2017 Fall

"Phase Equilibria in Materials"

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Office hours: by an appointment

- Binary System mixture/solution/compound
- Gibbs Free Energy in Binary System

$$G_1 = X_A G_A + X_B G_B$$
 J/mol $G_2 = G_1 + \Delta G_{mix}$ J/mol

Ideal solution (
$$\Delta H_{mix} = 0$$
) $\Delta G^{mix} = RT(X_A \ln X_A + X_B \ln X_B)$

$$G = X_A G_A + X_B G_B + RT(X_A \ln X_A + X_B \ln X_B)$$

Regular solution
$$\Delta H_{mix} = P_{AB} \epsilon$$
 where $\epsilon = \epsilon_{AB} - \frac{1}{2} (\epsilon_{AA} + \epsilon_{BB})$

$$G = X_A G_A + X_B G_B + \Omega X_A X_B + RT(X_A \ln X_A + X_B \ln X_B)$$

- Chemical potential and Activity

emical potential and Activity
$$\mu_{A} = \left(\frac{\partial G'}{\partial n_{A}}\right)_{T, P, n_{B}} \qquad \bullet \quad \mu_{A} = G_{A} + RT \ln a_{A} \quad \ln\left(\frac{a_{A}}{X_{A}}\right) = \frac{\Omega}{RT}(1 - X_{A})^{2}$$

$$\frac{a_{A}}{Y_{A}} = \gamma_{A} = \text{activity coeff}$$

$$\mu_{A} = G_{A} + RT \ln a_{A} \quad \ln \left(\frac{a_{A}}{X_{A}} \right) = \frac{\Omega}{RT} (1 - X_{A})$$

 $\frac{a_A}{X_A} = \gamma_A = \text{activity coefficient}$

 μ 는 조성에 의해 결정되기 때문에 $dn_{\scriptscriptstyle \Delta}$ 가 매우 작아서 조성변화 없어야

- Chemical equilibrium → Gibbs phase rule

Regular Solutions

$$G_2 = G_1 + \Delta G_{mix}$$

$$G = X_A G_A + X_B G_B +$$

 ΔH_{mix}

$$\Omega X_A X_B$$

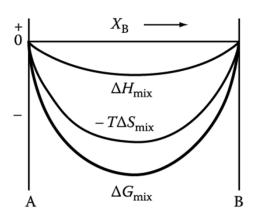
 $-T\Delta S_{mix}$

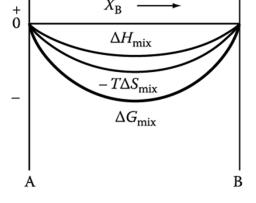
$$G = X_A G_A + X_B G_B + \Omega X_A X_B + RT (X_A \ln X_A + X_B \ln X_B)$$

Reference state

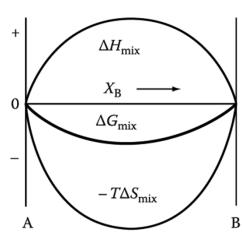
Pure metal $G_A^0 = G_B^0 = 0$

$$\Delta G_{mix} = \Delta H_{mix} - T \Delta S_{mix}$$

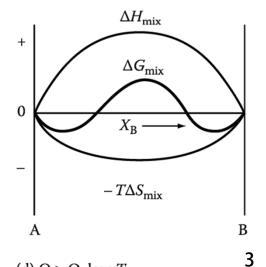




(a)
$$\Omega$$
 < O, high T



(b)
$$\Omega$$
 < O, low T



(c)
$$\Omega$$
 > O, high T

(d)
$$\Omega$$
 > O, low T

At T_c the term $d^2(\Delta G_m)/d(X_A)^2$ will be zero.

Since

$$\frac{d^{2}(\Delta G_{m})}{d(X_{A})^{2}} = -2NC + NkT_{c} \left(\frac{1}{X_{A}} + \frac{1}{1 - X_{A}} \right) = 0$$

then

$$2C = \frac{kT_c}{X_A(1 - X_A)} \quad \text{or} \quad T_c = \frac{2CX_A(1 - X_A)}{k}$$

The term T_c will be a maximum when $X_A = (1 - X_A) = 0.5$. It follows that

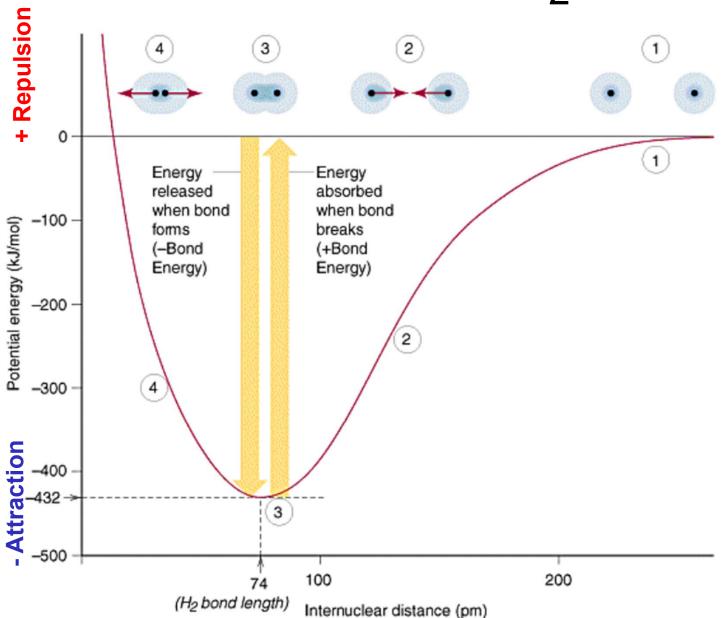
$$T_c = \frac{C}{2k}. (101)$$

A high value of the critical temperature is associated with a high positive value for $C = z[H_{AB} - \frac{1}{2}(H_{AA} + H_{BB})]$.

The stronger the attraction between similar atoms, the higher T_c . In those binary phase diagrams with a miscibility gap in the solid state the gap has not the symmetrical form shown in Fig. 21. This is primarily because the initial simplifying assumption that the energy is the sum of interaction between pairs of atoms is never absolutely valid. The systems Pd-Ir*, Pt-Ir** and Pt-Au*** all have miscibility gaps in the solid state with varying degrees of asymmetry. Most binary phase diagrams with a positive value of ΔH_m do not show a miscibility gap with a closure at temperature T_c since melting occurs before T_c is reached (for example the Ag-Cu system).

$$\Delta H_{\text{mix}} = P_{AB} \epsilon$$

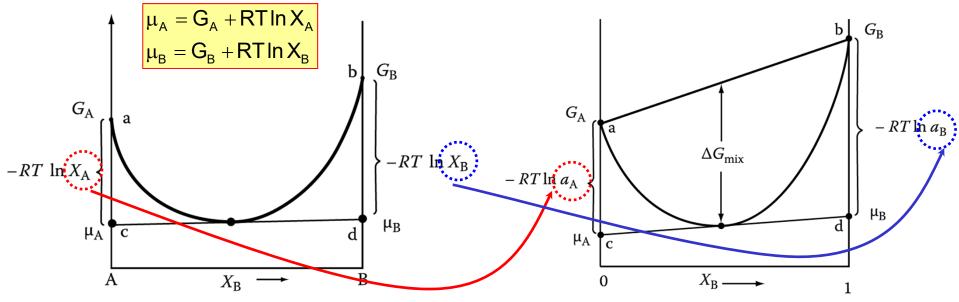
where
$$\varepsilon = \varepsilon_{AB} - \frac{1}{2}(\varepsilon_{AA} + \varepsilon_{BB})$$



Activity, a: effective concentration for mass action

ideal solution

regular solution



$$\mu_{A} = G_{A} + RT \ln a_{A}$$

$$\mu_{A} = G_{A} + \Omega (1 - X_{A})^{2} + RT \ln X_{A}$$

$$\mu_{B} = G_{B} + RT \ln a_{B}$$

$$\mu_{B} = G_{B} + \Omega (1 - X_{B})^{2} + RT \ln X_{B}$$

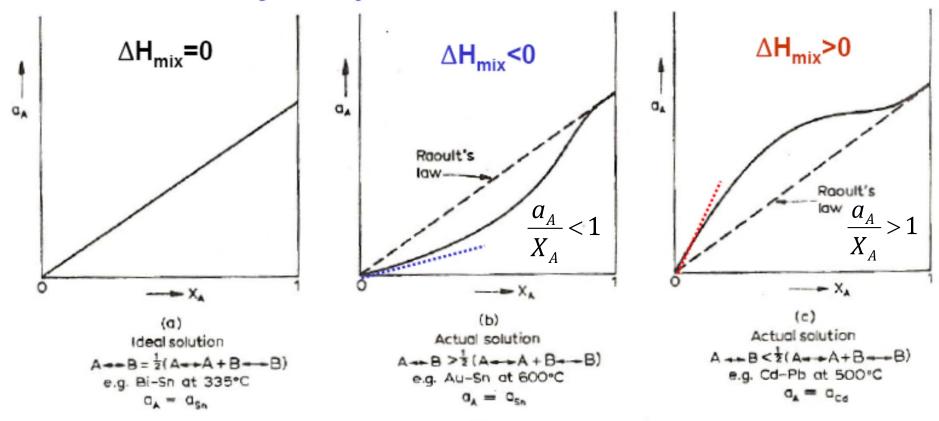
$$\ln\left(\frac{a_A}{X_A}\right) = \frac{\Omega}{RT}(1 - X_A)^2$$

$$\frac{a_A}{X_A} = \gamma_A = \text{activity coefficient}$$

$$\ln(\frac{a_B}{X_B}) = \frac{\Omega}{RT} (1 - X_B)^2$$

$$\gamma_{\rm B} = \frac{a_{\rm B}}{X_{\rm B}} \qquad \qquad 6$$

Activity-composition curves for solutions



For a dilute solution of B in A (X_B→0)

$$\gamma_B = \frac{a_B}{X_B} \cong constant \quad (Henry's Law)$$

$$\gamma_A = \frac{a_A}{X_A} \cong 1$$
 (Rault's Law)

The Gibbs Phase Rule

Degree of freedom (number of variables that can be varied independently)

= the number of variables – the number of constraints

- Number of phases : p, number of components : c,
- # of controllable variable : composition (c-1)p, temperature : p, pressure : p
- # of restrictions :

(p-1)c from chemical equilibrium
$$\mu_i^a = \mu_i^\beta = \mu_i^\gamma = \dots = \mu_i^p$$

p-1 from thermal equilibrium
$$T^a = T^\beta = T^\gamma = \dots = T^p$$

p-1 from mechanical equilibrium
$$P^a = P^\beta = P^\gamma = \cdots = P^p$$

Number of variable can be controlled with maintaining equilibrium

$$f = (c-1)p + p + p - (p-1)c - (p-1) - (p-1) = c - p + 2$$
$$f = c - p + 2$$

- If pressure is constant: f = (c-1)p + p - (p-1)c - (p-1) = c - p + 1

Q1: What is "Real Solution"?

