

**2017 Fall**

# **“Phase Transformation *in* Materials”**

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## Contents for previous class

- **Binary System** mixture/ solution / compound
- **Gibbs Free Energy in Binary System**

$$G_1 = X_A G_A + X_B G_B \quad J/mol$$

$$G_2 = G_1 + \Delta G_{mix} \quad J/mol$$

**Ideal solution** ( $\Delta H_{mix} = 0$ )  $\Delta G^{mix} = RT(X_A \ln X_A + X_B \ln X_B)$

$$G = X_A G_A + X_B G_B + RT(X_A \ln X_A + X_B \ln X_B)$$

**Regular solution**

$$\Delta H_{mix} = P_{AB} \epsilon \quad \text{where } \epsilon = \epsilon_{AB} - \frac{1}{2}(\epsilon_{AA} + \epsilon_{BB})$$

$$G = X_A G_A + X_B G_B + \Omega X_A X_B + RT(X_A \ln X_A + X_B \ln X_B)$$

- **Chemical potential and Activity**

$$\mu_A = \left( \frac{\partial G'}{\partial n_A} \right)_{T, P, n_B}$$

$$\mu_A = G_A + RT \ln a_A \quad \ln \left( \frac{a_A}{X_A} \right) = \frac{\Omega}{RT} (1 - X_A)^2$$

$$\frac{a_A}{X_A} = \gamma_A = \text{activity coefficient}$$

$dn_A \sim$  small enough ( $\because \mu_A$  depends on the composition of phase)

## 1.3 Binary Solutions

### 1) Ideal solution

$$G = H - TS = E + PV - TS$$

## Chemical potential

The increase of the total free energy of the system by the increase of very small quantity of A,  $dn_A$ , will be proportional to  $\mu_A$ .

→  $dn_A \sim$  small enough  
( $\because \mu_A$  depends on the composition of phase)

$$dG' = \mu_A dn_A \quad (T, P, n_B: \text{constant})$$

$\mu_A$  : partial molar free energy of A  
or chemical potential of A

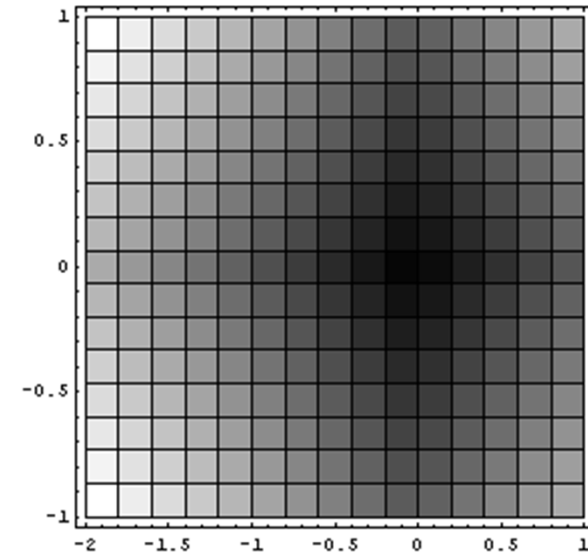
$$\mu_A = \left( \frac{\partial G'}{\partial n_A} \right)_{T, P, n_B}$$

$$\mu_B = \left( \frac{\partial G'}{\partial n_B} \right)_{T, P, n_A}$$

For A-B binary solution,  $dG' = \mu_A dn_A + \mu_B dn_B$

For variable T and P

$$dG' = -SdT + VdP + \mu_A dn_A + \mu_B dn_B$$



### 1.3 Binary Solutions

#### 1) Ideal solution

$$G = X_A G_A + X_B G_B + RT(X_A \ln X_A + X_B \ln X_B)$$

$$= (G_A + RT \ln X_A) X_A + (G_B + RT \ln X_B) X_B = \mu_A X_A + \mu_B X_B$$

$$\mu_A = G_A + RT \ln X_A$$

$$\mu_B = G_B + RT \ln X_B$$

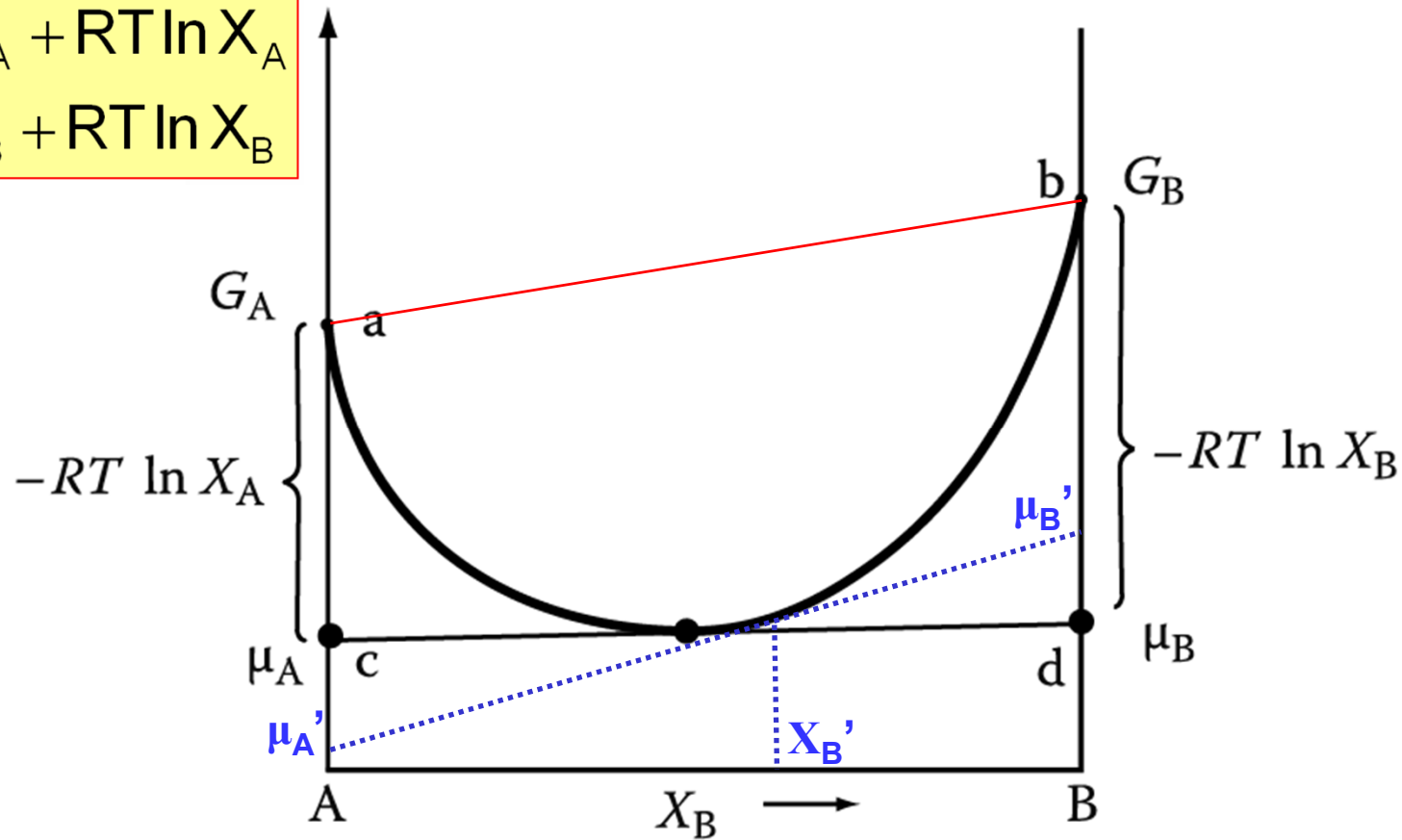


Fig. 1.12 The relationship between the free energy curve and Chemical potentials for an ideal solution.



## Contents for today's class II

- Ideal Solution/ **Regular Solution**/ Real solution

## 1.3 Binary Solutions

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$

**Q1: What is “Regular Solution”?**

## 1.3 Binary Solutions

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$

### Regular Solutions

Ideal solution :  $\Delta H_{mix} = 0$   $\Rightarrow$  This type of behavior is exceptional in practice and usually mixing is endothermic or exothermic.

Quasi-chemical model assumes that **heat of mixing,  $\Delta H_{mix}$** , is only due to **the bond energies between adjacent atoms**.

**Assumption:** the volumes of pure A and B are equal and do not change during mixing so that the interatomic distance and bond energies are independent of composition.

### Structure model of a binary solution

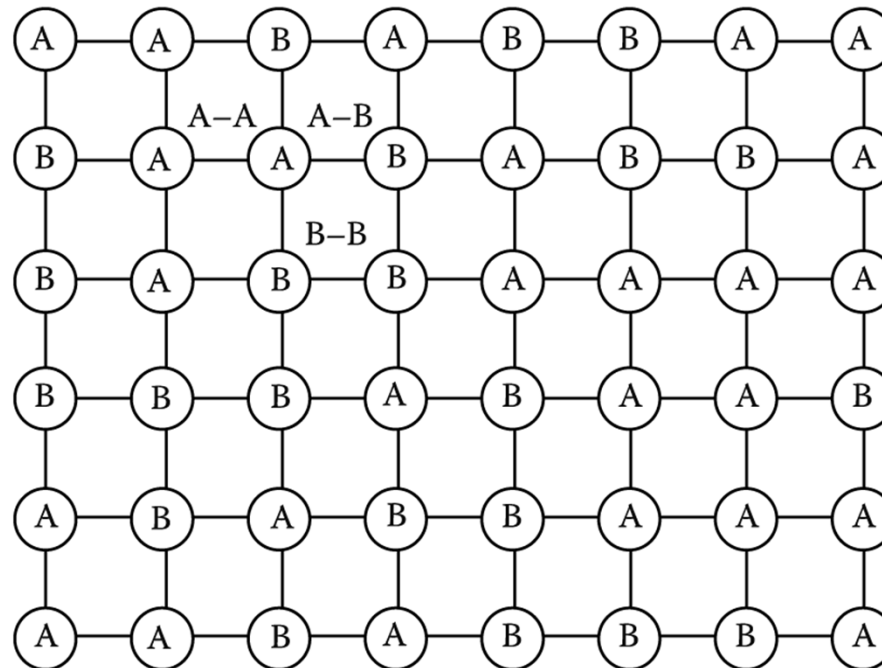


Fig. 1.13 The different types of interatomic bond in a solid solution.

