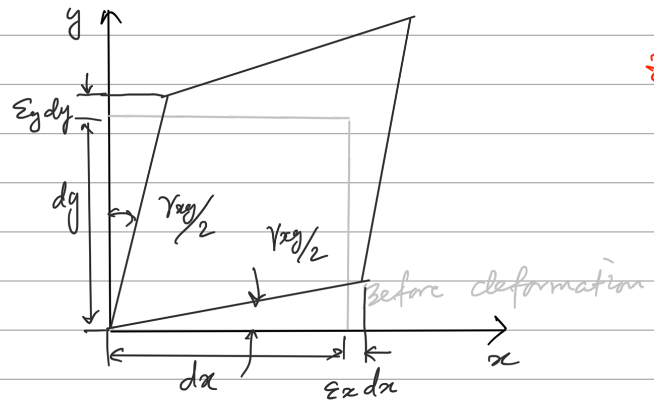
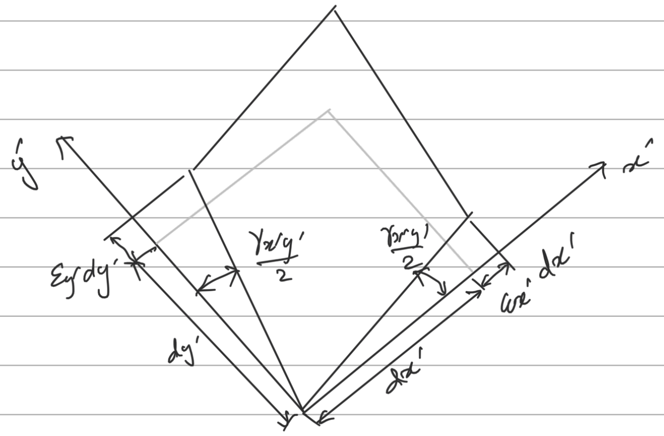
Chapter 10. Strain transformation

Plane strain condition $\qquad \varepsilon_x \quad \varepsilon_y \quad \cancel{\varepsilon_z} \rightarrow 0$

$\gamma_{xy} \quad \cancel{\gamma_{xz}} \rightarrow 0 \quad \cancel{\gamma_{yz}} \rightarrow 0$

Zero out-of-plane strains





$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

c.f.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

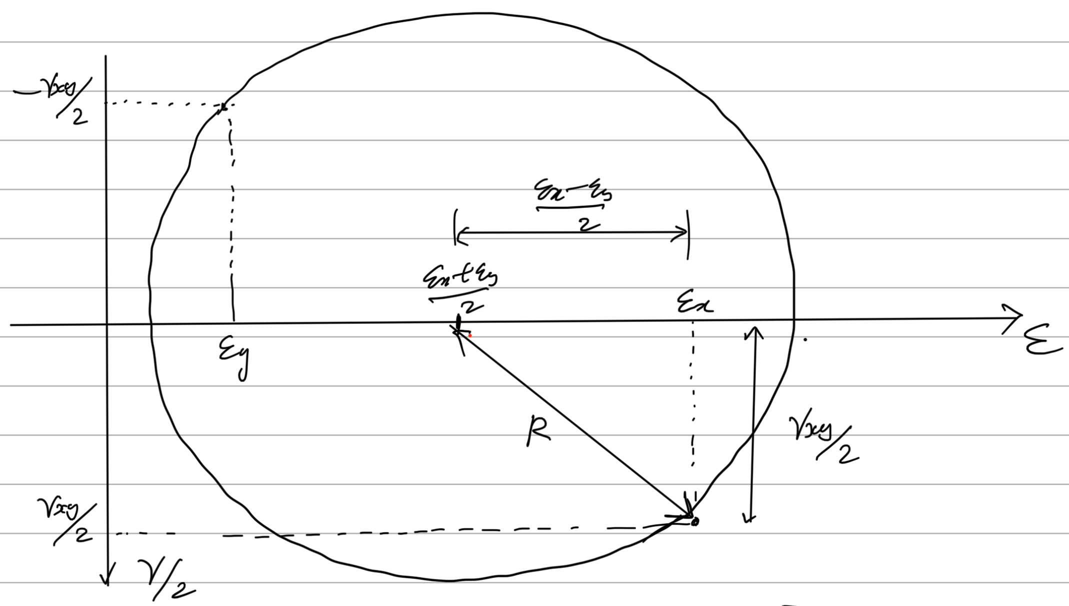
infinitesimal strain

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \Rightarrow \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

γ_{xy}: engineering shear strain ε_{xy}: true shear strain (tensorial)

Mohr's circle for plane strain



$$R = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Example 10.5.