1.3 Binary Solutions

$$\Delta G_{mix} = \Delta H_{mix} - T \Delta S_{mix}$$

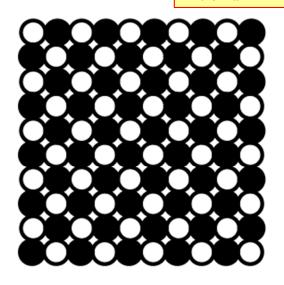
Real solution: sufficient disorder + lowest internal E

Ideal or Regular solution: over simplification of reality

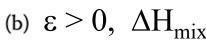
Config. Entropy $S = k \ln w$ + mixing enthalpy $\Delta H_{mix} = \Omega X_A X_B$ where $\Omega = N_a z \epsilon$

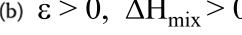
 $S_{thermal} = 0$

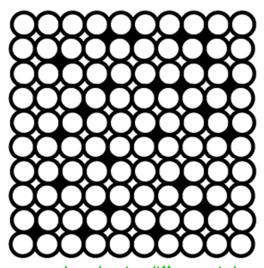












when the size difference is large strain effect

Ordered alloys

$$P_{AB} \uparrow \longrightarrow Internal E \downarrow$$

$$P_{AA}$$
, P_{BB}

Interstitial solution

* The degree of ordering or clustering will decrease as temp. increases due to the increasing importance of entropy.

High temp. → Entropy effect 1 Solution stability 1 X_B X_{B} — ΔH_{mix} $=\Delta H_{mix} - T\Delta S_{mix}$ ΔH_{mix} TAS mix -T & Smix ΔG_{mix} ΔG_{mix} (b) $\Omega < 0$, low T $\Omega < 0$, high T (a) ΔH_{mix} ΔH_{mix} ΔG_{mix} X_{B} ΔG_{mix} $-T_{\Delta}S_{mix}$ $-T \triangle S_{mix}$ (c) $\Omega > 0$, high T (d), $\Omega > 0 \text{ low } T$

1.3 Binary Solutions

Real solution: sufficient disorder + lowest internal E

2) In systems where there is a size difference between the atom e.g. interstitial solid solutions,

$$\rightarrow \Delta E = \Delta H_{mix} + elastic strain$$

→ quasi- chemical model ~ underestimate ΔE due to no consideration of elastic strain field



Q2: Short range order in solid solution?

1.3 Binary Solutions

Ordered phase $\varepsilon < 0$, $\Delta H_{mix} < 0$

SRO (Short Range Ordering) or LRO (Long Range Ordering)

• $\Omega < 0 \Rightarrow$ contain short-range order (SRO)

 $\Delta\Omega = N_a z \epsilon$ SRO parameter = s _ degree of ordering

$$s = \frac{P_{AB} - P_{AB}(random)}{P_{AB}(max) - P_{AB}(random)}$$

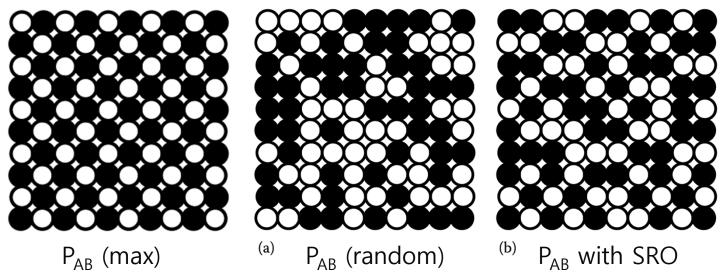
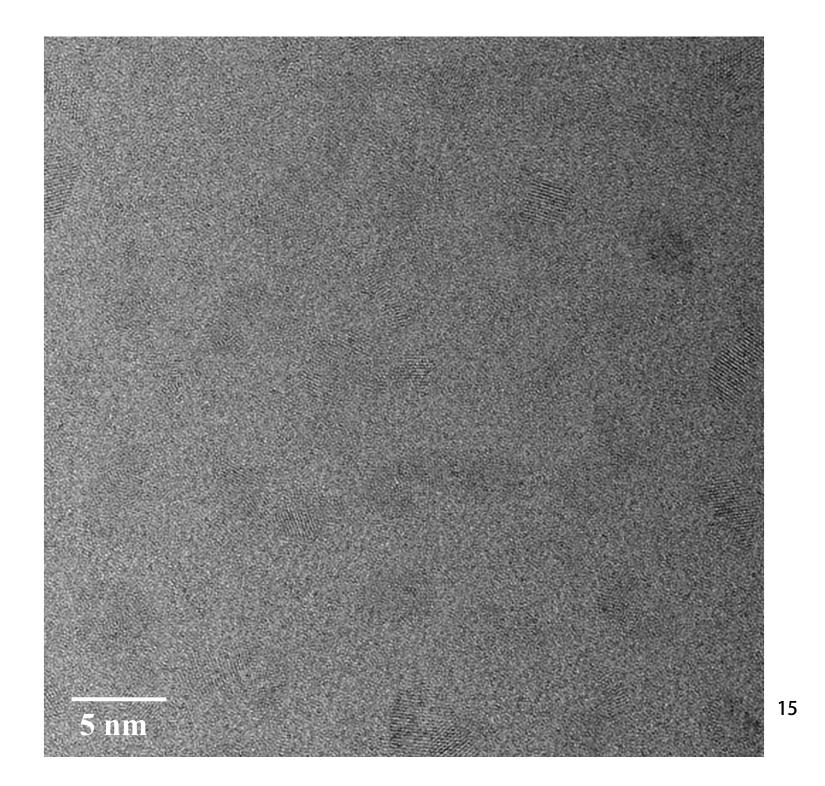
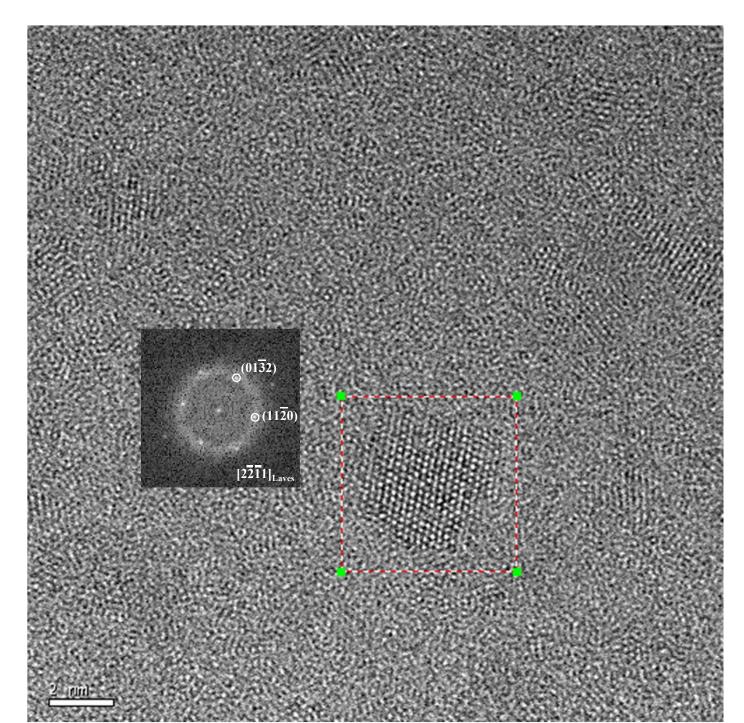


Fig. 1.19 (a) Random A-B solution with a total of 100 atoms and $X_A = X_B = 0.5$, $P_{AB} \sim 100$, S=0. (b) Same alloy with short-range order $P_{AB} = 132$, $P_{AB} (max) \sim 200$, S=(132-100)/(200-100)=0.32. 14





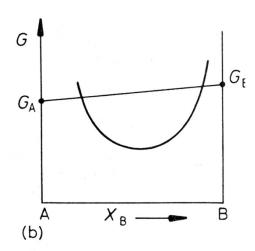
Ordered phase: "Long range order (LRO)"

(1) superlattice, 2 intermediate phase, 3 intermetallic compound)

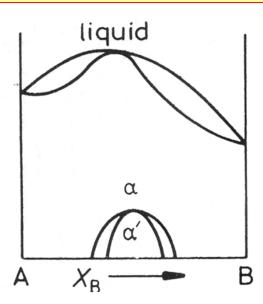
- * Solid solution \rightarrow ordered phase
 - → random mixing
 - → entropy ↑ negative enthalpy \downarrow

$$\Delta H_{mix}^S < 0$$

Large composition range \rightarrow G \downarrow



diate phases: (a) for an intermetallic com-(b) for an intermediate phase with a wide



* Compound : AB, A₂B...

- \rightarrow entropy \downarrow
- → covalent, ionic contribution.
- → enthalpy more negative ↓

$$\Delta H_{mix}^{S} << 0$$

Small composition range

→ G ↓

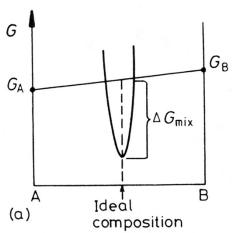
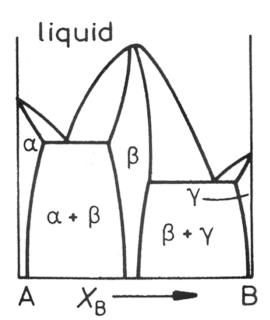


Fig. 1.23 Free energy curves for intermed pound with a very narrow stability range, (stability range.



Q3: Superlattice

* The degree of ordering or clustering will decrease as temp. increases due to the increasing importance of entropy.

