Chapter 12. Deflection of beams

Loading $\quad M$

$\downarrow$

stresses $\quad \sigma = E\varepsilon = -\frac{M}{I}y$

$\downarrow$

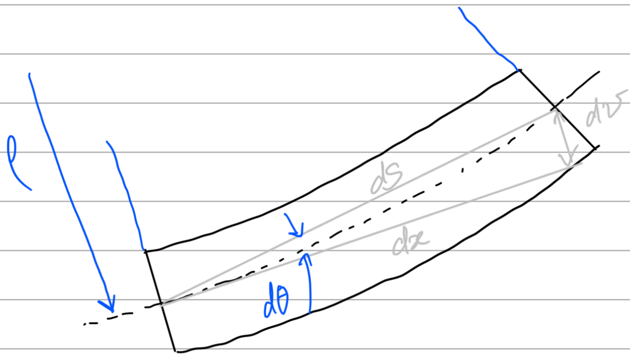
strains $\quad \varepsilon = -\kappa y = -\frac{M}{EI}y$

$\downarrow$

deformation $\quad \kappa = \frac{M}{EI}$

Q. How is beam deflection related to κ (or $1/\rho$)

v : deflection of the centroid axis due to bending

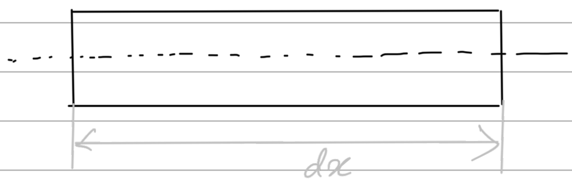


① $\rho\, d\theta = ds$

② $\tan d\theta = \frac{dv}{dx}$

$\rightarrow d\theta = \operatorname{atan} \frac{dv}{dx}$

③ $ds^2 = dx^2 + dv^2$



ρ, $\cancel{d\theta}$, $\cancel{ds}$, dx, dv

From ②

$$\frac{d\theta}{dx} = \frac{d}{dx}\left(\operatorname{atan}\frac{dv}{dx}\right) \qquad \left(\frac{d}{dx}(\operatorname{atan} x) = \frac{1}{1+x^2}\right)$$

$$= \frac{1}{1+(dv/dx)^2} \cdot \frac{d}{dx}\left(\frac{dv}{dx}\right)$$

From ①

$$d\theta = \frac{1}{\rho} ds$$

$$\rightarrow \frac{1}{\rho}\frac{ds}{dx} = \frac{d^2v/dx^2}{1+(dv/dx)^2}$$

From ③ $\quad \frac{ds^2}{dx^2} = 1 + \frac{dv^2}{dx^2} \rightarrow \frac{ds}{dx} = \sqrt{1+(dv/dx)^2}$

$$\therefore \quad \kappa = 1/\rho = \frac{d^2v/dx^2}{\left[1 + (dv/dx)^2\right]^{3/2}}$$

If $\quad dv/dx \simeq 0$

$$\kappa = 1/\rho = d^2v/dx^2 = \frac{M}{EI}$$

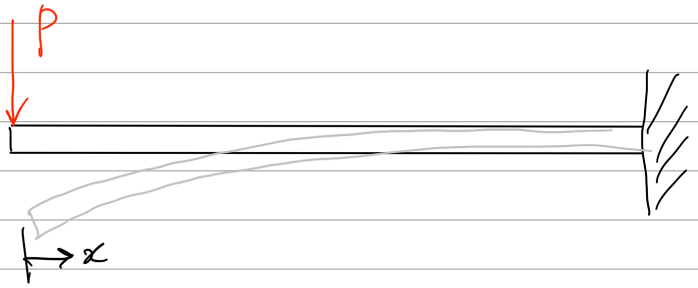
$$\therefore \quad EI \frac{d^2v}{dx^2} = M(x)$$

$$EI \frac{d^3v}{dx^3} = V(x) \qquad \left(\frac{dM}{dx} = V\right)$$

$$EI \frac{d^4v}{dx^4} = w(x) \qquad \left(\frac{dV}{dx} = w\right)$$

flexural rigidity

Example. 12.2.



F.B.D.