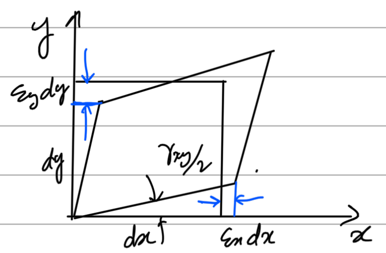
State of plane strain at a point

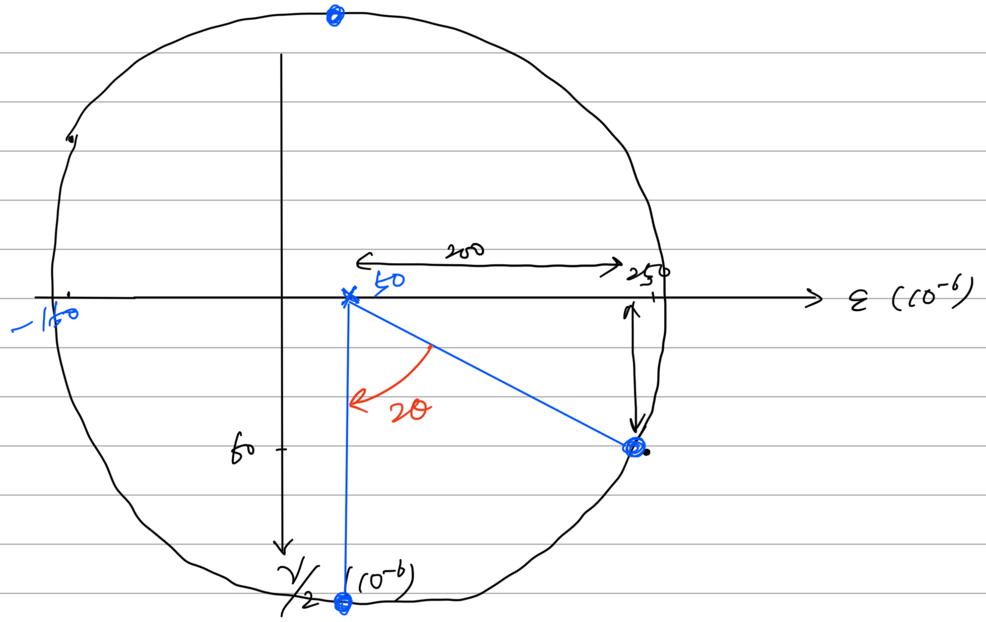
: $\varepsilon_x = 250 \times 10^{-6}$

$\varepsilon_y = -150 \times 10^{-6}$

$\gamma_{xy} = 120 \times 10^{-6}$

Determine maximum in-plane shear strains and the orientation of the element upon which they act.

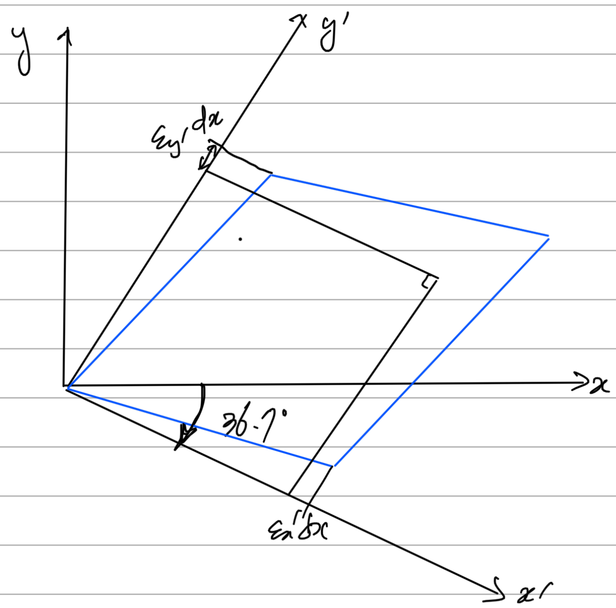




$$\left(\frac{\gamma_{x'y'}}{2}\right)_{max} = R = \sqrt{200^2 + 60^2} = 208.8 \times 10^{-6}$$