High temp. → Entropy effect 1 → Solution stability 1 X_B $X_{\rm B}$ — ΔH_{mix} $=\Delta H_{mix} - T\Delta S_{mix}$ ΔH_{mix} TAS mix -T & Smix ΔG_{mix} ΔG_{mix} (b) $\Omega < 0$, low T $\Omega < 0$, high T (a) ΔH_{mix} ΔH_{mix} ΔG_{mix} X_{B} △G mix $-T_{\Delta}S_{mix}$ $-T\Delta S_{mix}$ В (d), $\Omega > 0 \text{ low } T$ (c) $\Omega > 0$, high T

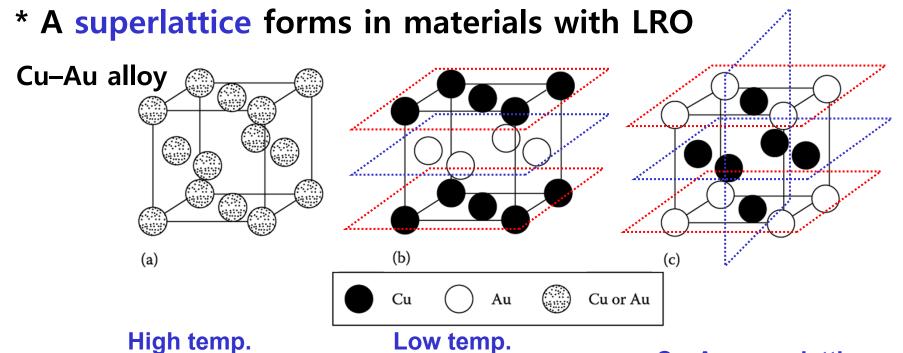
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1.3 Binary Solutions

Ordered phase $\varepsilon < 0$, $\Delta H_{mix} < 0$

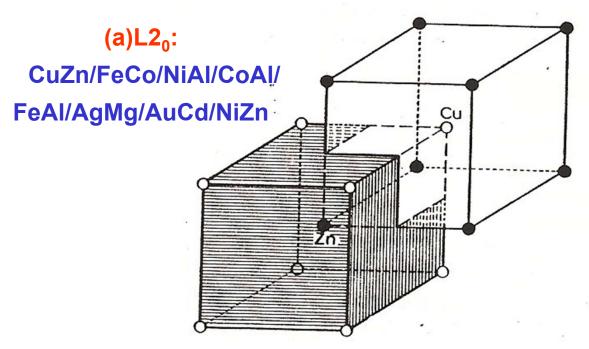
- * In solutions with compositions that are close to a simple ratio of A:B atoms another type of order can be found.
- * This is known as long-range order (LRO) CuAu, Cu₃Au and many other intermetallics show LRO.

(The atom sites are no longer equivalent but can be labelled as A-sites and B-sites.)



Superlattice formation: order-disorder transformation

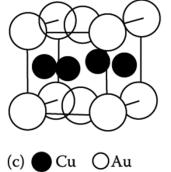
- $\varepsilon < 0$, $\Delta H_{mix} < 0$
- between dissimilar atoms than between similar atoms
- Large electrochemical factor: tendency for the solute atoms to avoid each other and to associate with the solvent atoms
- Size factor just within the favorable limit: lead to atomic rearrangement so as to relieve the lattice distortion imposed by the solute atoms

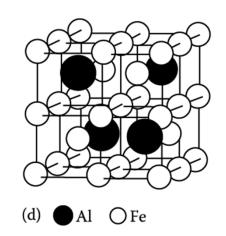


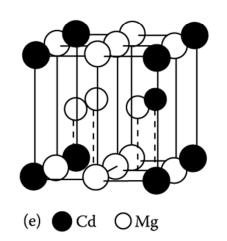
1.3 Binary Solutions Five common ordered lattices

(a)L2₀: (b) L1₂: (c) L1₀: $\label{eq:cuzn/feco/NiAl/CoAl/Cu3Au/Ni3Mn/Ni3Fe/Ni3Al/CuAu/CoPt/FePt} CuZn/FeCo/NiAl/CoAl/ Cu_3Au/Ni_3Mn/Ni_3Fe/Ni_3Al/ CuAu/CoPt/FePt$ FeAI/AgMg/AuCd/NiZn Pt₃Fe/Au₃Cd/Co₃V/TiZn₃









(d) $D0_3$:

Fe₃Al/Cu₃Sb/Mg₃Li/Fe₃Al/ Fe₃Si/Fe₃Be/Cu₃Al

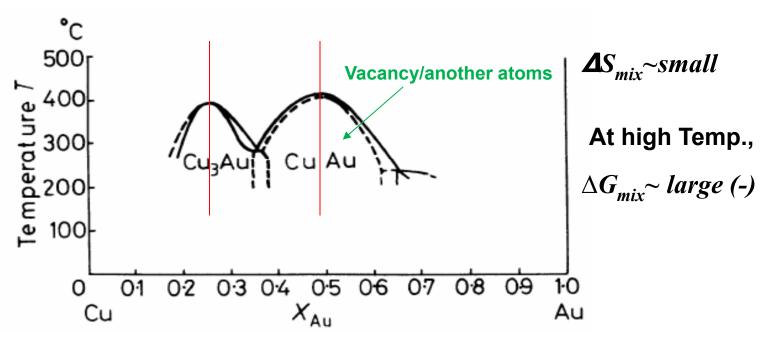
(e) D0₁₉:

Mg₃Cd/Cd₃Mg/Ti₃Al/Ni₃Sn/Ag₃In/ Co₃Mo/Co₃W/Fe₃Sn/Ni₃In/Ti₃Sn

23

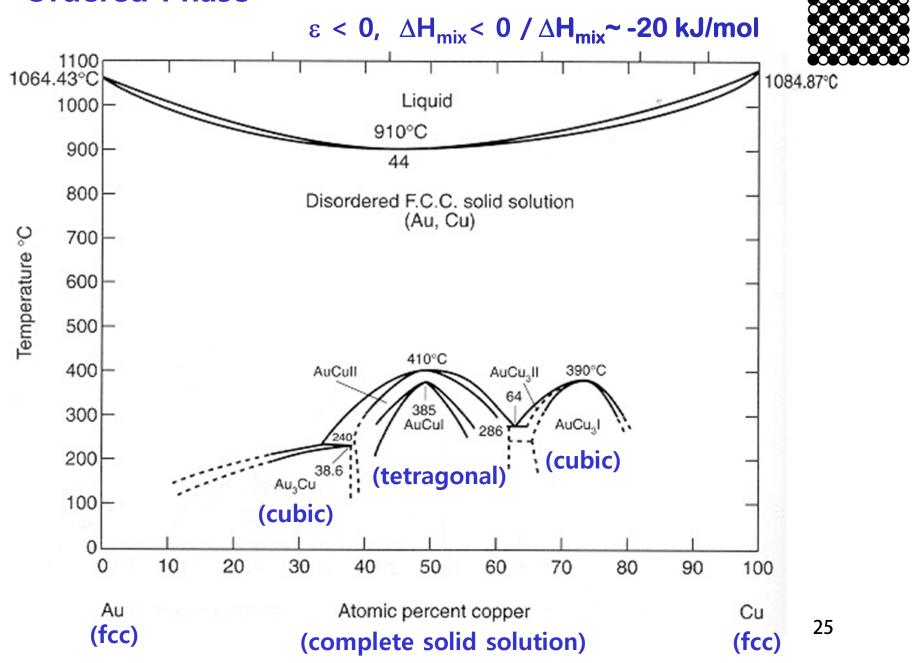
$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$
 Ordered phase $\varepsilon < 0$, $\Delta H_{mix} < 0$

Fig. 1.21. Part of the Cu-Au phase diagram showing the regions where the Cu₃Au and CuAu superlattices are stable.



- The entropy of mixing of structure with LRO is extremely small and the degree of order decrease with increasing temperature until above some <u>critical</u> <u>temperature</u> there is no LRO at all.
- This temperature is a maximum when the composition is the ideal required for the superlattice.
- The critical temperature for loss of LRO increases with increasing Ω or ΔH_{mix} , and in many systems the ordered phase is stable up to the melting point.

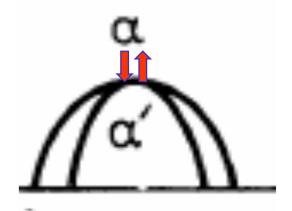
Ordered Phase



Q4: Order-disorder transition

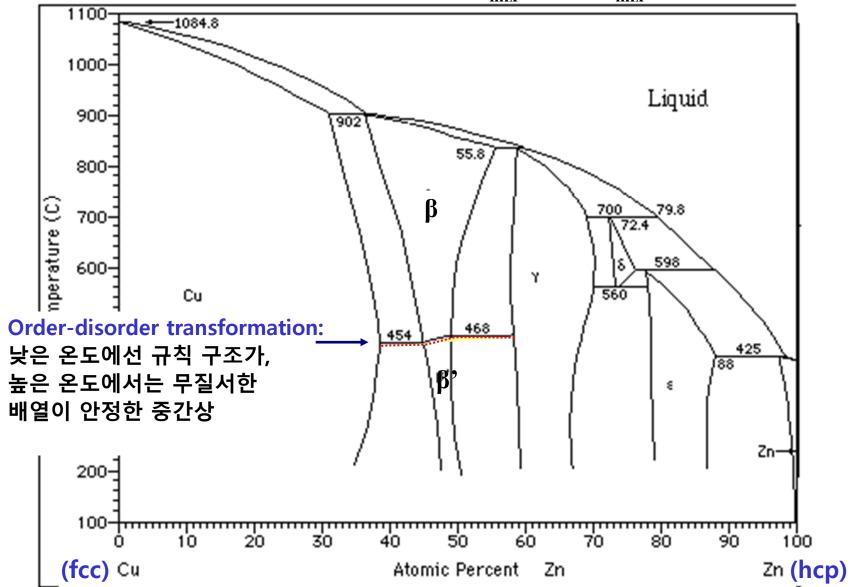
Order-disorder phase transformation

- Not classical phase change=~not depend on diffusion process
- change of temperature allowed a continuous re-arrangement of atoms without changing the phase = "2nd order transition"
- boundary: ordered lattice & disordered lattice/phase rule could not applied there are cases in which an ordered phase of one composition exists in equilibrium with a disordered phase of a different composition.
- Simple composition of the type AB or AB₃ can the transformation (i.e. at the temperature maximum) be considered diffusionless.



