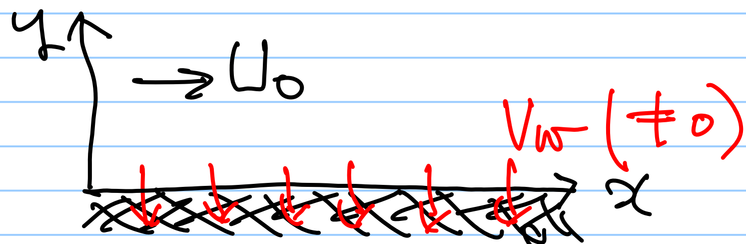
$$C_M = \frac{-2M}{\frac{1}{2}\rho\omega^2 r_0^5} \simeq \frac{3.87}{\sqrt{Re}}, \qquad Re = \frac{\omega r_0^2}{\nu}.$$

⑨ Flow over a porous wall.



· Steady, uniform flow over the surface.

· constant pressure.

porous(permeable) wall.

· $u = u(y)$ only, incompressible

$$\cancel{\frac{\partial u}{\partial x}}^{\,0} + \frac{\partial v}{\partial y} = 0. \rightarrow \underline{v = \text{const}} = -V_w$$

$$v\frac{du}{dy} = \nu\frac{d^2u}{dy^2},$$

BC : $u(\infty) = U_0$.

$v(x,0) = \underline{-V_w}$. $u(0) = 0$

$$-V_w \frac{du}{dy} = \nu \frac{d^2u}{dy^2} \rightarrow u(y) = U_0\left(1 - \exp\left(-\frac{V_w}{\nu}y\right)\right).$$

"Layer where viscosity affects", by considering the value of y at which $u/U_0 = 1 - 1/e^2$ ($\simeq 0.86$) to be δ.

$\hookrightarrow \delta = 2\frac{\nu}{V_w} \neq f(x) \rightarrow$ constant!

① the distance away from the wall at which the uniform flow is recovered, is proportional to ν and inversely proportional to V_w (suction vel.).

② convection of vorticity (mtm) toward the wall due to suction is balanced by the diffusion of vorticity (mtm) away from the wall.

LOW-REYNOLDS NUMBER SOLUTIONS.

- High vs Low-Re flows,
- steady flow around a body (L) with U_0.

$$\rho \bar{u}\cdot\nabla\bar{u} = -\nabla p + \mu\nabla^2\bar{u}.$$

① High Re : $\rho\bar{u}\cdot\nabla\bar{u} \simeq -\nabla p \rightarrow \rho U_0 \cdot \dfrac{U_0}{L} \sim \dfrac{P}{L}$,

$$\Downarrow$$

$$P \sim \rho U_0^2.$$

$u^* \equiv u/U_0$, $p^* = P/\rho U_0^2$, ...

$$\bar{u}^*\cdot\nabla^*\bar{u}^* = -\nabla^* p^* + \cancel{\frac{1}{Re}\nabla^{*2}\bar{u}^*} \quad Re \gg 1.$$

② Low Re : $\nabla p = \mu \nabla^2 \bar{u} \rightarrow \frac{p}{L} \sim \mu \frac{U_0}{L^2}, \; p \sim \frac{\mu U_0}{L}$.

$$Re \cdot \cancel{\bar{u}^* \cdot \nabla^* \bar{u}^*} = -\nabla^* p^* + \nabla^{*2} \bar{u}^*$$

(Re<<1)

: creeping flow, $\nabla^* p^* = \nabla^{*2} \bar{u}^*$.

· Incompressible creeping flow.

$$\begin{cases} \nabla \cdot \bar{u} = 0 \\ \nabla p = \mu \nabla^2 \bar{u} \end{cases} \rightarrow \begin{array}{l} \nabla \times (\nabla p = \mu \nabla^2 \bar{u}) \rightarrow \nabla^2 \bar{\omega} = 0 \\ \nabla \cdot (\nabla p = \mu \nabla^2 \bar{u}) \rightarrow \nabla^2 p = 0. \end{array}$$

vorticity

· Types of creeping flow

① immersed bodies : sphere, cylinder, etc.