## Gibbs Free Energy of Regular Solutions

**Q2: How can you estimate  
“ $\Delta H_{\text{mix}}$  of regular solution”?**

$$\Delta H_{\text{mix}} = \Omega X_A X_B \text{ where } \Omega = N_a z \epsilon$$

# 1.3 Binary Solutions

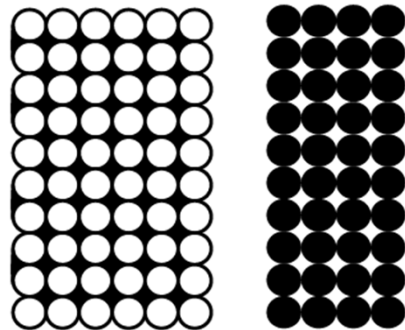
## Regular Solutions

	Bond energy	Number of bond
A-A	$\epsilon_{AA}$	$P_{AA}$
B-B	$\epsilon_{BB}$	$P_{BB}$
A-B	$\epsilon_{AB}$	$P_{AB}$

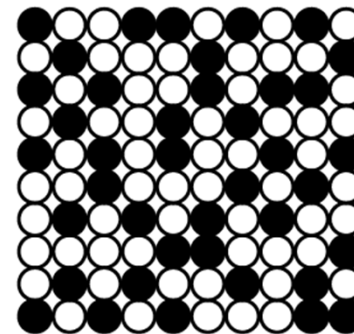
Internal energy of the solution

$$E = P_{AA} \epsilon_{AA} + P_{BB} \epsilon_{BB} + P_{AB} \epsilon_{AB}$$

Before mixing



After mixing



$$\Delta H_{\text{mix}} = P_{AB} \epsilon$$

where  $\epsilon = \epsilon_{AB} - \frac{1}{2}(\epsilon_{AA} + \epsilon_{BB})$  9

## 1.3 Binary Solutions

### Regular Solutions

Completely random arrangement

$$\varepsilon = 0 \quad \rightarrow \quad \varepsilon_{AB} = \frac{1}{2}(\varepsilon_{AA} + \varepsilon_{BB})$$

$$\Delta H_{\text{mix}} = 0 \quad \text{ideal solution}$$

$$\Delta S^{\text{mix}} = -R(X_A \ln X_A + X_B \ln X_B)$$

$$P_{AB} = N_a z X_A X_B \quad \text{bonds per mole}$$

$N_a$  : Avogadro's number

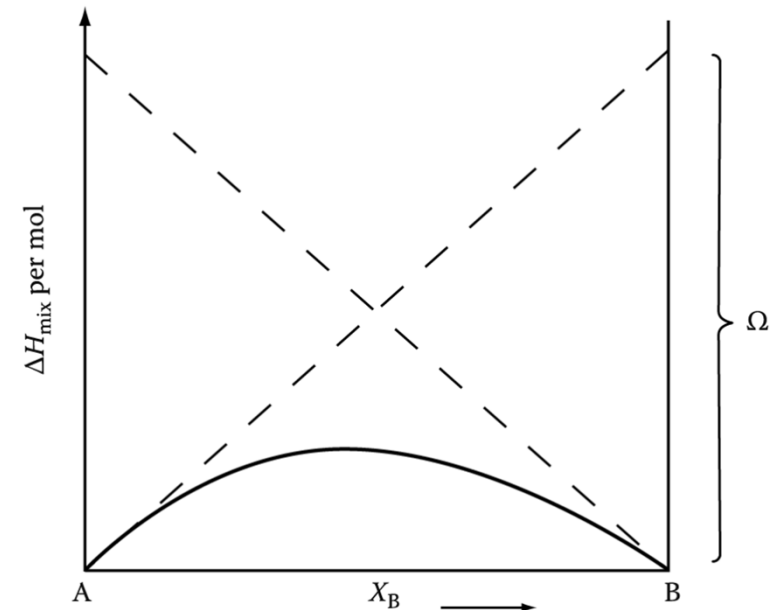
$z$  : number of bonds per atom

$$(1) \quad \varepsilon < 0 \rightarrow P_{AB} \uparrow \quad (2) \quad \varepsilon > 0 \rightarrow P_{AB} \downarrow$$

$$(3) \quad \varepsilon \approx 0 \quad \rightarrow \quad \Delta H_{\text{mix}} = P_{AB} \varepsilon$$

$$\Delta H_{\text{mix}} = \Omega X_A X_B \quad \text{where} \quad \Omega = N_a z \varepsilon$$

**Regular solution**



**If  $\Omega > 0$ ,**

Fig. 1.14 The variation of  $\Delta H_{\text{mix}}$  with composition for a regular solution.

## Gibbs Free Energy of Regular Solutions

Q3: How can you estimate

“Molar Free energy for regular solution”?

$$G_2 = G_1 + \Delta G_{mix}$$

$$G = X_A G_A + X_B G_B + \Omega X_A X_B + RT (X_A \ln X_A + X_B \ln X_B)$$

# Regular Solutions

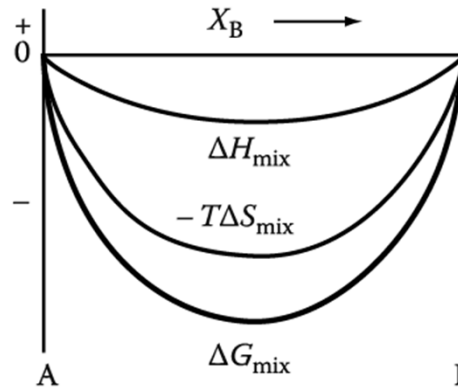
$$G_2 = G_1 + \Delta G_{mix}$$

$$G = X_A G_A + X_B G_B + \Omega X_A X_B + RT (X_A \ln X_A + X_B \ln X_B)$$

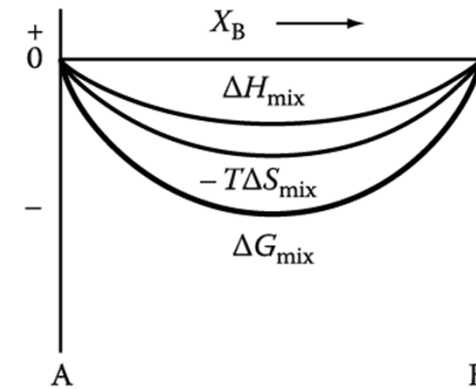
Reference state

Pure metal  $G_A^0 = G_B^0 = 0$

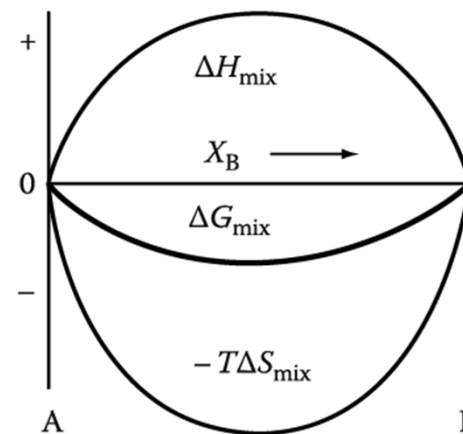
$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$



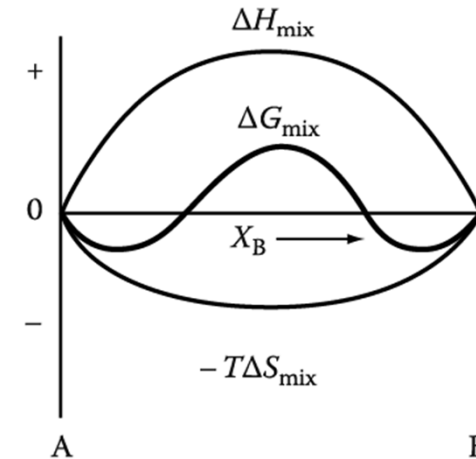
(a)  $\Omega < 0$ , high  $T$



(b)  $\Omega < 0$ , low  $T$



(c)  $\Omega > 0$ , high  $T$



(d)  $\Omega > 0$ , low  $T$



## Gibbs Free Energy of Binary Solutions

**Q4: “Correlation between chemical potential and free energy”?**

## Correlation between chemical potential and free energy

For 1 mole of the solution (T, P: constant)

$$G = E + PV - TS \quad G = \mu_A X_A + \mu_B X_B \quad \text{Jmol}^{-1}$$

$$G = H - TS \quad G = X_A G_A + X_B G_B + \Omega X_A X_B + RT (X_A \ln X_A + X_B \ln X_B)$$

$$\longrightarrow X_A X_B = X_A X_B (X_A + X_B) = X_A^2 X_B + X_B^2 X_A$$

$$= X_A (G_A + \Omega(1 - X_A)^2 + RT \ln X_A) + X_B (G_B + \Omega(1 - X_B)^2 + RT \ln X_B)$$