 $\varepsilon < 0$, $\Delta H_{mix} < 0 / \Delta H_{mix} \sim -21 \text{ kJ/mol}$

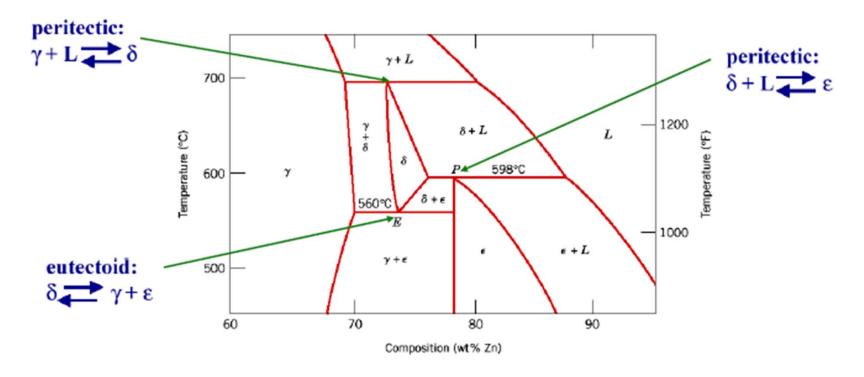
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- \cdot α and η are terminal solid solutions
- · β , β ', γ , δ and ϵ are intermediate solid solutions.

Cu-Zn Phase Diagram

Eutectoid and Peritectic Reactions



Eutectoid: one solid phase transforms into two other solid phases upon cooling

Solid &

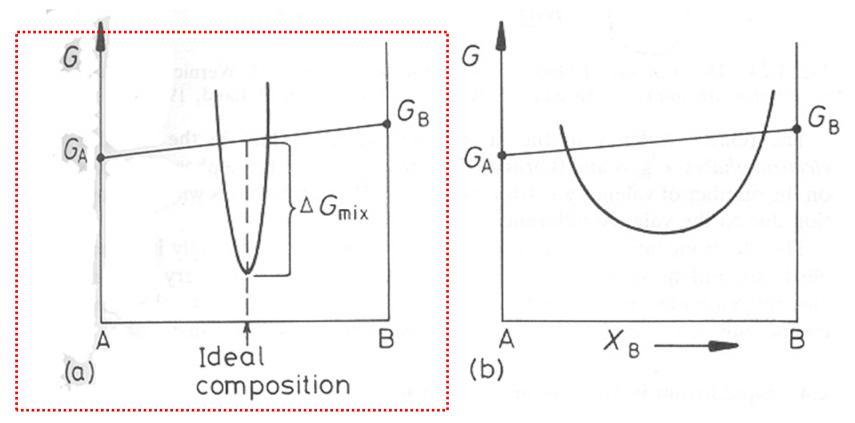
β+α

Peritectic: one solid and one liquid phase transform into another solid phase

upon cooling

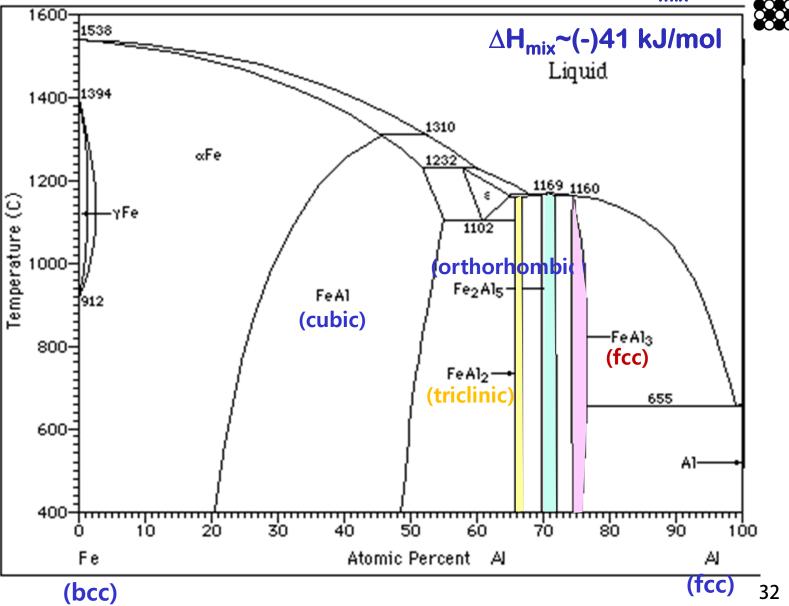
<u>Peritectoid</u>: two other solid phases transform into another solid phase upon cooling

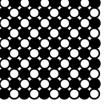
Q5: Intermediate phase vs Intermetallic compound

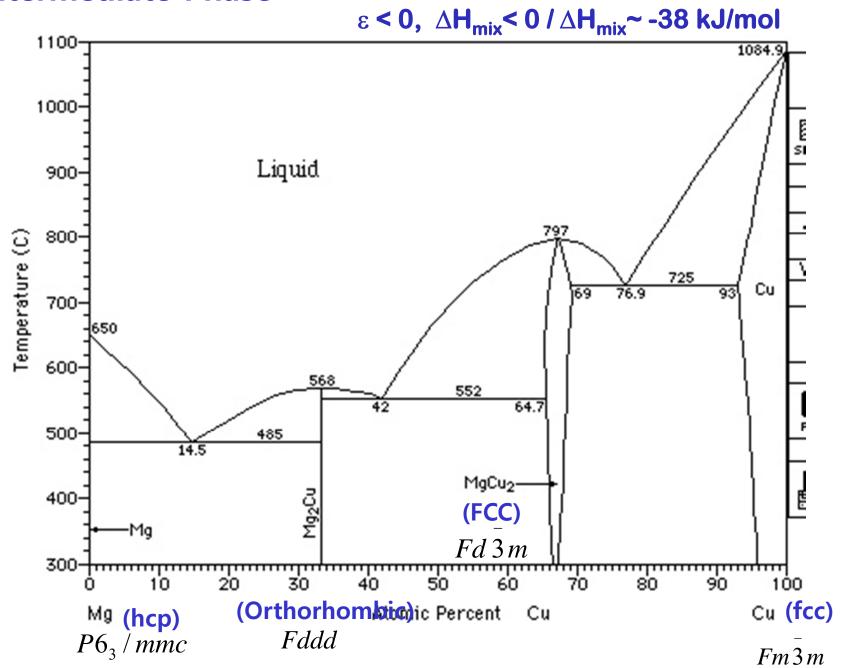


- * Many intermetallic compounds have stoichiometric composition $A_m B_n$ and a characteristic free energy curve as shown in Fig (a).
- * In other structure, <u>fluctuations in composition can be tolerated</u> by some atoms occupying 'wrong' <u>position</u> or by atom sites being left vacant, and in these cases the curvature of the G curve is much less, Fig (b).

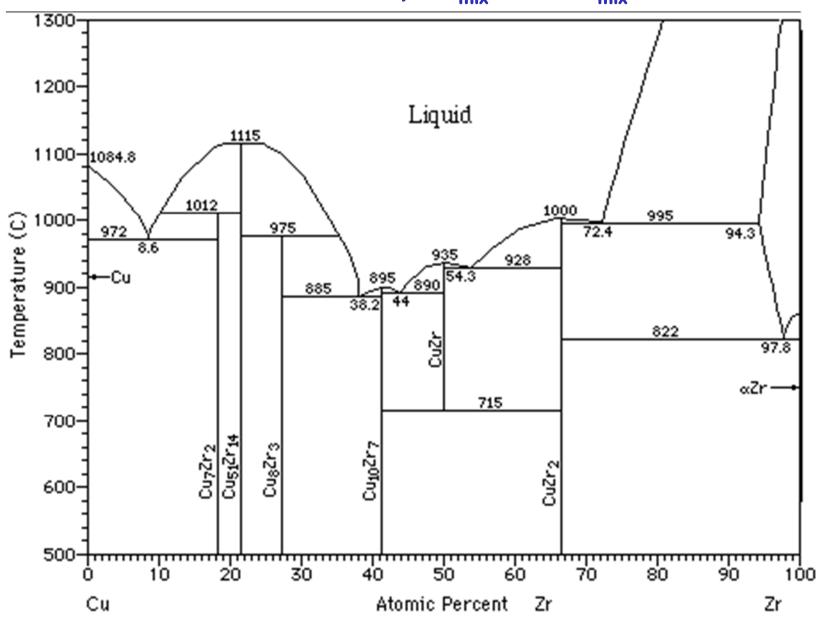
 ε < 0, ΔH_{mix} < 0







 ε << 0, ΔH_{mix} << 0 / ΔH_{mix} ~ -142 kJ/mol



Q6: Main factors determining the structure of intermediate phase

1.3 Binary Solutions

Intermediate Phase

_"different crystal structure as either of the pure component"

3 main factors

determining the structure of Intermediate phase?

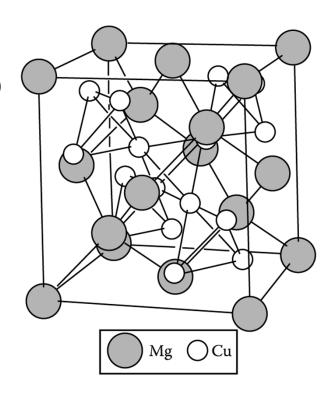
1) Relative atomic size

- Laves phase (size difference: 1.1~1.6 ex: MgCu₂) fill space most efficiently ~ stable
- Interstitial compound: MX, M₂X, MX₂, M₆X M= Cubic or HCP ex: Zr, Ti, V, Cr, etc, X= H, B, C, and N

2) Relative valency electron

- electron phases ex_α & β brass
 # of valency electrons per unit cell
 → depending on compositional change
- 3) Electronegativity

- <u>very different electronegativites</u> → ionic bond_normal valency compounds ex Mg₂Sn



MgCu₂ (A Laves phase)

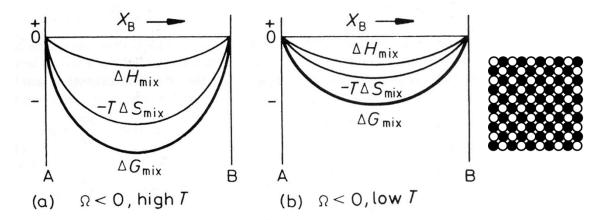
"Clustering"? → Phase separation

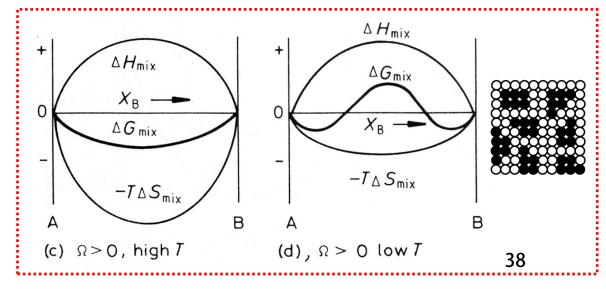
Q7: Metastable vs Stable miscibility gap

* The degree of ordering or clustering will decrease as temp. increases due to the increasing importance of entropy.