② narrow passage : lubrication theory
(journal bearing)

③ porous media : heat transfer
ground water movement.

④ fully-developed duct flow.

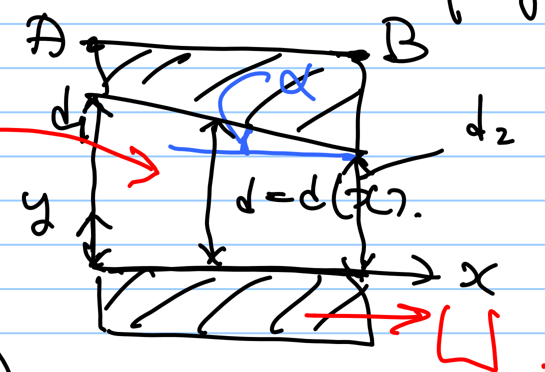
① Reynolds lubrication theory.

- creeping flow : $\rho u \frac{\partial u}{\partial x} \ll \mu \frac{\partial^2 u}{\partial y^2}$.

$\rightarrow \rho U \frac{U}{l} \sim \mu \frac{U}{d^2}$.

$\rightarrow \frac{\rho U l}{\mu} \left(\frac{d}{l}\right)^2 \ll 1$.

So if $d/l \ll 1$. $\rightarrow$ Re can be relatively large, but still we have a creeping flow!



· $\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$. let $-\frac{\partial p}{\partial x} = G$.

$\hookrightarrow$ $u(y) = \frac{G}{2\mu} y(d-y) + U\left(1 - \frac{y}{d}\right)$.

· volume flow rate,

$$Q = \int_0^{d} u \, dy = \frac{G d^3}{12\mu} + \frac{1}{2} U d = \text{constant}.$$

height @ any x.

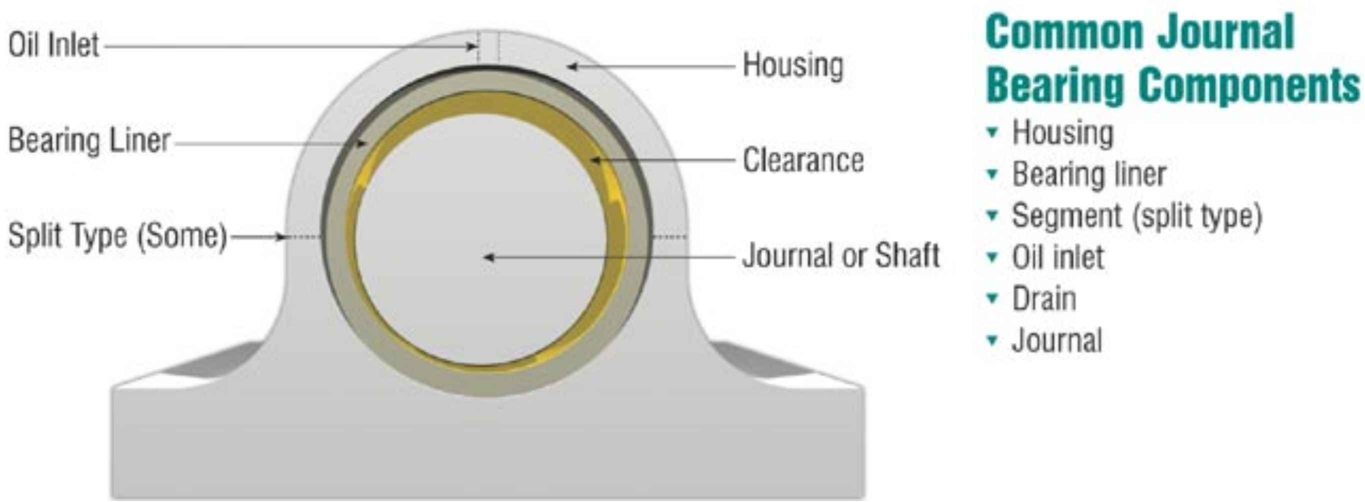
$$\Rightarrow \frac{dp}{dx} = 6\mu\left(\frac{U}{d^2} - \frac{2Q}{d^3}\right), \quad d \doteqdot d_1 - \alpha x.$$

By calculating $\int_0^x \frac{dp}{dx}\,dx$, $P - P_0$ can be obtained.