$$\therefore (\gamma_{x'y'})_{max} = 418 \times 10^{-6}$$

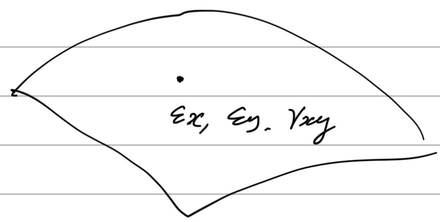
$$\tan 2\theta = \frac{200}{60} \rightarrow \theta = 36.7°$$

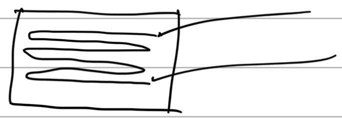


10.5. Strain rosettes

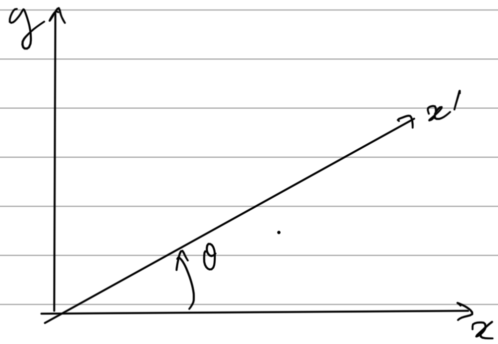
strain state measurement

: using 3 strain gages
(Rosette)

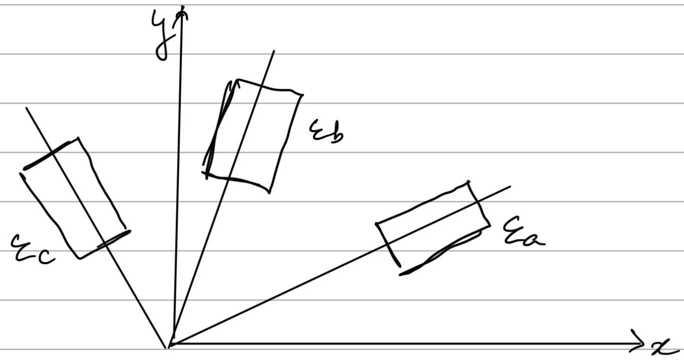




Electrical resistance change
$\Leftrightarrow$ strain change



$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

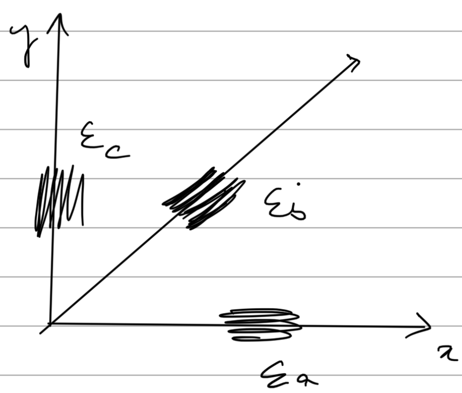


$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$
$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$
$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

measurements unknowns choose

Three equations & three unknowns

45° rosette



$\theta = 0$: $\varepsilon_a = \varepsilon_x$

$\theta = 45°$: $\varepsilon_b = \frac{1}{2}\varepsilon_x + \frac{1}{2}\varepsilon_y + \frac{1}{2}\gamma_{xy}$

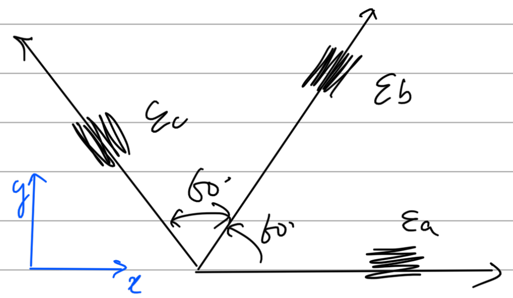
$\theta = 90°$: $\varepsilon_c = \varepsilon_y$

$\therefore$ $\varepsilon_x = \varepsilon_a$

$\varepsilon_y = \varepsilon_c$

$\gamma_{xy} = 2\varepsilon_b - (\varepsilon_a + \varepsilon_c)$

60° rosette → HW.