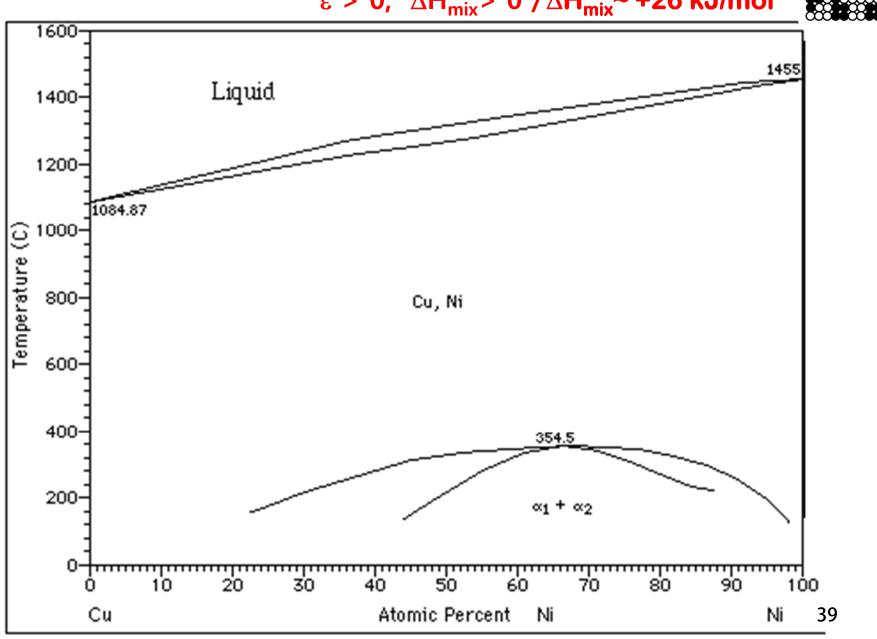
High temp. \longrightarrow Entropy effect \uparrow \longrightarrow Solution stability \uparrow



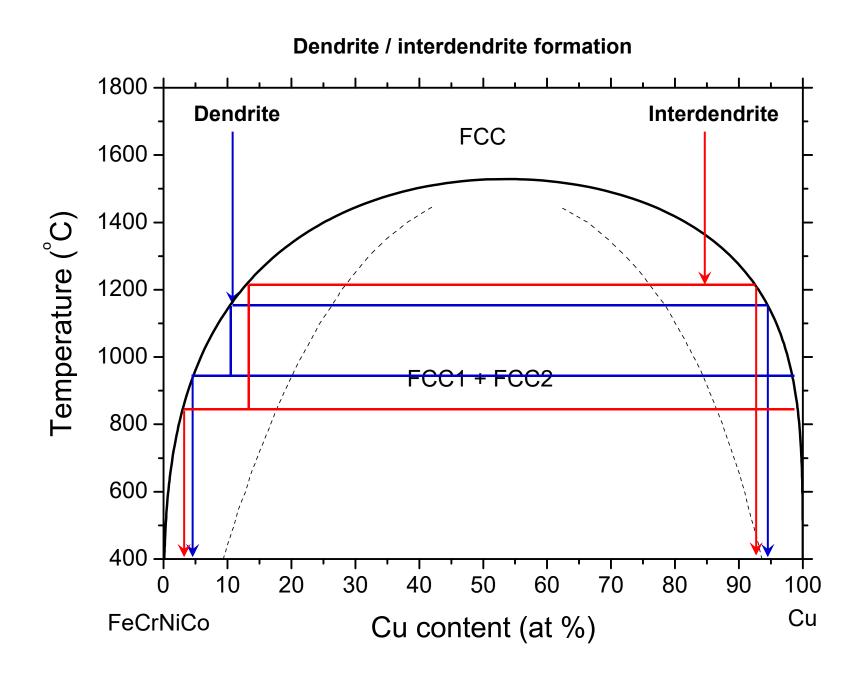




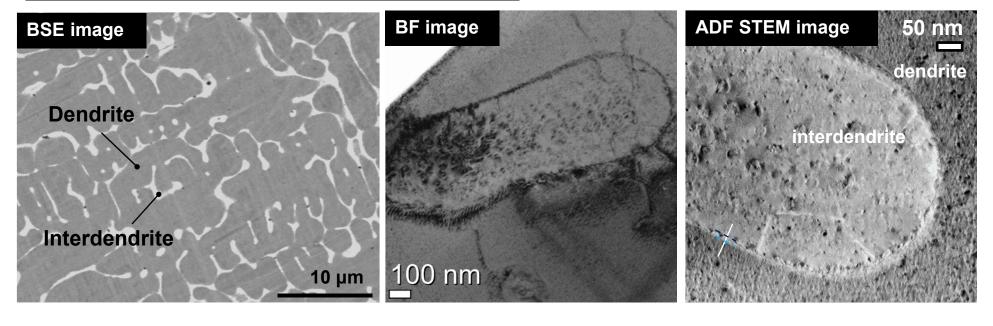


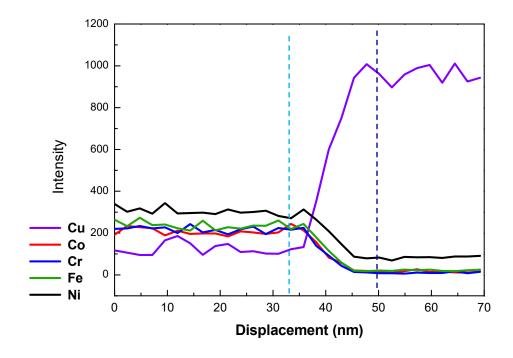


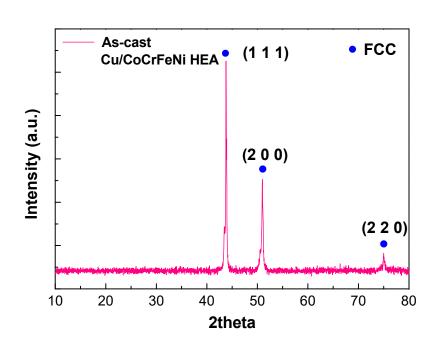
Cooling process in the miscibility gap



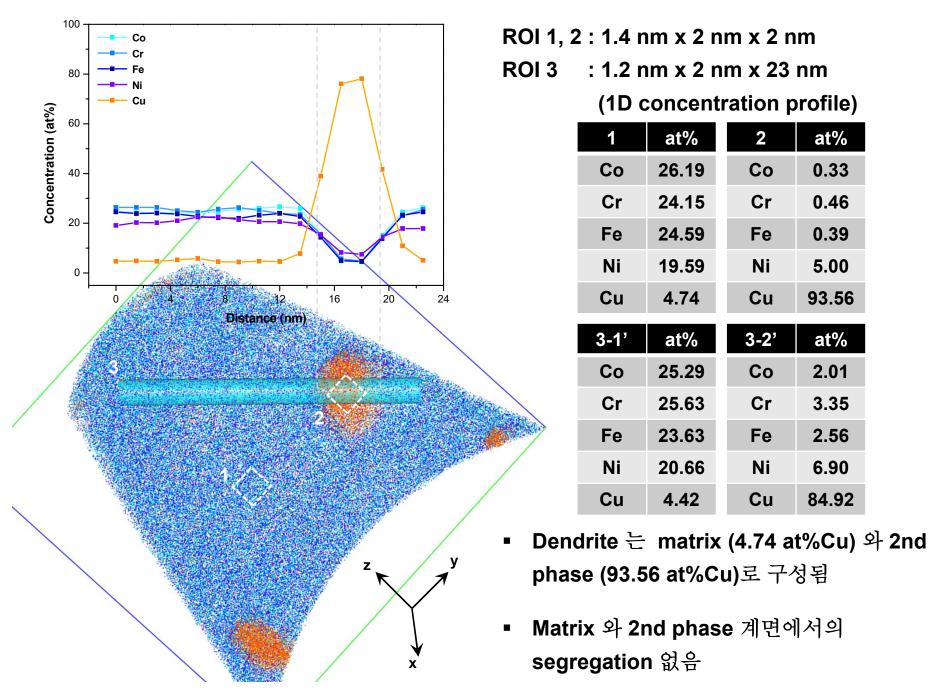
Microstructure of as-cast CoCrFeNiCu HEA



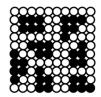




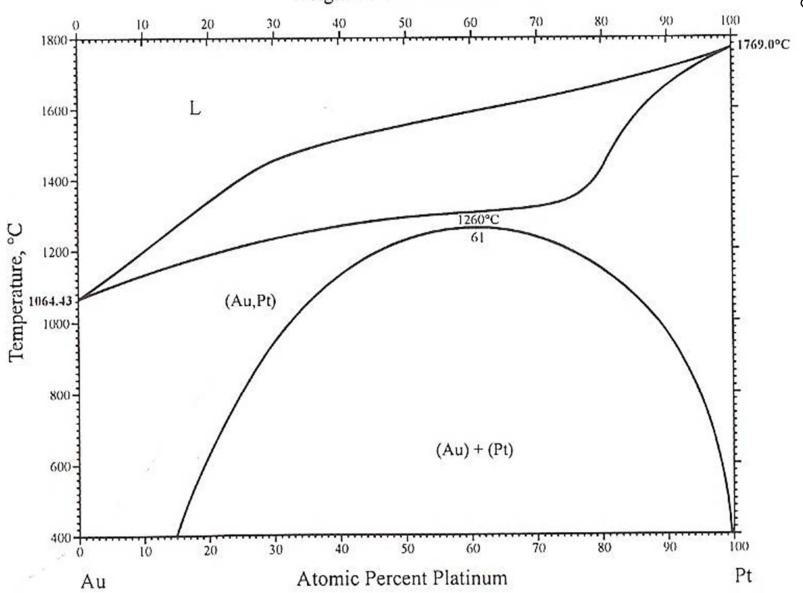
Compositional analysis of as-cast CoCrFeNi/Cu HEA (dendrite)