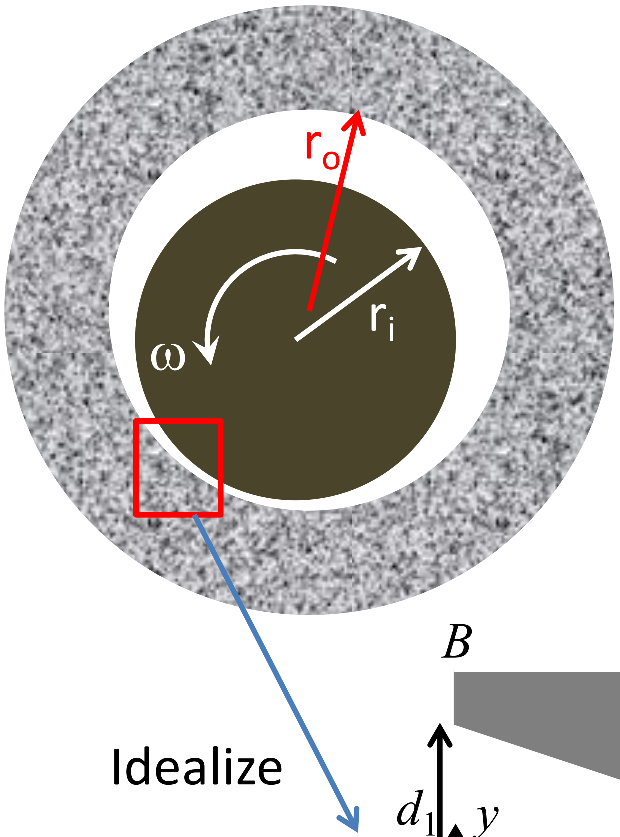
(P_0 = pressure at $x=0$)

if $P_A = P_B$, $P - P_0 = \frac{6\mu U}{\alpha} \cdot \frac{(d_1 - d)(d - d_2)}{d^2(d_1 + d_2)}$

- **Hydrodynamic journal (shaft) bearing**
  - operating with hydrodynamic lubrication
  - bearing is separated from the journal by the lubricant oil film
  - If there is no force applied to the journal its position will remain concentric to the bearing position. However a loaded journal displaces from the concentric position and forms a converging gap between the bearing and journal surfaces.

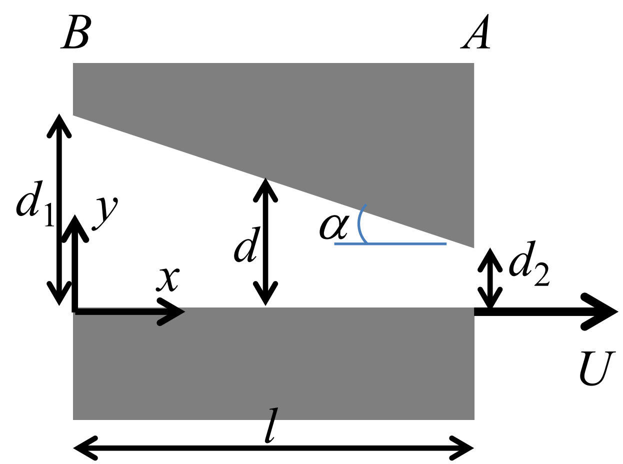


Reynolds Lubrication Theory



- $r_i/r_o \sim 1$
- Two bodies slide past each other w/ thin layer of fluid between
- High pressure can be built up in the fluid
- Combined Couette-Poiseuille flow