Regular solution

$$\mu_A = G_A + \Omega (1 - X_A)^2 + RT \ln X_A$$

$$\mu_B = G_B + \Omega (1 - X_B)^2 + RT \ln X_B$$

$$\mu_A = G_A + RT \ln X_A$$

$$\mu_B = G_B + RT \ln X_B$$

Ideal solution

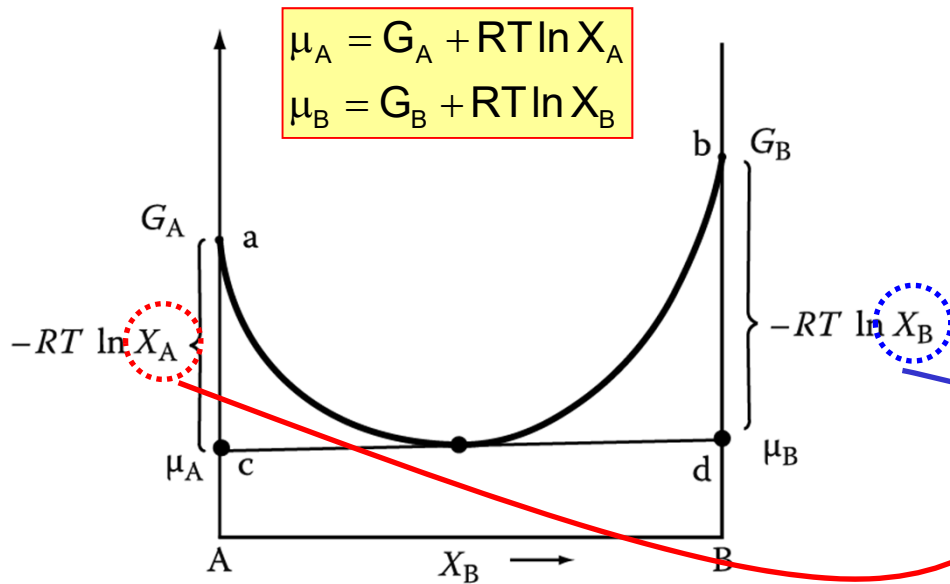


## Gibbs Free Energy of Binary Solutions

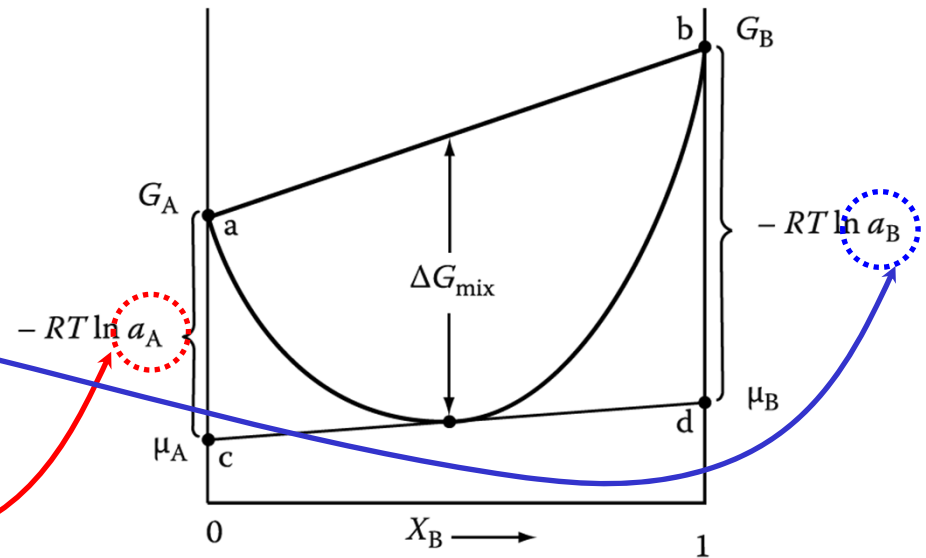
**Q5: What is “activity”?**

# Activity, a : effective concentration for mass action

**ideal solution**



**regular solution**



$$\mu_A = G_A + RT \ln a_A$$

$$\mu_B = G_B + RT \ln a_B$$

$$\mu_A = G_A + \Omega (1 - X_A)^2 + RT \ln X_A$$

$$\mu_B = G_B + \Omega (1 - X_B)^2 + RT \ln X_B$$

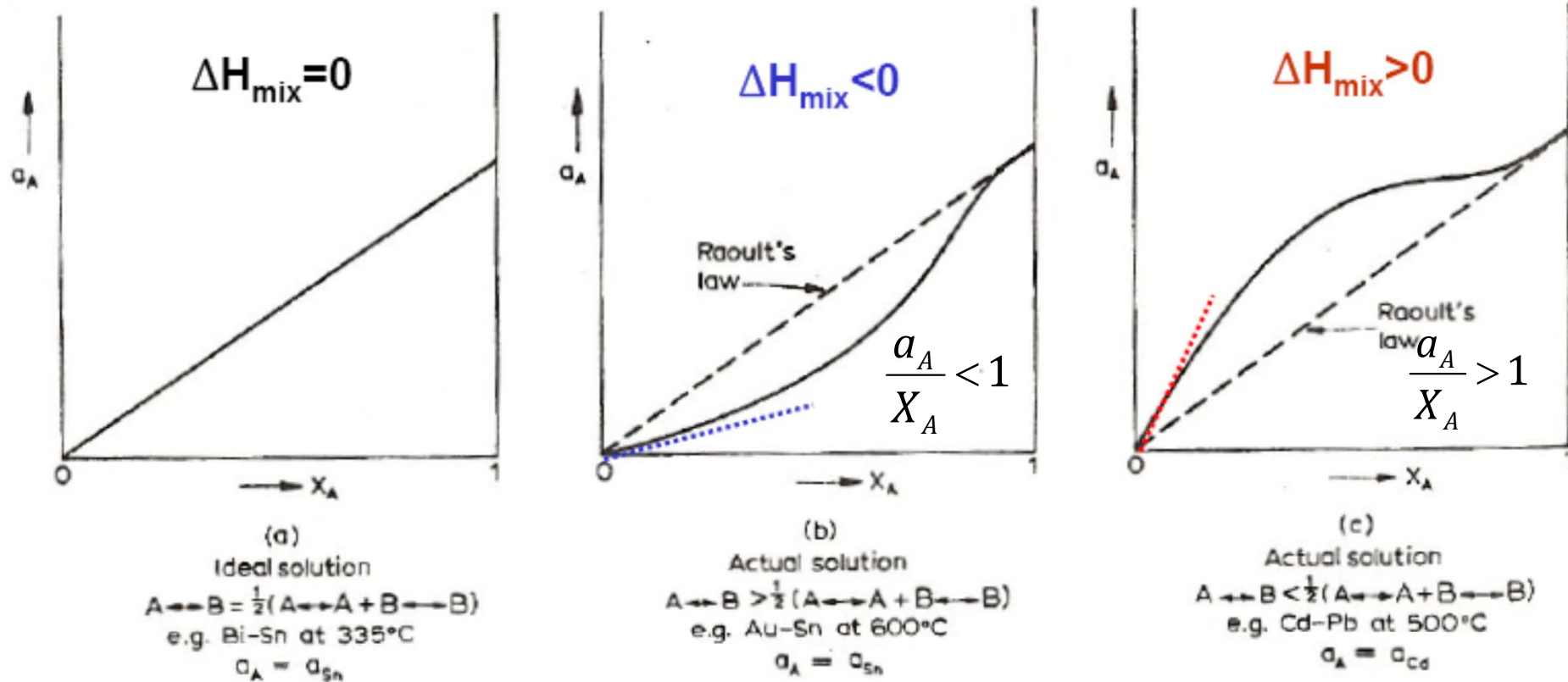
$$\ln \left( \frac{a_A}{X_A} \right) = \frac{\Omega}{RT} (1 - X_A)^2$$

$$\ln \left( \frac{a_B}{X_B} \right) = \frac{\Omega}{RT} (1 - X_B)^2$$

$$\frac{a_A}{X_A} = \gamma_A = \text{activity coefficient}$$

$$\gamma_B = \frac{a_B}{X_B}$$

## Activity-composition curves for solutions



### Degree of non-ideality

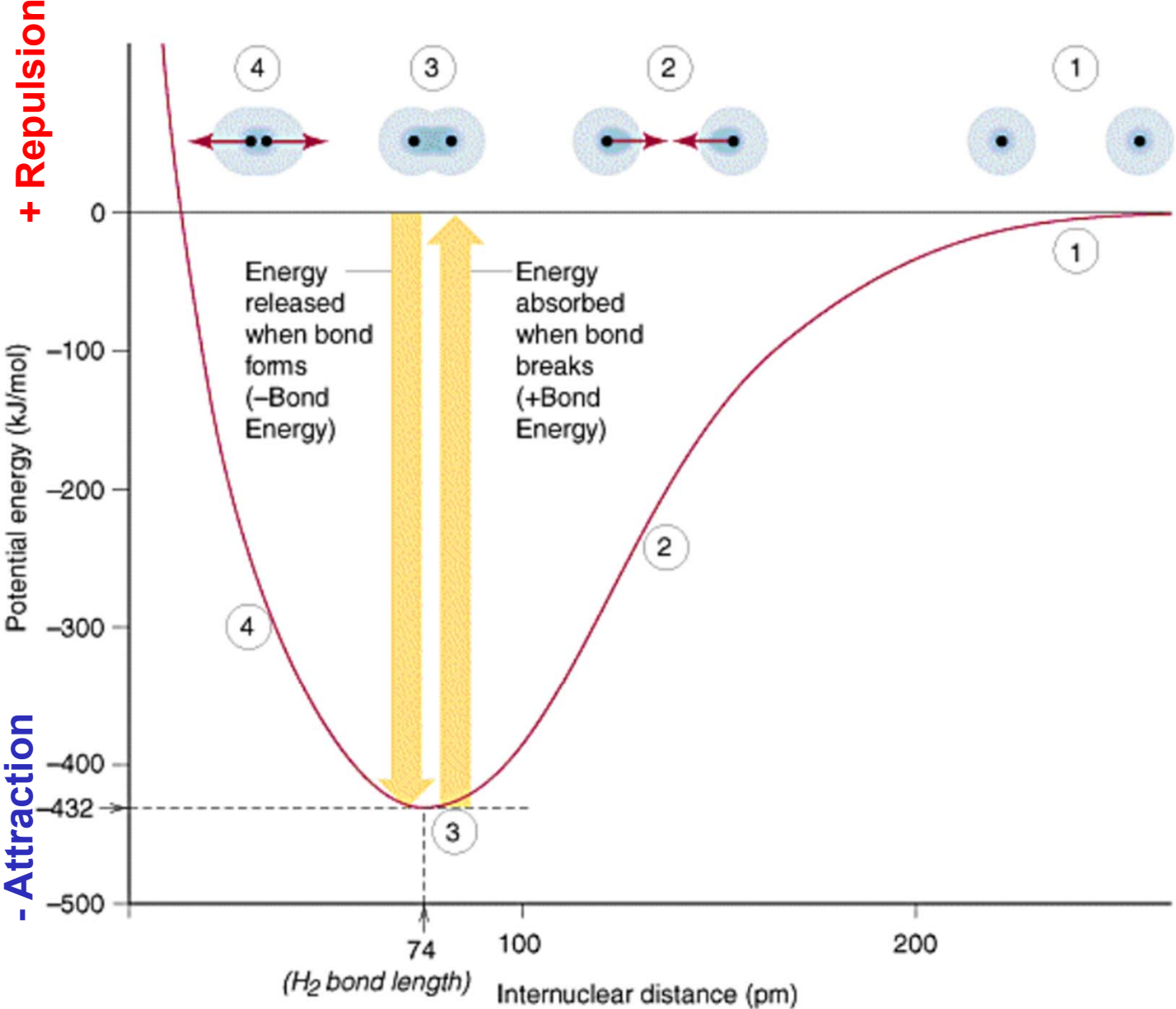
- For a dilute solution of B in A ( $X_B \rightarrow 0$ )