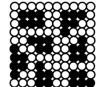


$\varepsilon > 0$, $\Delta H_{mix} > 0 / \Delta H_{mix} \sim +17 \text{ kJ/mol}$

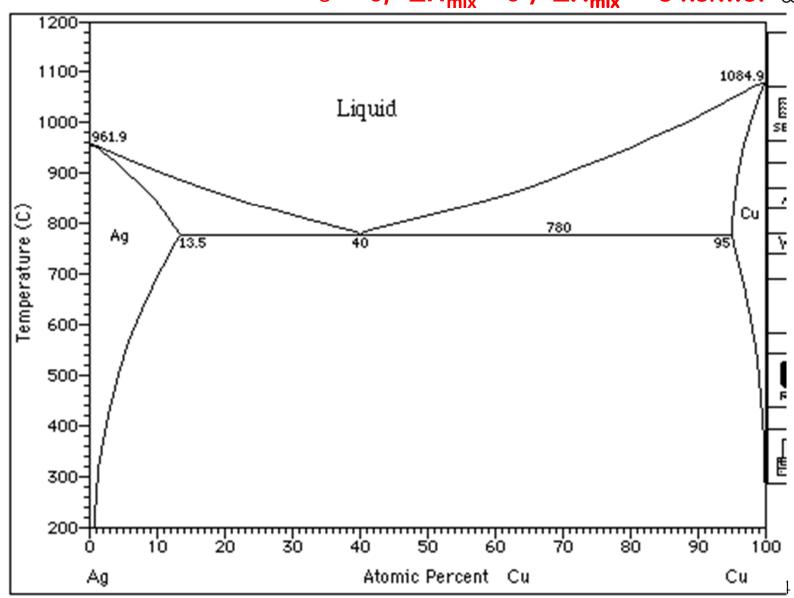


Weight Percent Platinum



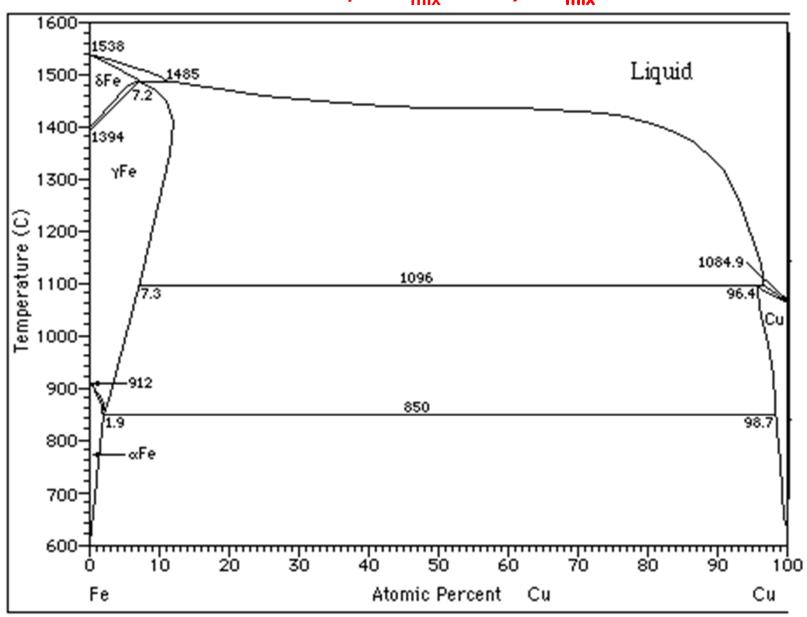


$\varepsilon > 0$, $\Delta H_{mix} > 0$ / $\Delta H_{mix} \sim +5$ kJ/mol



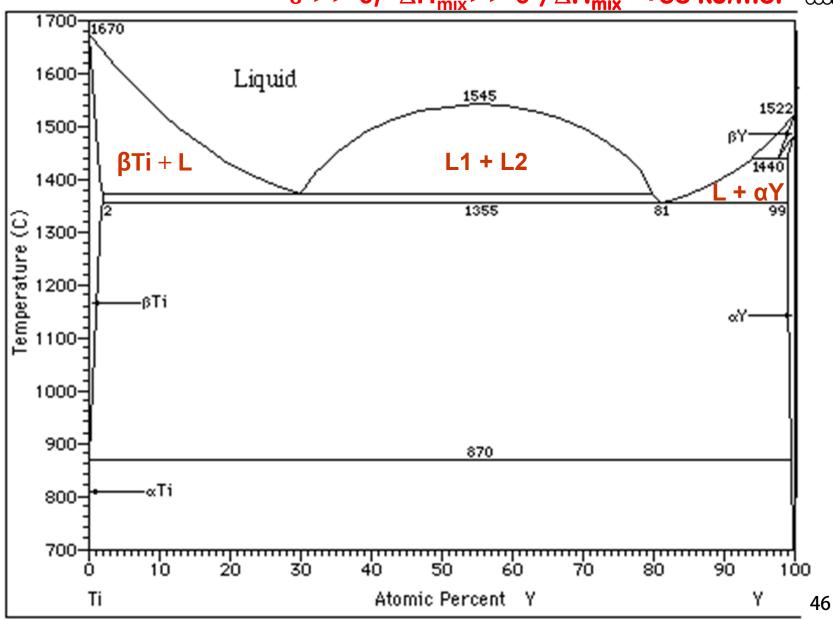


$\varepsilon >> 0$, $\Delta H_{mix} >> 0 / \Delta H_{mix} \sim +60 \text{ kJ/mol}$





$\varepsilon >> 0$, $\Delta H_{mix} >> 0 / \Delta H_{mix} \sim +58 \text{ kJ/mol}$



"Clustering"? → Phase separation

Q8: Spinodal decomposition

5.5.5 Spinodal Decomposition

Spinodal mode of transformation has no barrier to nucleation

: describing the transformation of a system of two or more components in a metastable phase into two stable phases

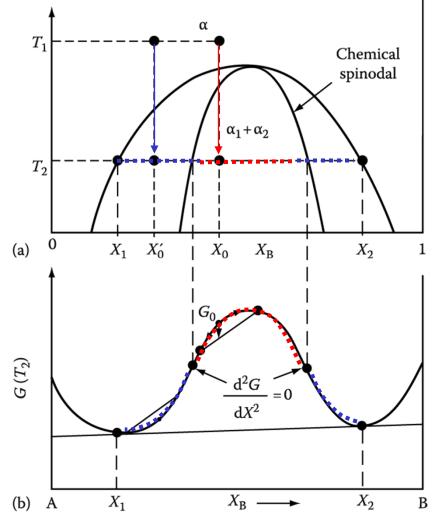


Fig. 5.38 Alloys between the spinodal points are unstable and can decompose into two coherent phasees α_1 and α_2 without overcoming an activation energy barrier. Alloys between the coherent miscibility gaps and the spinodal are metastable and can decompose only after nucleation of the other phase.

How does it differ between inside and outside the inflection point of Gibbs free energy curve?

1) Within the spinodal $\frac{d^2G}{dX^2} < 0$

: phase separation by small fluctuations in composition/ "up-hill diffusion"

2) If the alloy lies outside the spinodal, small variation in composition leads to an increase in free energy and the alloy is therefore metastable.

The free energy can only be decreased if nuclei are formed with a composition very different from the matrix.

→ nucleation and growth 48
: "down-hill diffusion"

a) Composition fluctuations within the spinodal

b) Normal down-hill diffusion outside the spinodal

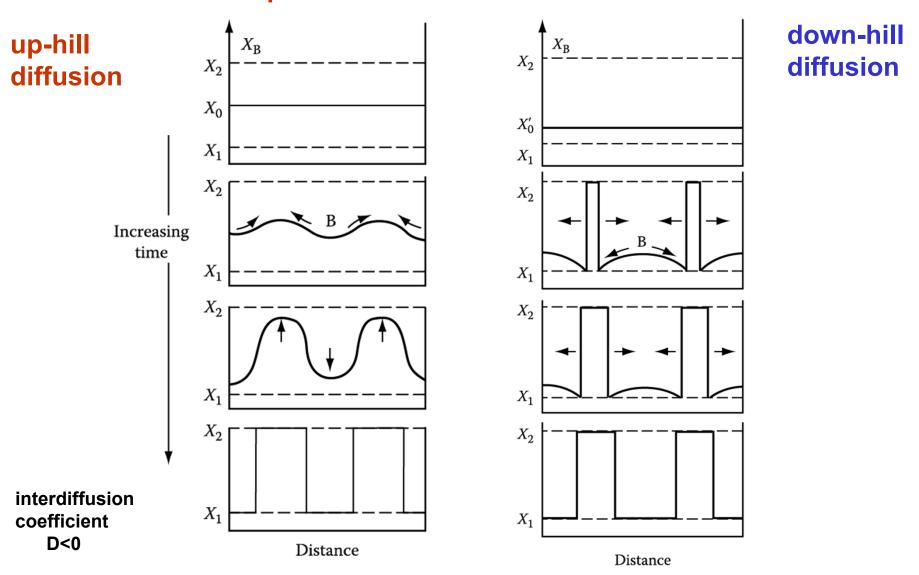


Fig. 5.39 & 5.40 schematic composition profiles at increasing times in (a) an alloy quenched into the spinodal region (X_0 in Figure 5.38) and (b) an alloy outside the spinodal points (X_0 in Figure 5.38)