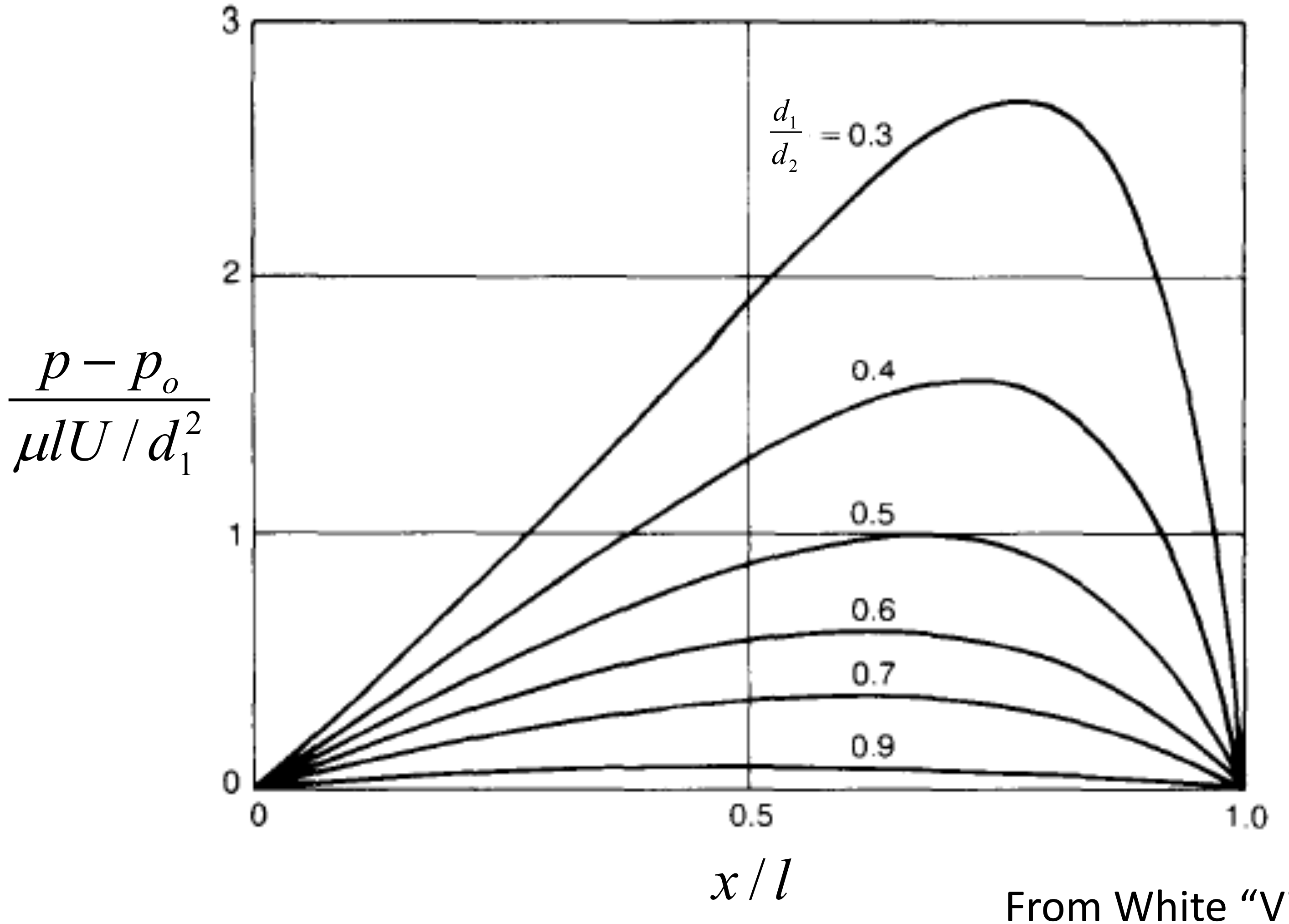
Idealize



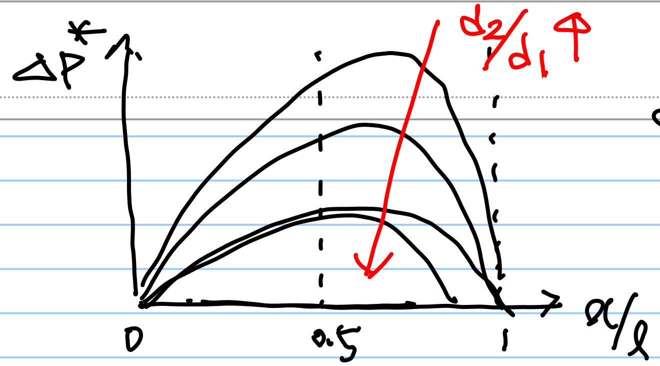
- $d << l$
- $d/dz = 0$

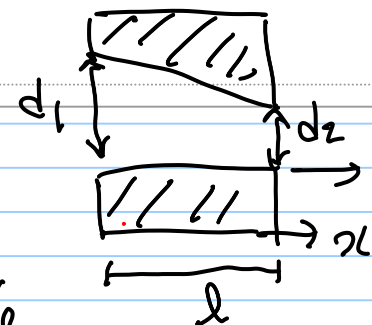
Reynolds Lubrication Theory

Pressure distribution in a two-dimensional linear-gap bearing



From White "Viscous Fluid Flow"



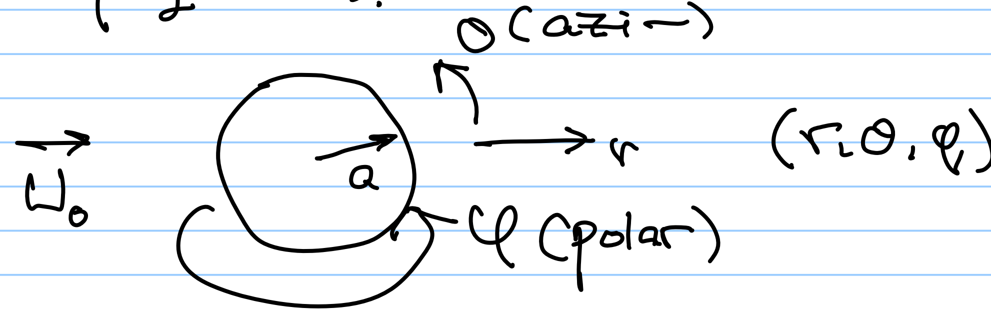


(2) Flow around a sphere (creeping flow).

· $\nabla^2 \bar{\omega} = 0$

· spherical coord.

· axisymmetric.



$\rightarrow$ The only non-zero component of vorticity

$$\therefore\ \omega_\varphi = \frac{1}{r}\left[\frac{\partial}{\partial r}(r u_\theta) - \frac{\partial u_r}{\partial \theta}\right]$$

· From the stokes stream function, ψ:

$$u_r \equiv \frac{1}{r^2 \sin\theta} \cdot \frac{\partial \psi}{\partial \theta}, \quad u_\theta \equiv -\frac{1}{r \sin\theta} \cdot \frac{\partial \psi}{\partial r}.$$

$$\rightarrow \left[\frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2} \cdot \frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta} \cdot \frac{\partial}{\partial \theta}\right)\right]^2 \psi = 0. \quad \left(\nabla^4 \psi = 0\right)$$