$$\gamma_B = \frac{a_B}{X_B} \cong \text{constant} \quad (\text{Henry's Law})$$

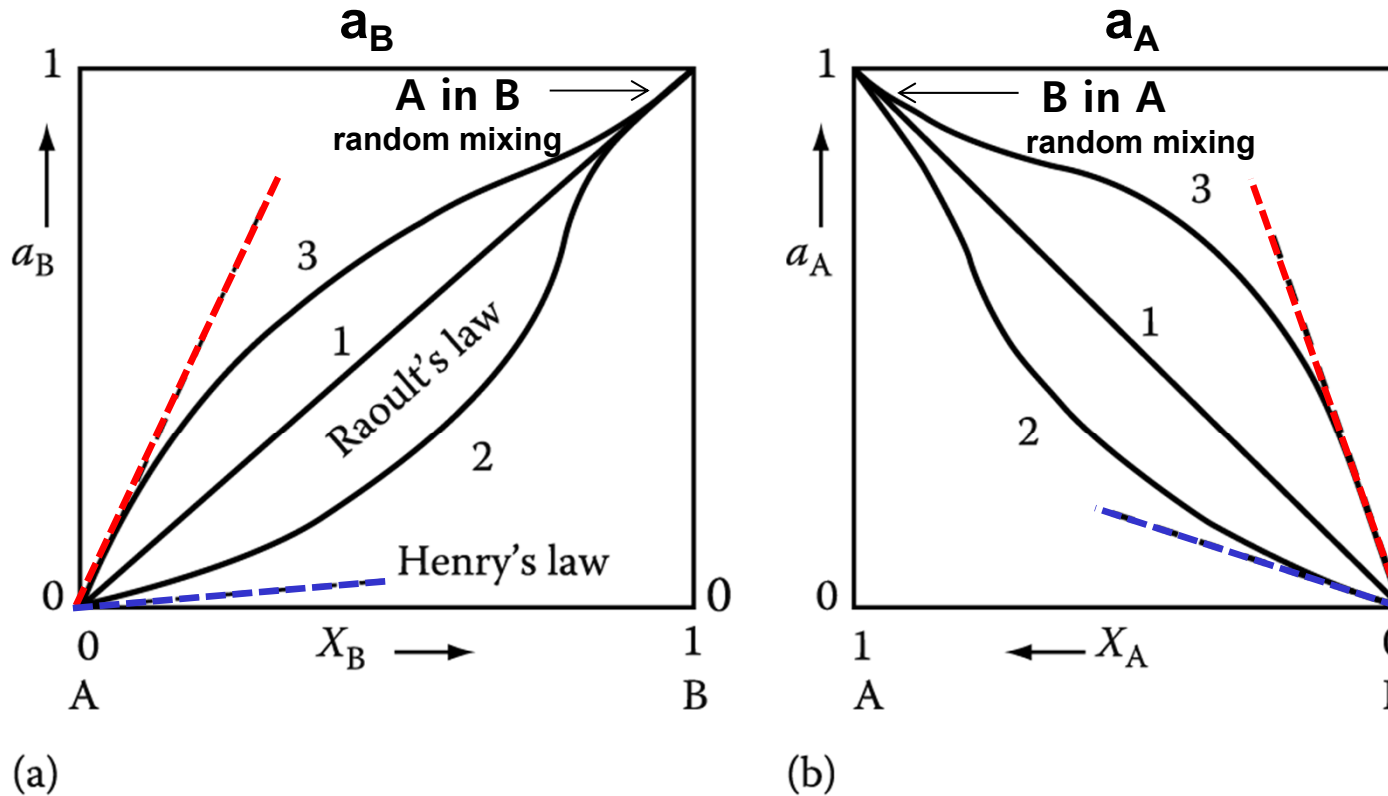
$$\gamma_A = \frac{a_A}{X_A} \cong 1 \quad (\text{Raoult's Law})$$

$$\Delta H_{\text{mix}} = P_{AB} \epsilon$$

where  $\epsilon = \epsilon_{AB} - \frac{1}{2}(\epsilon_{AA} + \epsilon_{BB})$



## Variation of activity with composition (a) $a_B$ , (b) $a_A$



**Line 1 :** (a)  $a_B = X_B$ , (b)  $a_A = X_A$

**Line 2 :** (a)  $a_B < X_B$ , (b)  $a_A < X_A$

**Line 3 :** (a)  $a_B > X_B$ , (b)  $a_A > X_A$

**ideal solution...Raoult's law**

$$\Delta H_{\text{mix}} < 0 \quad \leftarrow \quad \ln\left(\frac{a_A}{X_A}\right) = \frac{\Omega}{RT}(1-X_A)^2$$

$$\Delta H_{\text{mix}} > 0$$

## Gibbs Free Energy of Binary Solutions

**Q6: “Chemical equilibrium of multi-phases”?**



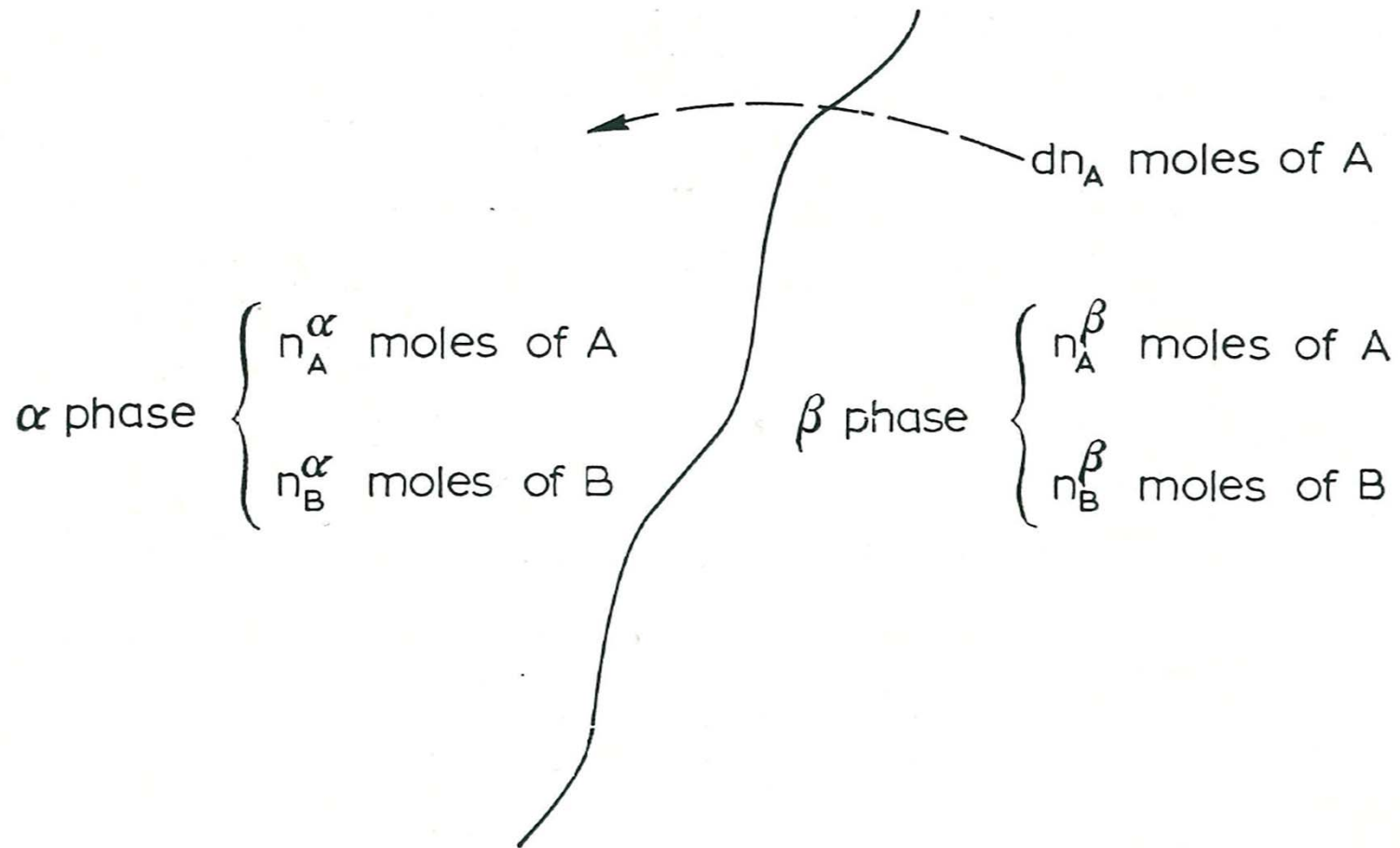


Fig. 10. Transfer of  $dn_A$  moles of component A from the  $\beta$  to the  $\alpha$  phase.