Q9: Phase separation

Positive heat of mixing relation among constituent elements

► Alloy design considering heat of mixing relation among constituent elements

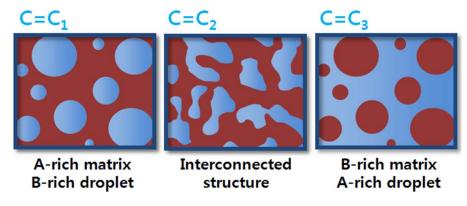
$$\Delta H_{mix} \gg 0$$
 between A & B

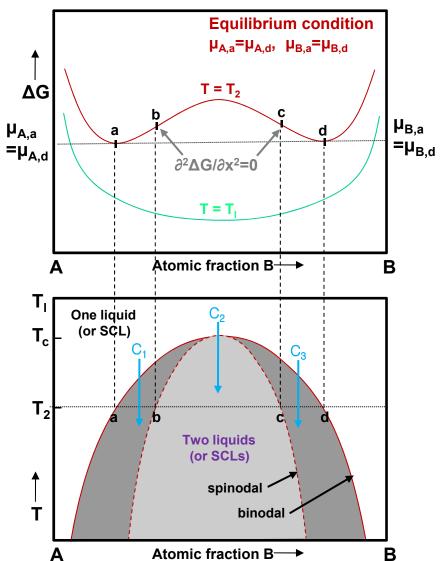
creates (meta)stable miscibility gap in limited composition range

1

Phase separation to A-rich & B-rich phase

▶ Different two-phase structure by initial composition before phase separation



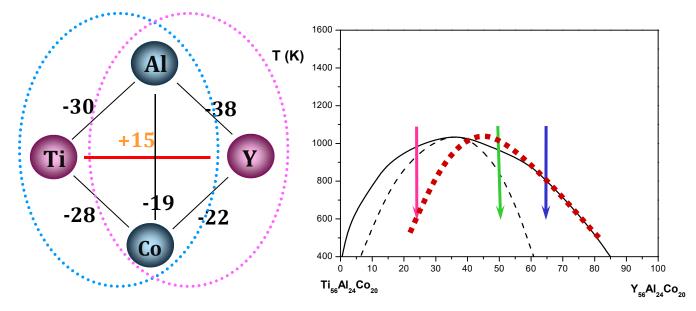


Nucleation and growth

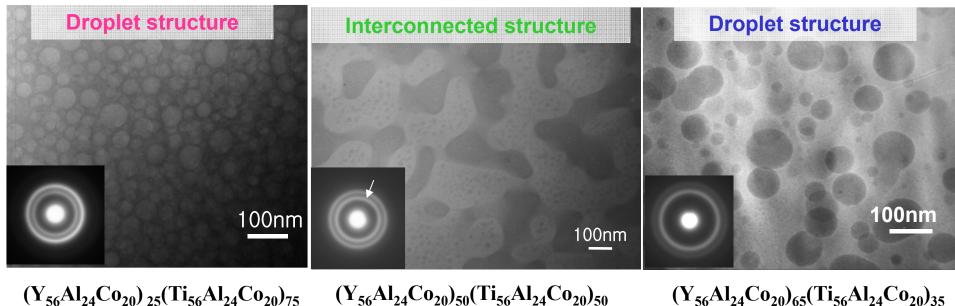
→ Spinodal decomposition without any barrier to the nucleation process

* Ti-Y-Al-Co system

 $(Y_{56}Al_{24}Co_{20})_{25}(Ti_{56}Al_{24}Co_{20})_{75}$



 $(Y_{56}Al_{24}Co_{20})_{65}(Ti_{56}Al_{24}Co_{20})_{35}$



* La-Zr-Al-Cu-Ni system

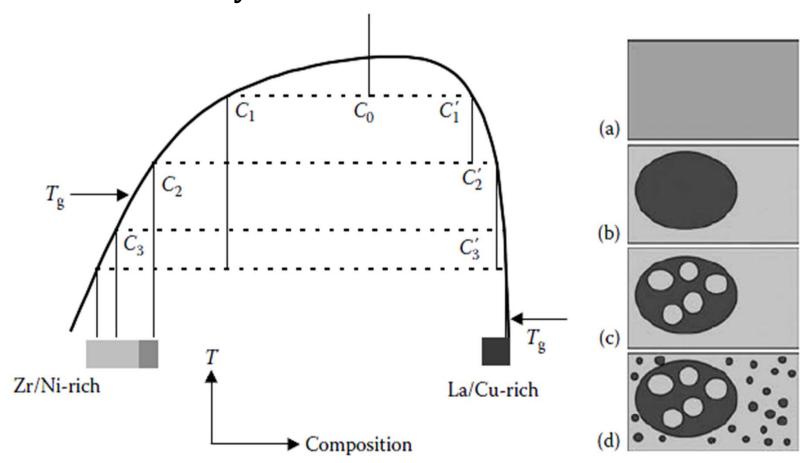
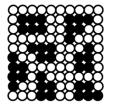
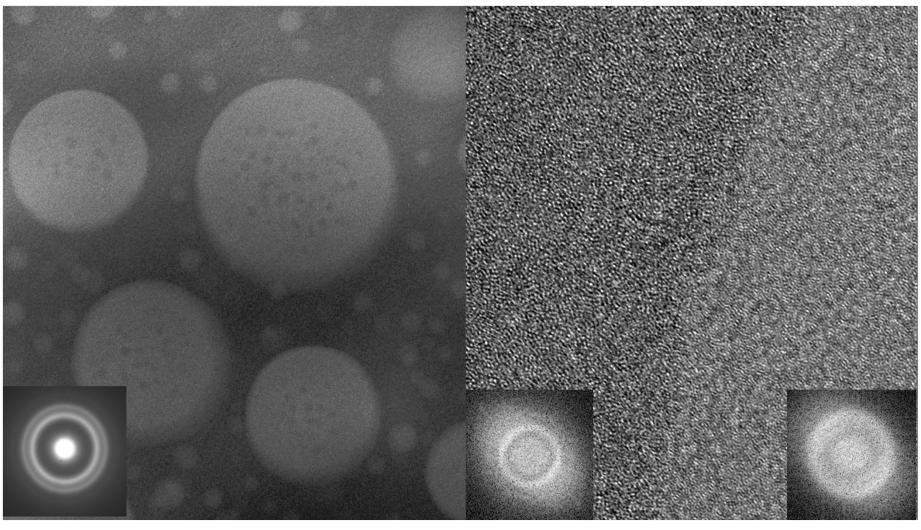


FIGURE 5.17

Schematic of the miscibility gap and the sequence of phase formation during cooling in the La–Zr–Al–Cu–Ni system. The positions of letters (a) to (d) in the diagram on the left correspond to the schematic microstructures (a) to (d) on the right. (Reprinted from Kündig, A.A. et al., Acta Mater., 52, 2441, 2004. With permission.)



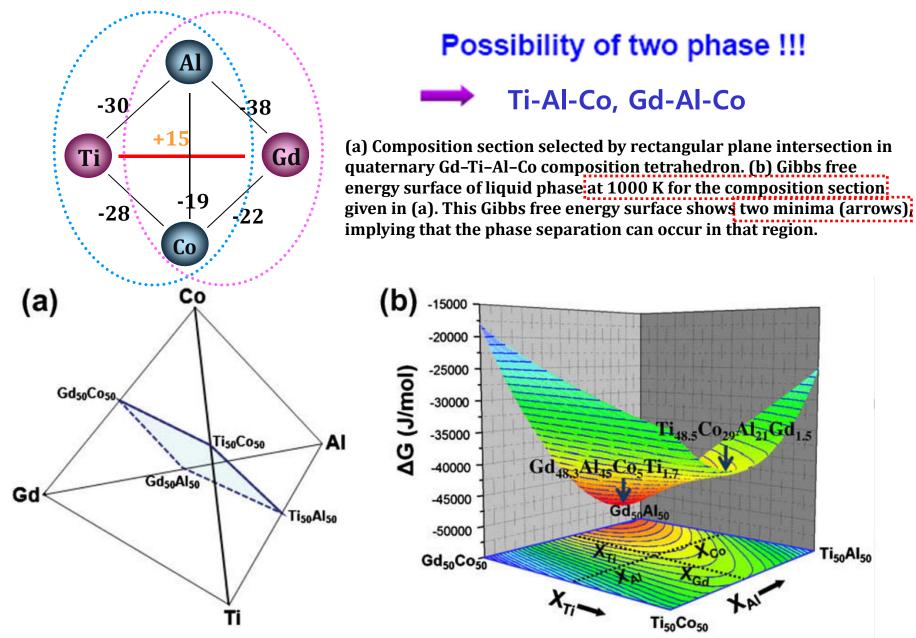
Phase separation in metallic glasses



Q10: Microstructure determining parameters of phase separation in metallic glasses

- (a) Composition
- (b) Critical temperature, T_c
- (c) Asymmetry of the spinodal curve/decomposition range
- (d) Glass-forming ability of the separated liquid

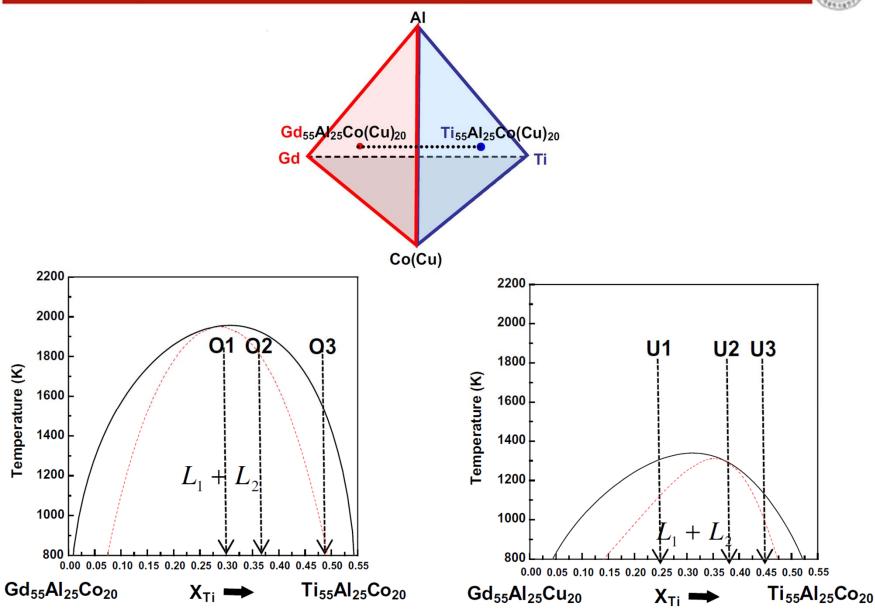
Synthesis of metallic glass composites using phase separation phenomenon