$M = -Px$

$$EI \frac{d^2v}{dx^2} = -Px$$

$$EI \frac{dv}{dx} = -\frac{Px^2}{2} + C_1$$

B.C. $\quad \left.\frac{dv}{dx}\right|_{x=L} = -\frac{PL^2}{2} + C_1 = 0$

$$EI \frac{dv}{dx} = \frac{P}{2}(L^2 - x^2)$$

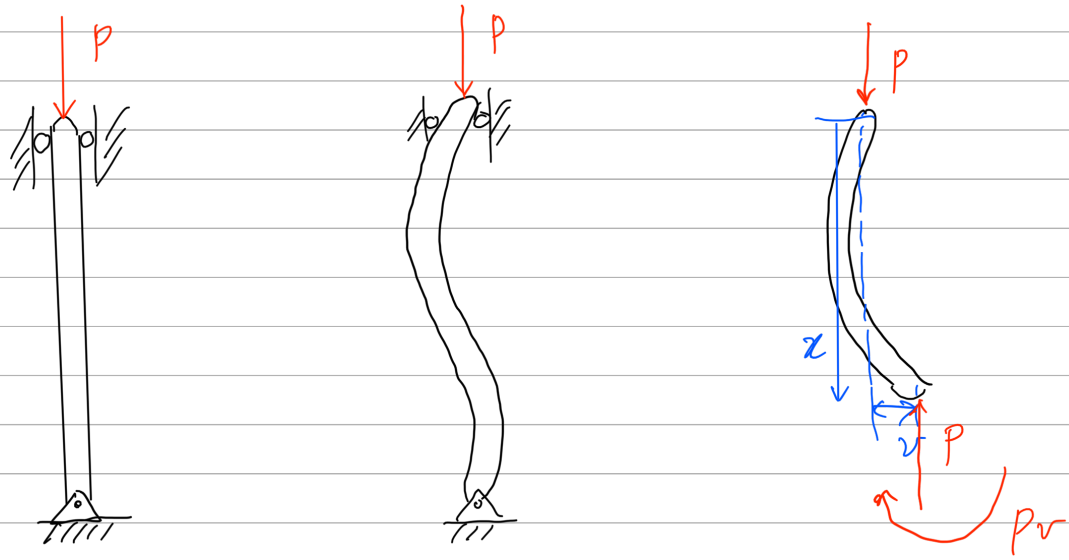
$$EI\, v = \frac{P}{2}\left(L^2x - \frac{x^3}{3}\right) + C_2$$

B.C. $\quad v|_{x=L} = \frac{P}{2} \cdot \frac{2}{3} L^3 + C_2 = 0$

Chapter 13. Buckling of Columns

- Material failure
  (e.g., Tresca, von Mises criteria)
- Structural failure
  : buckling (sudden collapse due to the loss of stability)

Euler buckling



$$EI\frac{d^2v}{dx^2} = -Pv.$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

$$v = C_1 \sin\sqrt{\frac{P}{EI}}\,x + C_2 \cos\sqrt{\frac{P}{EI}}\,x$$

Boundary condition

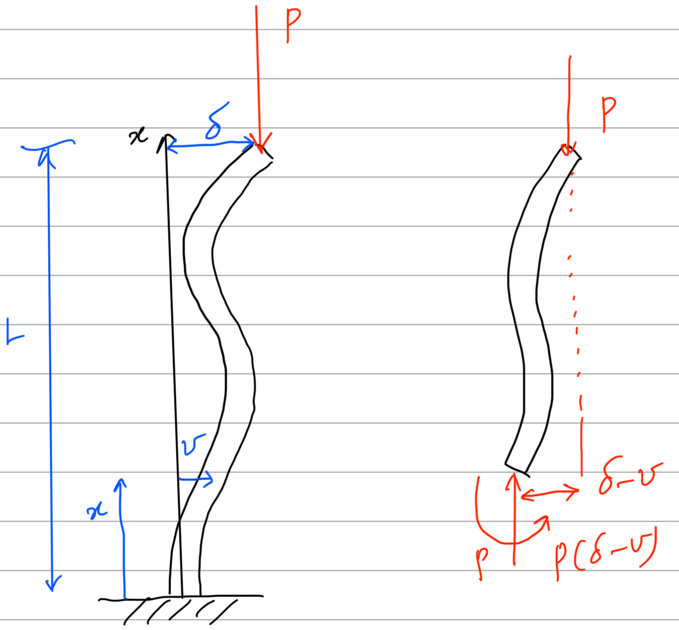
$$v(x=0) = C_2 = 0$$

$$v(x=L) = C_1 \sin\sqrt{\frac{P}{EI}}\cdot L = 0$$

$$\therefore \sqrt{\frac{P}{EI}}\cdot L = n\pi \rightarrow P = \frac{n^2\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad : \text{Euler load}$$

13.3. Columns having various types of supports



$$EI \frac{d^2 v}{dx^2} = P(\delta - v)$$

$$\frac{d^2 v}{dx^2} + \frac{P}{EI} v = \frac{P}{EI}\delta$$

$$\therefore \; v = C_1 \sin\sqrt{\frac{P}{EI}}\,x + C_2 \cos\sqrt{\frac{P}{EI}}\,x + \delta$$

B.C. $\quad v(x=0) = C_2 + \delta = 0 \quad \rightarrow \quad C_2 = -\delta$

$$\left.\frac{dv}{dx}\right|_{x=0} = C_1\sqrt{\frac{P}{EI}} = 0 \quad \rightarrow \quad C_1 = 0$$

$$v = \delta\left[1 - \cos\sqrt{\frac{P}{EI}}\,x\right]$$

$$v(x=L) = \delta\left[1 - \cos\sqrt{\frac{P}{EI}}\,L\right] = \delta$$

$$\therefore \; \sqrt{\frac{P}{EI}}\,L = \frac{n\pi}{2}, \quad n = 1, 3, 5, \cdots$$

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$