**Activity** of a component is just another means of describing **the state of the component in a solution.**

degree of non-ideality ?  $\rightarrow \frac{a_A}{X_A}$

$$\frac{a_A}{X_A} = \gamma_A, \quad a_A = \gamma_A X_A$$

$\gamma_A$  : activity coefficient

$$\ln\left(\frac{a_B}{X_B}\right) = \frac{\Omega}{RT} (1 - X_B)^2$$

$$\mu_A = G_A + RT \ln a_A$$

Activity or chemical potential of a component is important when several condensed phases are in equilibrium.

**Chemical Equilibrium ( $\mu$ ,  $a$ )  $\rightarrow$  multiphase and multicomponent**  
 $(\mu_i^\alpha = \mu_i^\beta = \mu_i^\gamma = \dots), (a_i^\alpha = a_i^\beta = a_i^\gamma = \dots)$

$$(\mu_A^\alpha = \mu_A^\beta = \mu_A^\gamma = \dots), (a_A^\alpha = a_A^\beta = a_A^\gamma = \dots)$$

$$(\mu_B^\alpha = \mu_B^\beta = \mu_B^\gamma = \dots), (a_B^\alpha = a_B^\beta = a_B^\gamma = \dots)$$

$\vdots$

- **Binary System** mixture/ solution / compound
- **Gibbs Free Energy in Binary System**

$$G_1 = X_A G_A + X_B G_B \quad J/mol$$

$$G_2 = G_1 + \Delta G_{mix} \quad J/mol$$

**Ideal solution ( $\Delta H_{mix}=0$ )**  $\Delta G^{mix} = RT(X_A \ln X_A + X_B \ln X_B)$

$$G = X_A G_A + X_B G_B + RT(X_A \ln X_A + X_B \ln X_B)$$

**Regular solution**  $\Delta H_{mix} = P_{AB} \varepsilon$  where  $\varepsilon = \varepsilon_{AB} - \frac{1}{2}(\varepsilon_{AA} + \varepsilon_{BB})$

$$G = X_A G_A + X_B G_B + \Omega X_A X_B + RT(X_A \ln X_A + X_B \ln X_B)$$

## - Chemical potential and Activity

$$\mu_A = \left( \frac{\partial G'}{\partial n_A} \right)_{T, P, n_B}$$

$$\bullet \quad \mu_A = G_A + RT \ln a_A \quad \ln \left( \frac{a_A}{X_A} \right) = \frac{\Omega}{RT} (1 - X_A)^2$$

$$\frac{a_A}{X_A} = \gamma_A = \text{activity coefficient}$$

$\mu$ 는 조성에 의해 결정되기 때문에  $dn_A$ 가 매우 작아서 조성변화 없어야

## Contents for today's class II

- Ideal Solution/ Regular Solution/ **Real solution**

## Q7: What is “Real Solution”?

**sufficient disorder + lowest internal E**

### 1.3 Binary Solutions

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$

**Real solution: sufficient disorder + lowest internal E**

**Ideal or Regular solution: over simplification of reality**

Config. Entropy

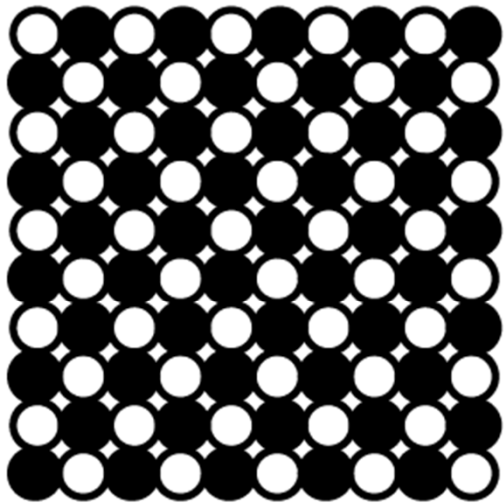
$$S = k \ln w$$

+ mixing enthalpy

$$\Delta H_{mix} = \Omega X_A X_B \text{ where } \Omega = N_a z \epsilon$$

$$S_{thermal} = 0$$

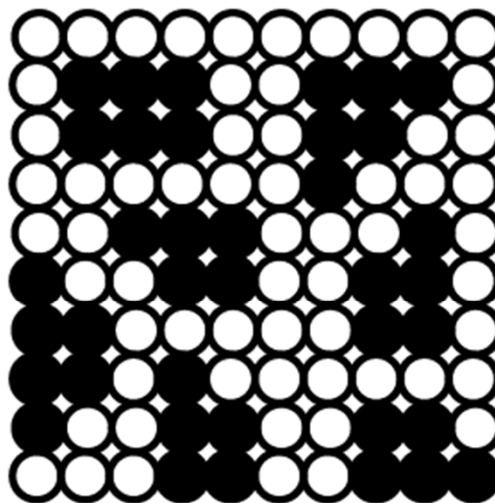
$$\epsilon \approx 0$$



(a)  $\epsilon < 0, \Delta H_{mix} < 0$

Ordered alloys

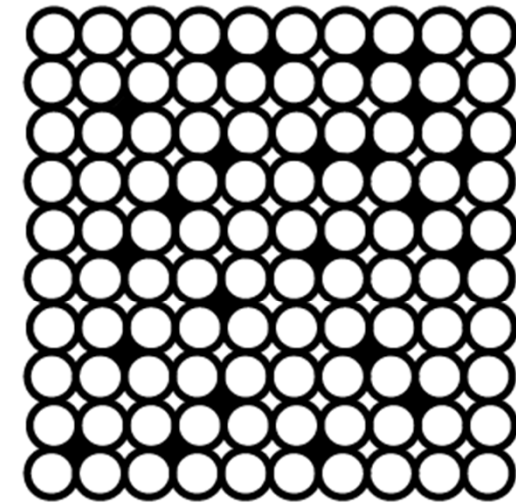
$P_{AB} \uparrow \longrightarrow$  Internal E  $\downarrow$



(b)  $\epsilon > 0, \Delta H_{mix} > 0$

Clustering

$P_{AA}, P_{BB} \uparrow$



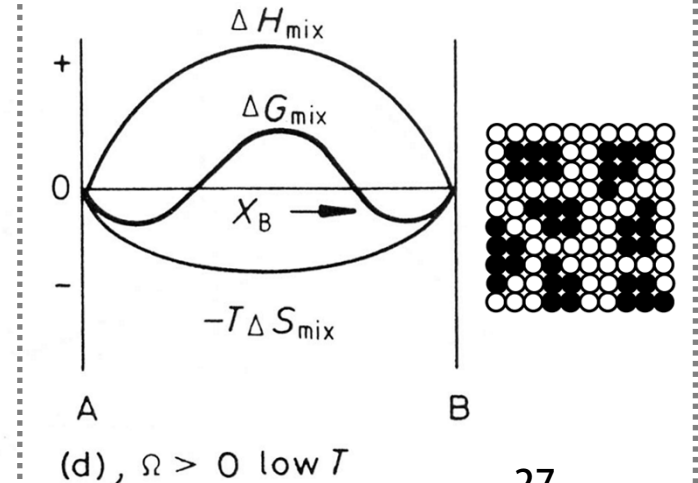
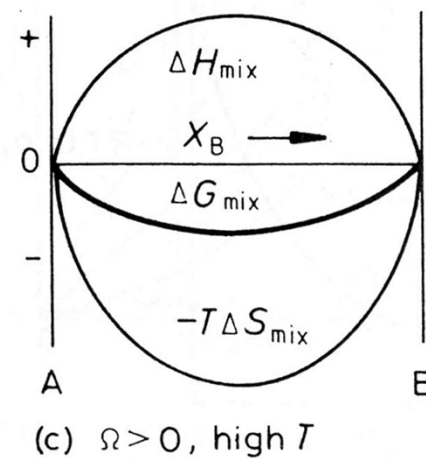
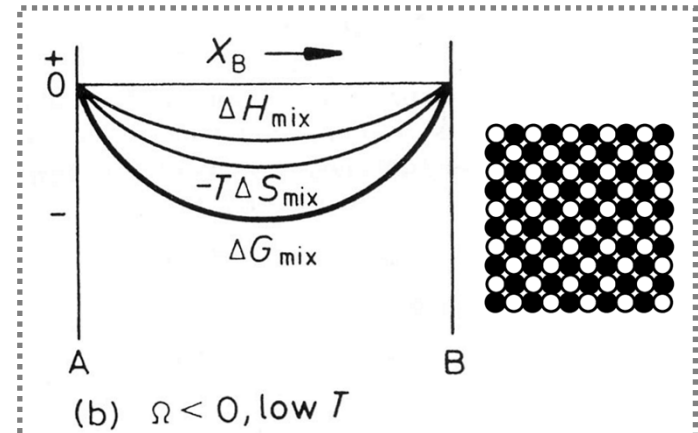
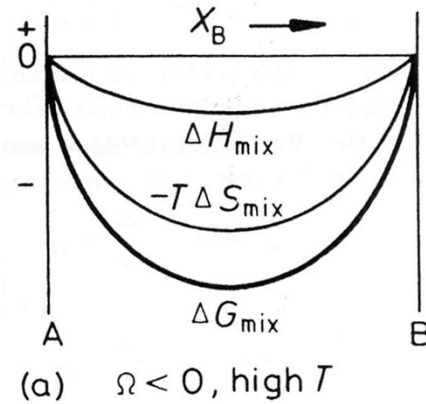
(c) when the size difference is large  
strain effect

Interstitial solution

\* The degree of **ordering or clustering** will decrease as temp. **increases** due to the increasing importance of **entropy**.