(a) Composition

Thermodynamic calculation using CALPHAD

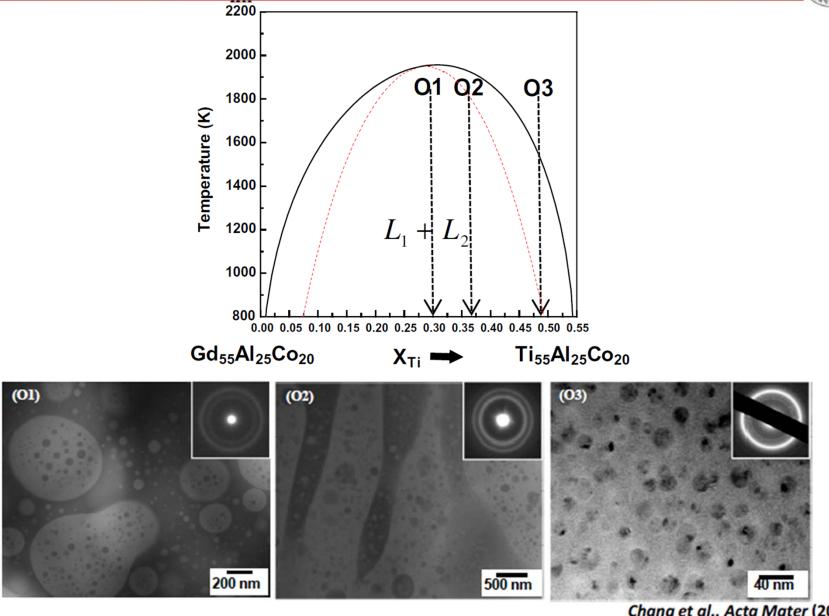




(a) Composition

Microstructure evolution (GdTiAlCo)



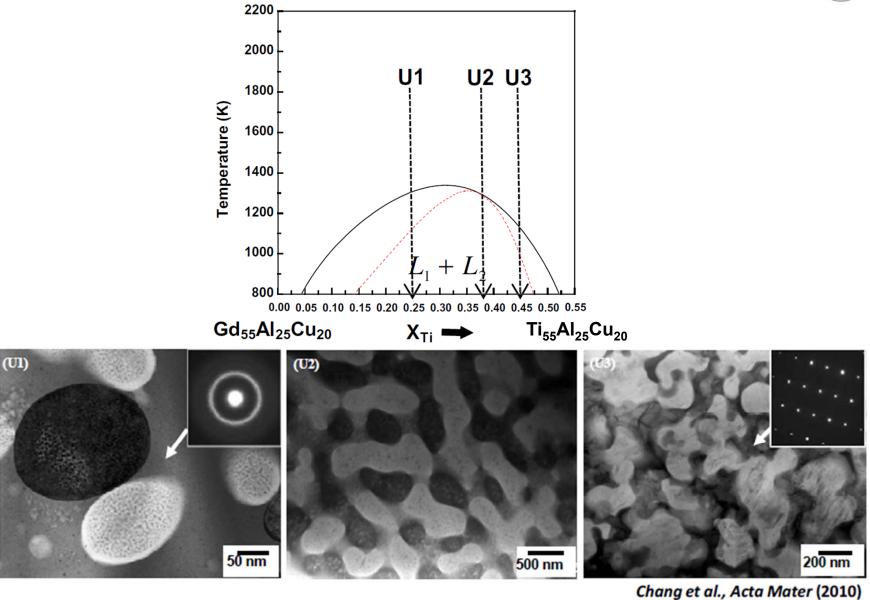


Chang et al., Acta Mater (2010)

(a) Composition

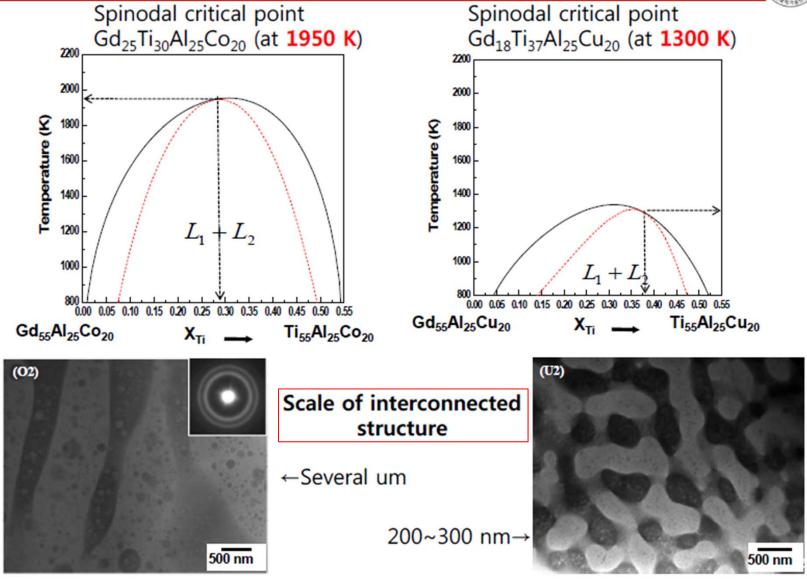
Microstructure evolution (GdTiAlCu)





(b) Critical temperature

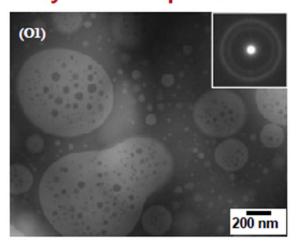


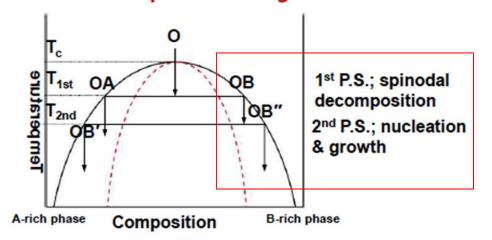


(c) Asymmetry of spinodal curve / Decomposition range

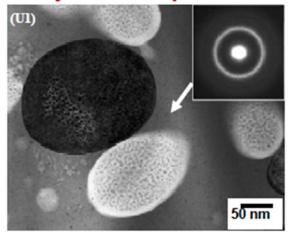


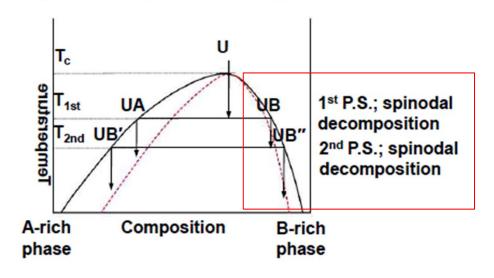
* Symmetric spinodal curve / smaller decomposition range



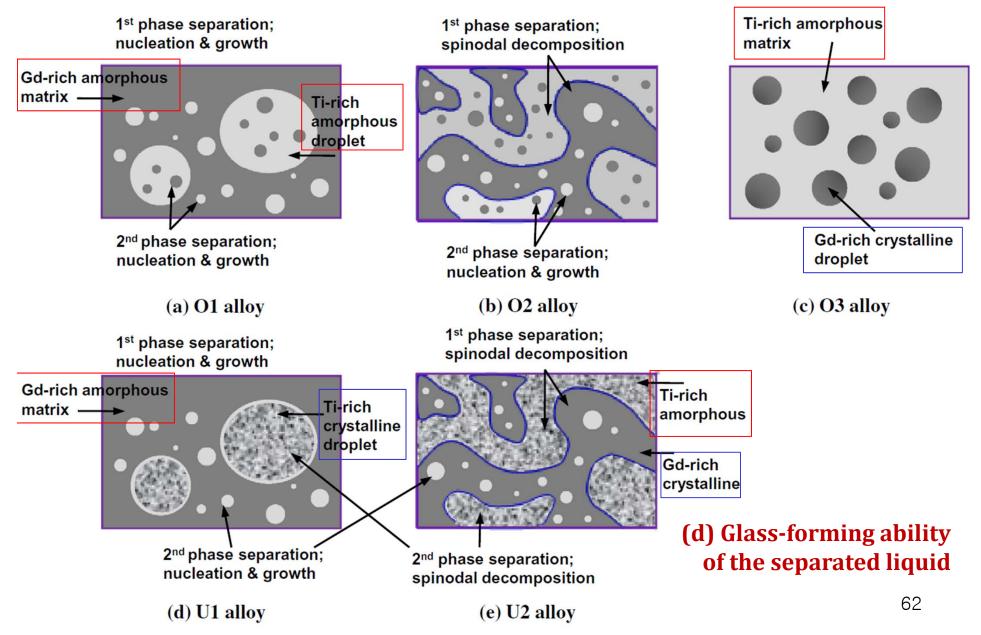


*Asymmetric spinodal curve / larger decomposition range





* Schematic drawings of the microstructures showing variation of microstructure depending on alloy composition and second phase separation mechanism.



Contents for today's class

- Binary System mixture/solution/compound

Ideal solution ($\Delta H_{mix} = 0$) Random distribution

$$\Delta H_{\text{mix}} = P_{AB} \epsilon$$

Regular solution
$$\Delta H_{\text{mix}} = P_{AB} \varepsilon$$
 where $\varepsilon = \varepsilon_{AB} - \frac{1}{2} (\varepsilon_{AA} + \varepsilon_{BB})$ $\varepsilon \approx 0$



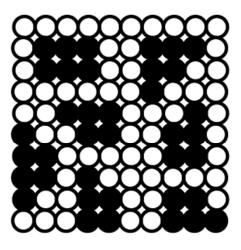
$$\Delta H_{\text{mix}} > 0 \text{ or } \Delta H_{\text{mix}} < 0$$

Real solution

(a)
$$\varepsilon$$
 < 0, ΔH_{mix} < 0 Ordered alloys

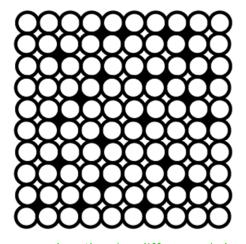
$$P_{AB} \uparrow \longrightarrow Internal E \downarrow \qquad P_{AA}, P_{BB} \uparrow$$

Ordered structure



(b)
$$\varepsilon > 0$$
, $\Delta H_{mix} > 0$

Clustering



(c) when the size difference is large strain effect

Interstitial solution