BC) $u_r = u_\theta = 0$ @ $r = a$. $\frac{\partial \psi}{\partial r} = \frac{\partial \psi}{\partial \theta} = 0$

@ $r \to \infty$, : uniform flow, $\psi(\infty, \theta) = \frac{1}{2} U_0 r^2 \sin^2\theta$

(Curie, potential flow sol.)

From this BC, we assume that

$$\psi(r, \theta) = f(r) \cdot \sin^2\theta.$$

$$\rightarrow f^{(4)} - \frac{4f''}{r^2} + \frac{8f'}{r^3} - \frac{8f}{r^4} = 0, \quad \text{Euler-Cauchy eq.}$$

$$f(r) = Ar^4 + Br^2 + Cr + \frac{D}{r}.$$

w/ BC's : $A = 0$, $B = U_0/2$, $C = -3U_0 a/4$, $D = U_0 a^3/4$.

$$\therefore \psi(r,\theta) = U_0 r^2 \sin^2\theta \left(\frac{1}{2} - \frac{3a}{4r} + \frac{a^3}{4r^3} \right)$$

uniform flow (irrotational)

$\boxed{\text{Stokeslet}}$ (rotational)

dipole (irrotational)

flow induced by a point force acting on a

- contains all the vorticity
- far-field dominant.

viscous fluid.