$\varepsilon_x = \varepsilon_a$

$\varepsilon_y = \frac{1}{3}(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a)$

$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c)$

Example 10.8 60° Rosette

$\varepsilon_a = 60 \times 10^{-6}$, $\varepsilon_b = 135 \times 10^{-6}$, $\varepsilon_c = 264 \times 10^{-6}$

Principal strains and directions

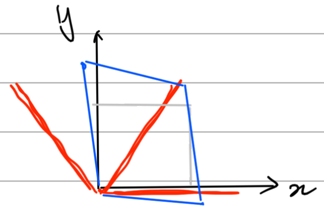
Using $\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$

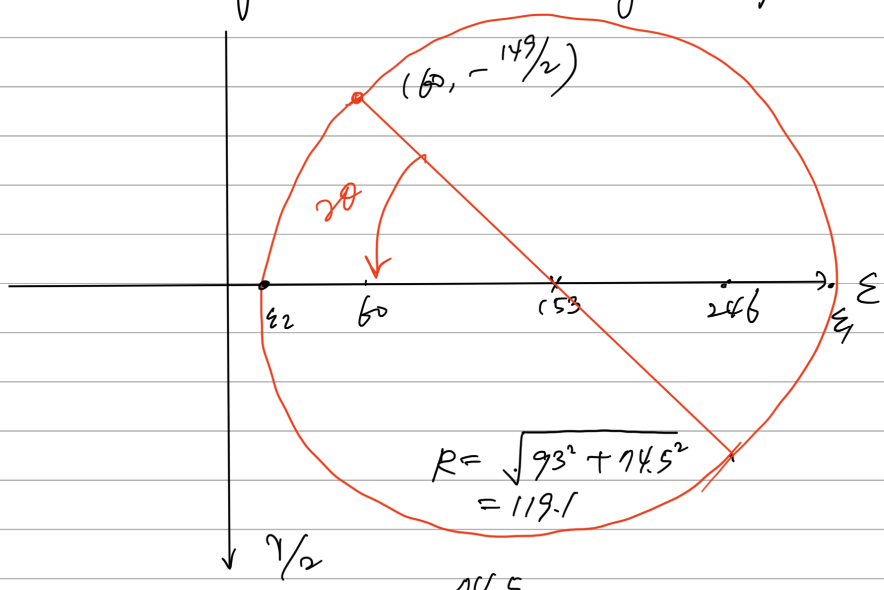
$60 = \varepsilon_x$

$135 = \varepsilon_x \cos^2 60° + \varepsilon_y \sin^2 60° + \gamma_{xy} \sin 60° \cos 60°$

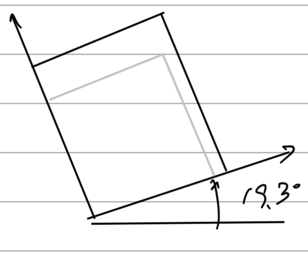
$264 = \varepsilon_x \cos^2 120° + \varepsilon_y \sin^2 120° + \gamma_{xy} \sin 120° \cos 120°$

$\therefore \varepsilon_x = 60 \times 10^{-6}$, $\varepsilon_y = 246 \times 10^{-6}$ $\gamma_{xy} = -149 \times 10^{-6}$





$R = \sqrt{93^2 + 74.5^2} = 119.1$



$\tan 2\theta = \frac{74.5}{93.} \rightarrow \theta = 19.3°$

$\varepsilon_1 = 153 + 119.1 = 272 \times 10^{-6}$

$\varepsilon_2 = 153 - 119.1 = 33.9 \times 10^{-6}$