High temp.  $\longrightarrow$  Entropy effect  $\uparrow$   $\longrightarrow$  Solution stability  $\uparrow$

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$



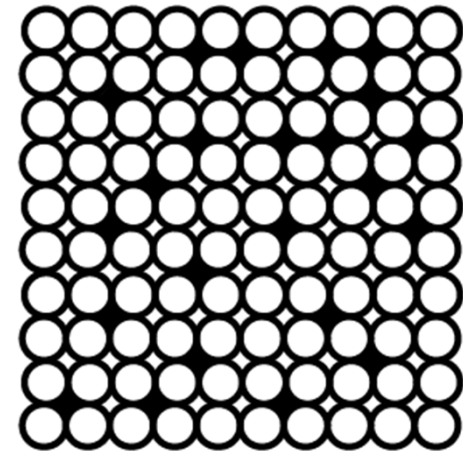
## 1.3 Binary Solutions

**Real solution: sufficient disorder + lowest internal E**

2) In systems where there is a size difference between the atom  
e.g. interstitial solid solutions,

→  $\Delta E = \Delta H_{\text{mix}} + \text{elastic strain}$

→ quasi-chemical model ~ underestimate  $\Delta E$   
due to no consideration of elastic strain field



→ **New mathematical models are needed to describe these solutions.**



**Q8: Ordered phase I:  
“Short range order (SRO)” in solution**

### 1.3 Binary Solutions

**Ordered phase**  $\varepsilon < 0, \Delta H_{\text{mix}} < 0$   $P_{AB} \uparrow$

**SRO** (Short Range Ordering) or **LRO** (Long Range Ordering)

- $\Omega < 0 \Rightarrow$  contain short-range order (SRO)

$\Delta\Omega = N_a z \varepsilon$  **SRO parameter = s** \_ degree of ordering

$$s = \frac{P_{AB} - P_{AB}(\text{random})}{P_{AB}(\text{max}) - P_{AB}(\text{random})}$$

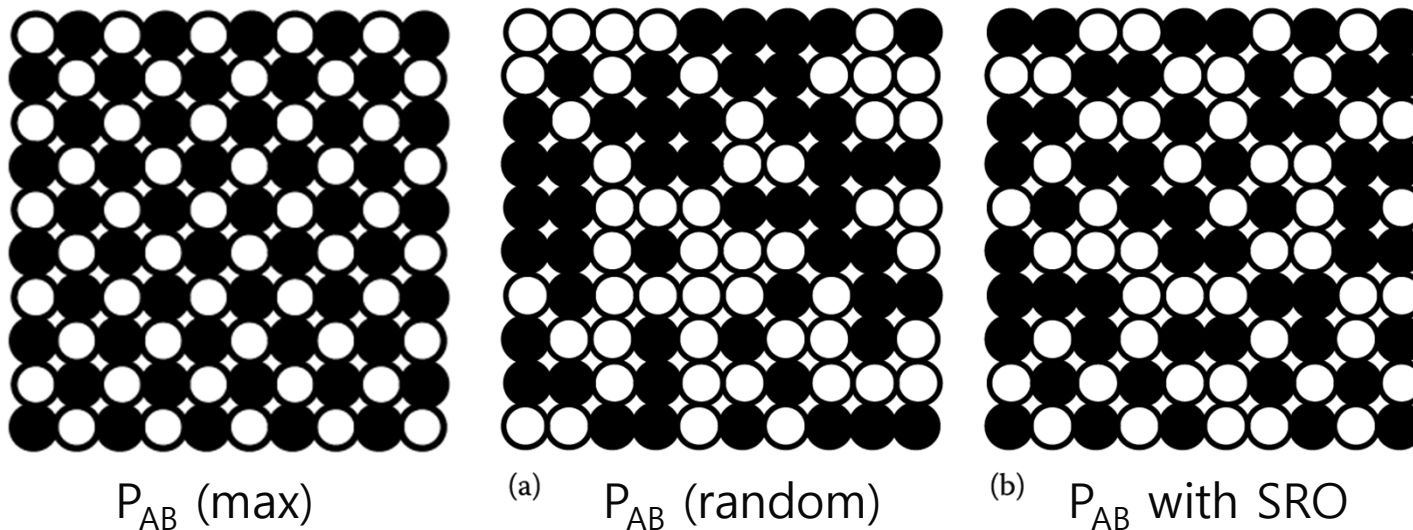
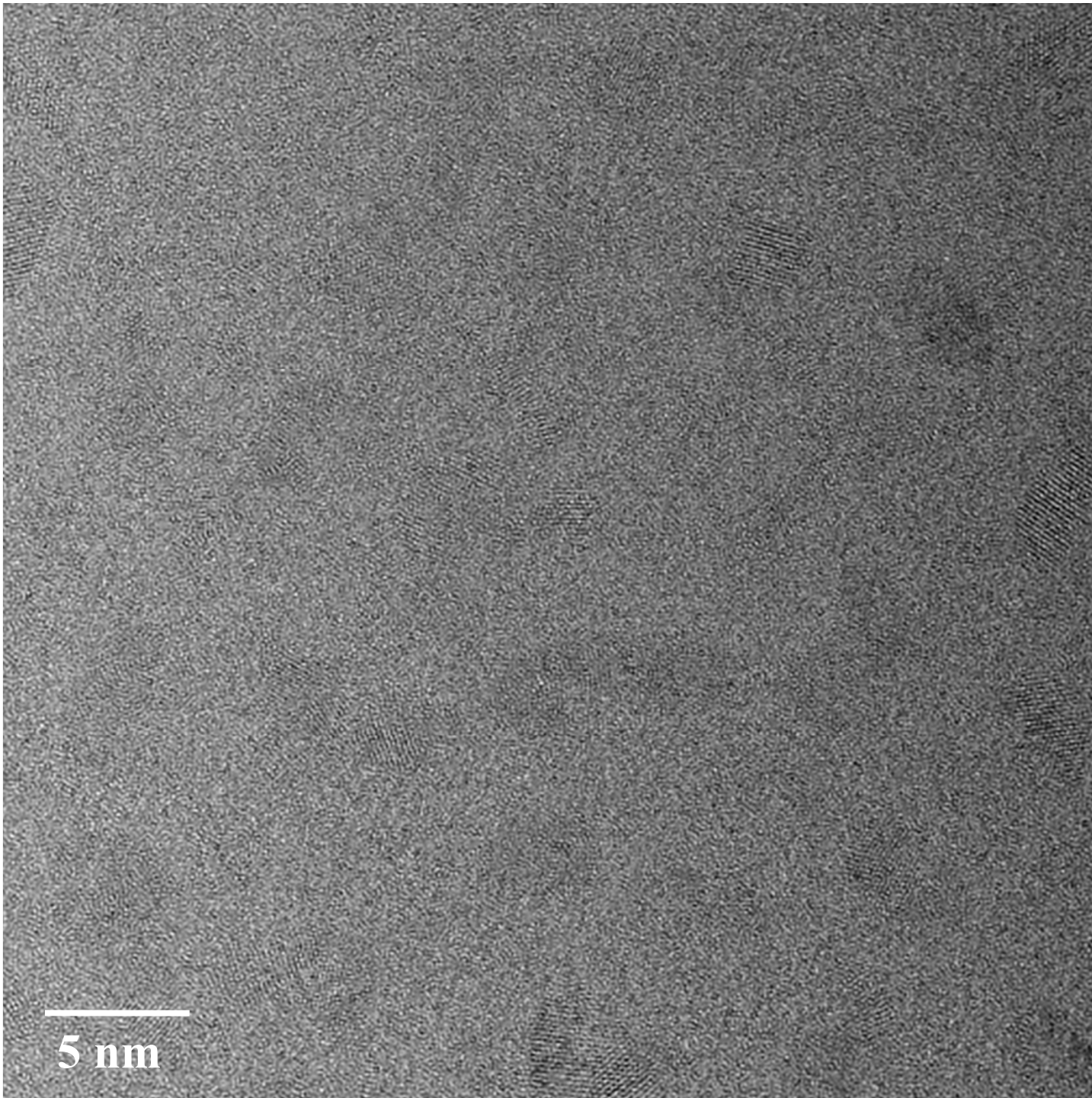
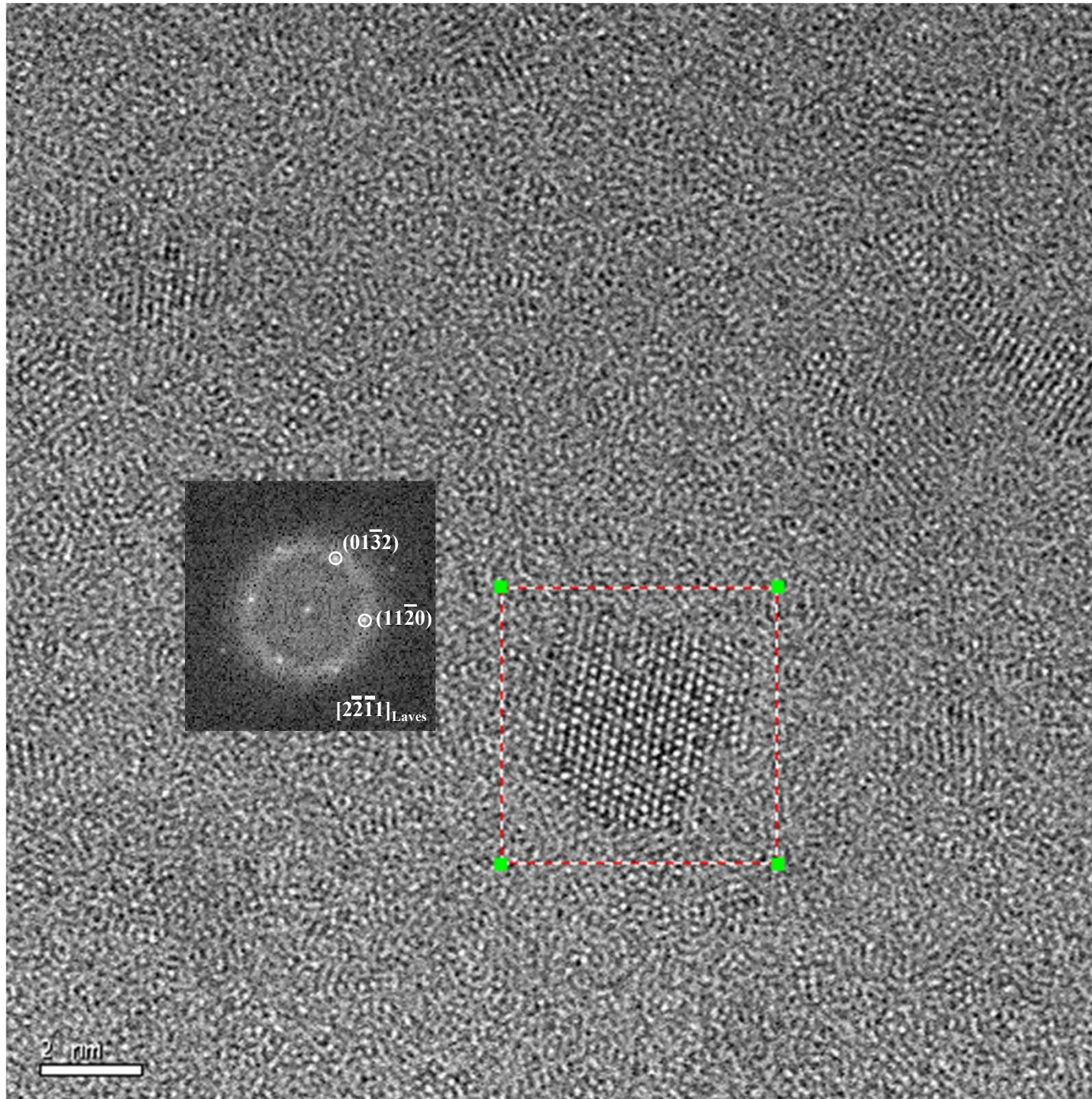