$$\psi = U_0 r^2 \sin^2\theta \left[\frac{1}{2} - \frac{3a}{4r} + \frac{a^3}{4r^3}\right].$$

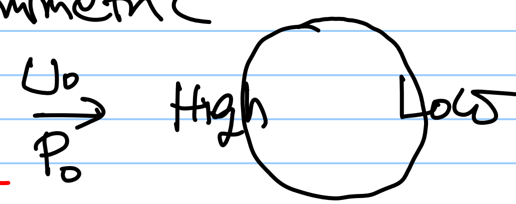
$$\rightarrow u_r = \frac{1}{r^2\sin\theta}\cdot\frac{\partial\psi}{\partial\theta} = U_0\cos\theta\left(1-\frac{3a}{2r}+\frac{a^3}{2r^3}\right)$$

$$u_\theta = -\frac{1}{r\sin\theta}\cdot\frac{\partial\psi}{\partial r} = -U_0\sin\theta\left(1-\frac{3a}{4r}-\frac{a^3}{4r^3}\right)$$

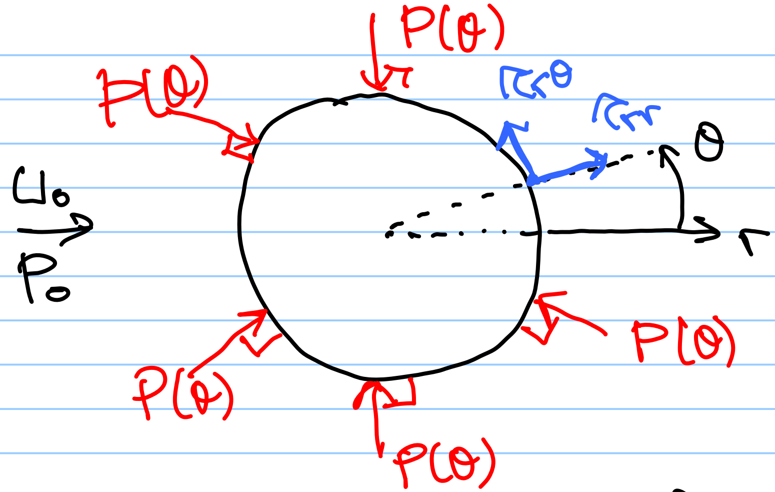
· mtm conservation: $\nabla p = \mu\nabla^2\bar{u}$ ← integrate!

$\therefore p = p_0 - \dfrac{3a\mu U_0\cos\theta}{2r^2}$ ∴ Not symmetric → pressure drag

pressure of free-stream (U_0).



· another source for drag → shear stress.



$$\tau_{rr} = 2\mu \frac{\partial u_r}{\partial r} = 2\mu U_0 \cos\theta \left[\frac{3a}{2r^2} - \frac{3a^3}{2r^4} \right]$$

$$\tau_{r\theta} = \mu \left[r \frac{\partial}{\partial r}\left(\frac{1}{r} u_\theta\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta} \right]$$

$$= -\frac{3\mu U_0 a^3 \cdot \sin\theta}{2r^4}$$

· total drag force, D

$$D = -\int_0^\pi P\Big|_{r=a} \cdot \cos\theta \cdot dA - \int_0^\pi \tau_{r\theta}\Big|_{r=a} \cdot \sin\theta \cdot dA + \cancel{\int_0^\pi \tau_{rr}\Big|_{r=a} \cdot \cos\theta \cdot dA}$$

0

$$= 2\pi\mu U_0 a + 4\pi\mu U_0 a = 6\pi\mu U_0 a.$$

$\therefore D \sim U_0$. (cf. $D \sim U_0^2$ for high Re#).

* Drag on an object in creeping flow.

· ~~inertia~~ (density) is ignored, $D = f(U_0, L, \mu, \cancel{\rho})$

· From π-theorem, s (parameter) $= 4$, u (units) $= 3$ (M, L, T)

$\rightarrow p = s - u = 1 \rightarrow$ number of non-dimensional group.

$\downarrow$

this should be constant.

$$\frac{D}{U_0 \mu L} = C$$

$\therefore D = C \cdot \mu U_0 L$, for a sphere,

$C = 6\pi$.