}{2} + \frac{1}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

$$I = \frac{1}{12} t d^3 + 2\left[\cancel{\frac{1}{12} b t^3} + tb \cdot \left(\frac{d}{2}\right)^2\right]$$

$$= \frac{td^2}{4}\left(\frac{1}{3}d + 2b\right)$$

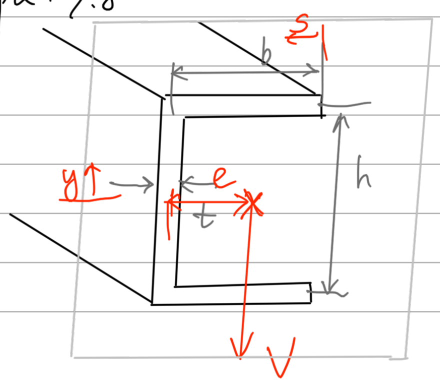
7.5. Shear center





Shear center : point through which a force can be applied which will cause a beam to bend without twist.

Example. 7.8



Shear flow

$$\tau = \frac{VQ}{It}$$

$$I = \frac{1}{12}th^3 + 2\left[\cancel{\frac{1}{12}bt^3} + bt\cdot\left(\frac{h}{2}\right)^2\right]$$

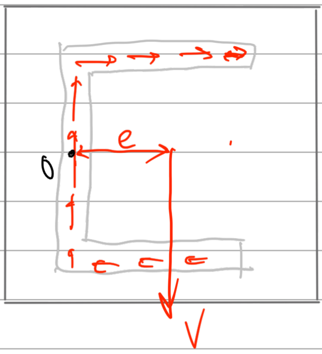
$$= \frac{th^2}{2}\left(\frac{h}{6} + b\right)$$

flange $\quad Q = \int y\, dA = \frac{h}{2}\cdot ts$

web $\quad Q = \int y\, dA = bt\cdot\frac{h}{2} + \int_y^{h/2} y\, dy\cdot t = \frac{bth}{2} + t\left[\frac{h^2}{4} - y^2\right]$

$$\therefore \tau = \frac{V\cdot\frac{h}{2}ts}{It} = \frac{Vh}{2I}s \quad \text{for flange}$$

$$= \frac{V\cdot\frac{bth}{2} + t\left[\frac{h^2}{4} - y^2\right]}{It} \quad \text{for web}$$



$$\sum M_o = Ve + 2\int_0^b \tau\cdot t\,ds\cdot\frac{h}{2}$$

$(\curvearrowright +)$

$$= V\cdot e + th\int_0^b \frac{Vh}{2I}s\,ds$$

$$= Ve + \frac{Vth^2}{2I}\frac{b^2}{2}$$

$$\therefore e = -\frac{th^2b^2}{4I} = -\frac{\cancel{th^2}b^2}{2\cancel{th^2}(h/6 + b)}$$

$$= -\frac{b^2}{2(h/6 + b)}$$

$\Rightarrow$ PPT slides