Fig. 1.19 (a) Random A-B solution with a total of 100 atoms and  $X_A = X_B = 0.5$ ,  $P_{AB} \sim 100$ ,  $S = 0$ .

(b) Same alloy with short-range order  $P_{AB} = 132$ ,  $P_{AB}(\text{max}) \sim 200$ ,  $S = (132 - 100) / (200 - 100) = 0.32$ . 30







**Q9: Ordered phase II:  
“Long range order (LRO)”**

**(①superlattice, ②intermediate phase, ③intermetallic compound)**

**\* Solid solution → ordered phase**

→ random mixing

→ entropy ↑

negative enthalpy ↓

$$\Delta H_{mix}^S < 0$$

Large composition range

→  $G \downarrow$

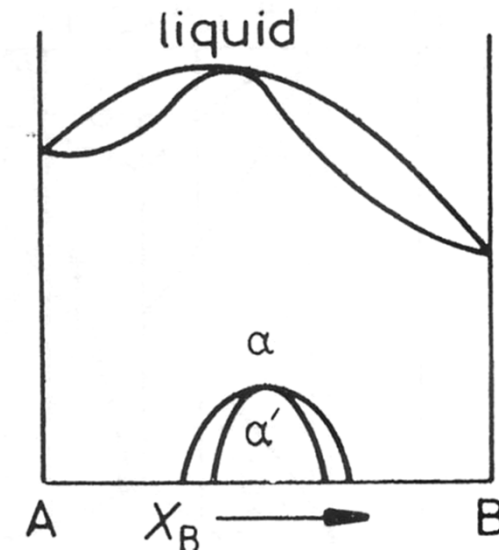
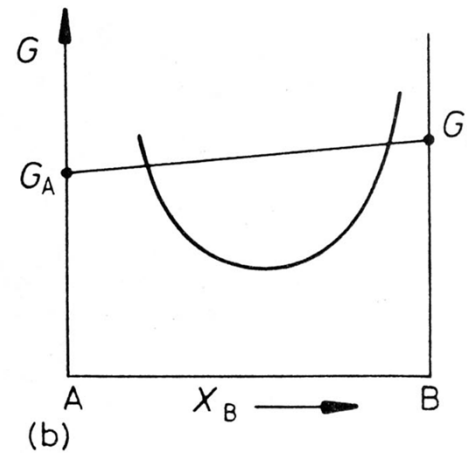


diagram: (a) for an intermetallic compound with a wide composition range, (b) for an intermediate phase with a wide composition range.

**\* Compound : AB, A<sub>2</sub>B...**

→ entropy ↓

→ covalent, ionic contribution.

→ enthalpy more negative ↓

$$\Delta H_{mix}^S \ll 0$$

Small composition range

→  $G \downarrow$

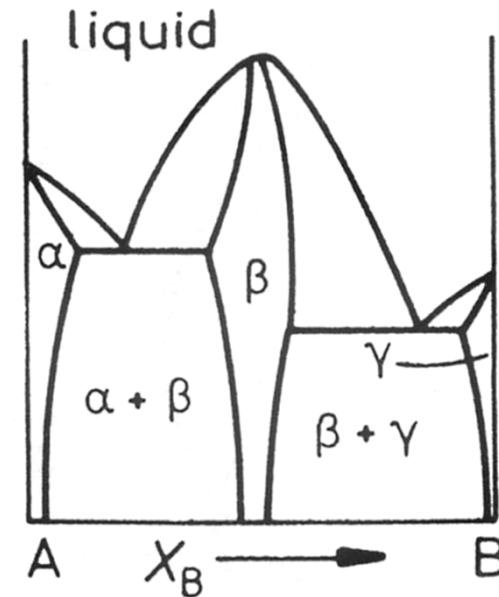
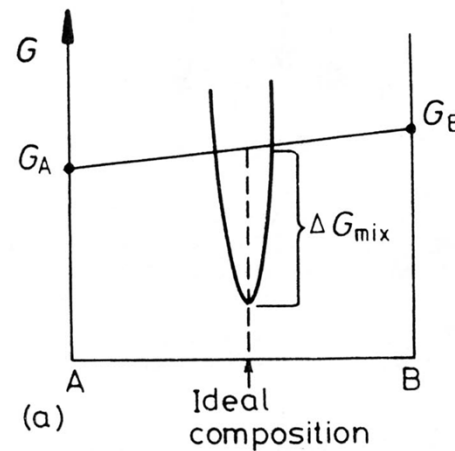
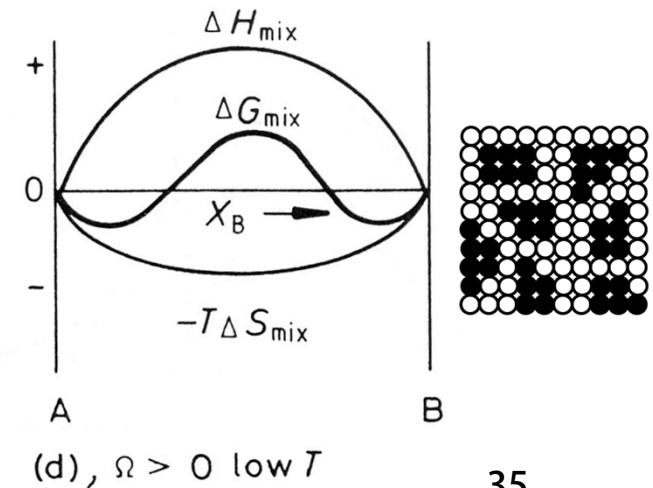
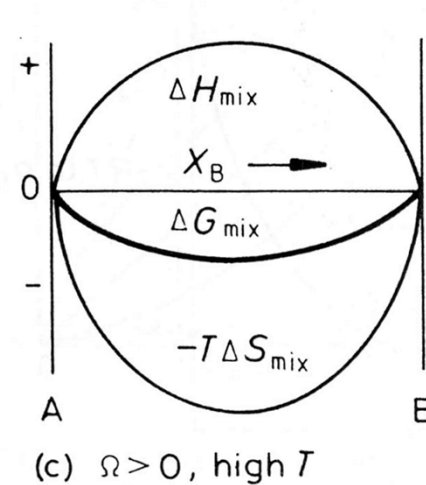
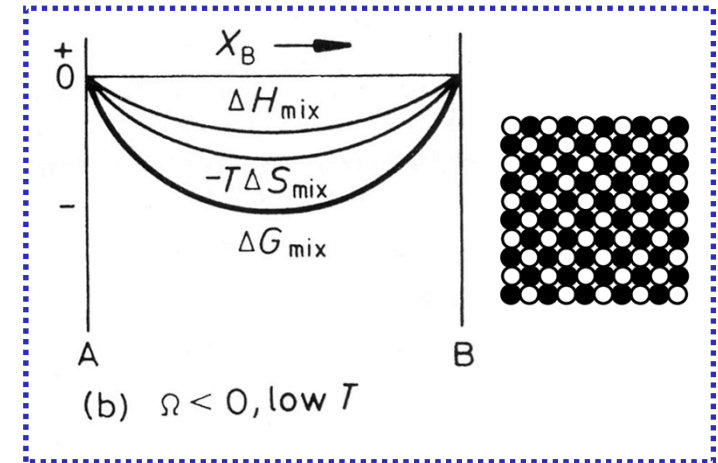
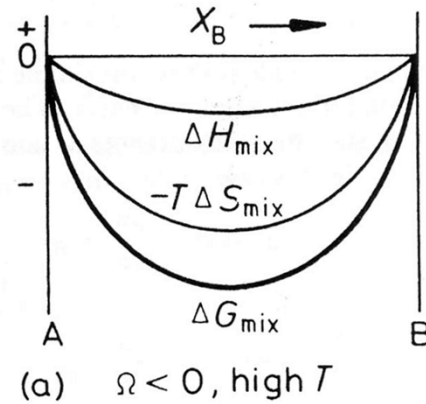


Fig. 1.23 Free energy curves for intermediate phase with a very narrow stability range, (a) for an intermediate phase with a very narrow stability range, (b) for an intermediate phase with a wide stability range.

\* The degree of **ordering or clustering** will decrease as temp. **increases** due to the increasing importance of **entropy**.

High temp.  $\longrightarrow$  Entropy effect  $\uparrow$   $\longrightarrow$  Solution stability  $\uparrow$

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$



## Q9-a: Superlattice

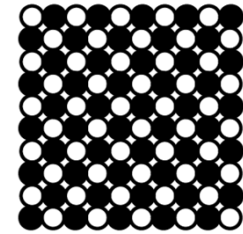


### 1.3 Binary Solutions

**Ordered phase**  $\epsilon < 0, \Delta H_{\text{mix}} < 0$

\* In solutions with compositions that are close to a simple **ratio of A:B atoms** another type of order can be found.

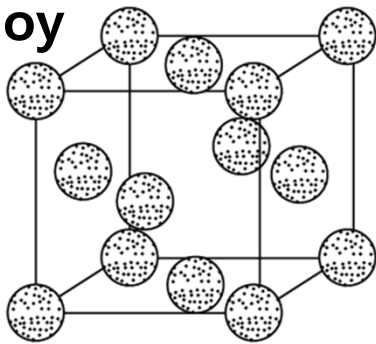
\* This is known as **long-range order (LRO)** CuAu, Cu<sub>3</sub>Au and many other intermetallics show LRO.



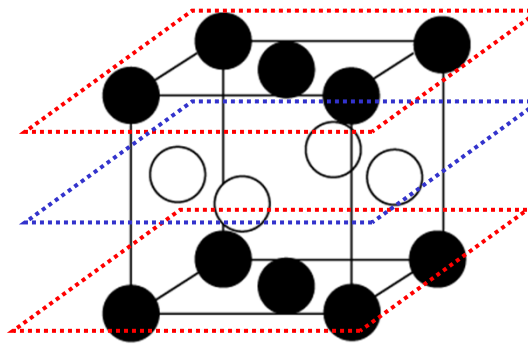
(The atom sites are no longer equivalent but can be labelled as A-sites and B-sites.)

\* **A superlattice** forms in materials with LRO

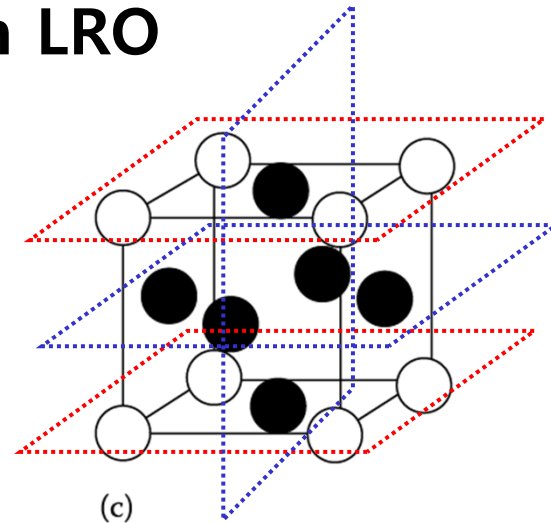
Cu–Au alloy



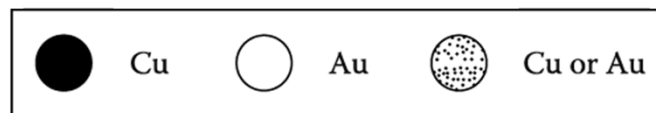
(a)



(b)



(c)



High temp.

Disordered Structure

Low temp.

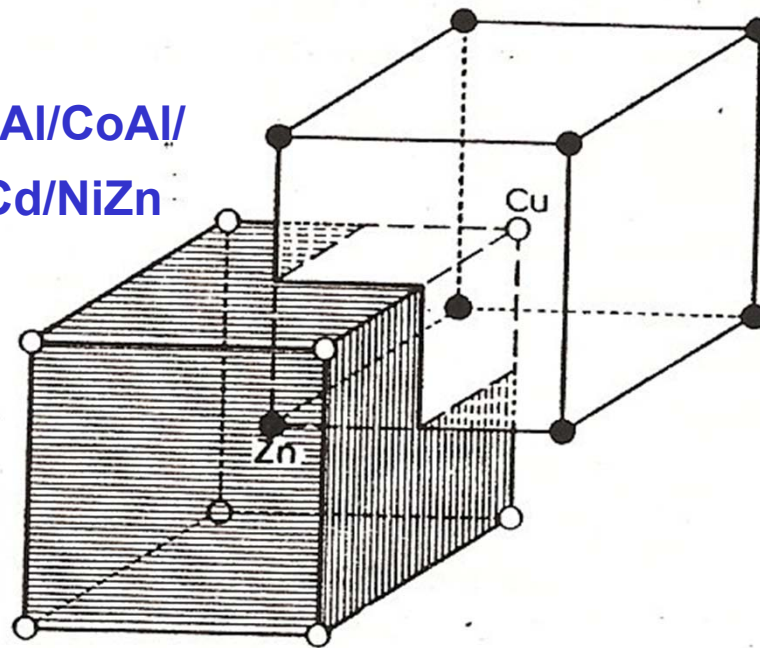
CuAu superlattice

Cu<sub>3</sub>Au superlattice

# Superlattice formation: order-disorder transformation

- $\epsilon < 0$ ,  $\Delta H_{\text{mix}} < 0$
- **between dissimilar atoms** than between similar atoms
- **Large electrochemical factor**: tendency for the solute atoms to avoid each other and to associate with the solvent atoms
- **Size factor just within the favorable limit**: lead to atomic rearrangement so as to relieve the lattice distortion imposed by the solute atoms

(a)  $L2_0$ :  
CuZn/FeCo/NiAl/CoAl/  
FeAl/AgMg/AuCd/NiZn



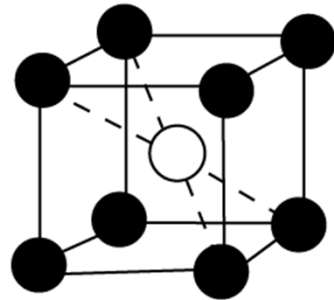
$\beta$  brass superlattice viewed as two inter-penetrating cubic lattices

# 1.3 Binary Solutions

## Five common ordered lattices

(a)  $L2_0$ :

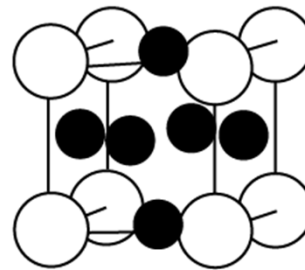
CuZn/FeCo/NiAl/CoAl/  
FeAl/AgMg/AuCd/NiZn



(a) ● Cu ○ Zn

(b)  $L1_2$ :

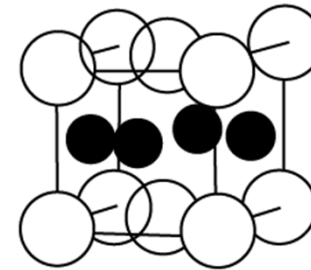
Cu<sub>3</sub>Au/Ni<sub>3</sub>Mn/Ni<sub>3</sub>Fe/Ni<sub>3</sub>Al/  
Pt<sub>3</sub>Fe/Au<sub>3</sub>Cd/Co<sub>3</sub>V/TiZn<sub>3</sub>



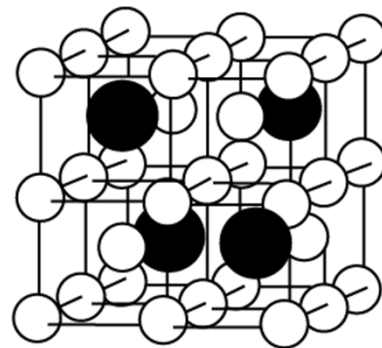
(b) ● Cu ○ Au

(c)  $L1_0$ :

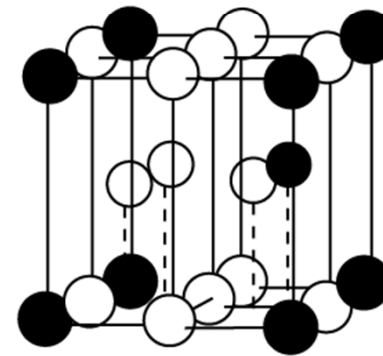
CuAu/CoPt/FePt



(c) ● Cu ○ Au



(d) ● Al ○ Fe



(e) ● Cd ○ Mg

(d)  $D0_3$ :

Fe<sub>3</sub>Al/Cu<sub>3</sub>Sb/Mg<sub>3</sub>Li/Fe<sub>3</sub>Al/  
Fe<sub>3</sub>Si/Fe<sub>3</sub>Be/Cu<sub>3</sub>Al

(e)  $D0_{19}$ :

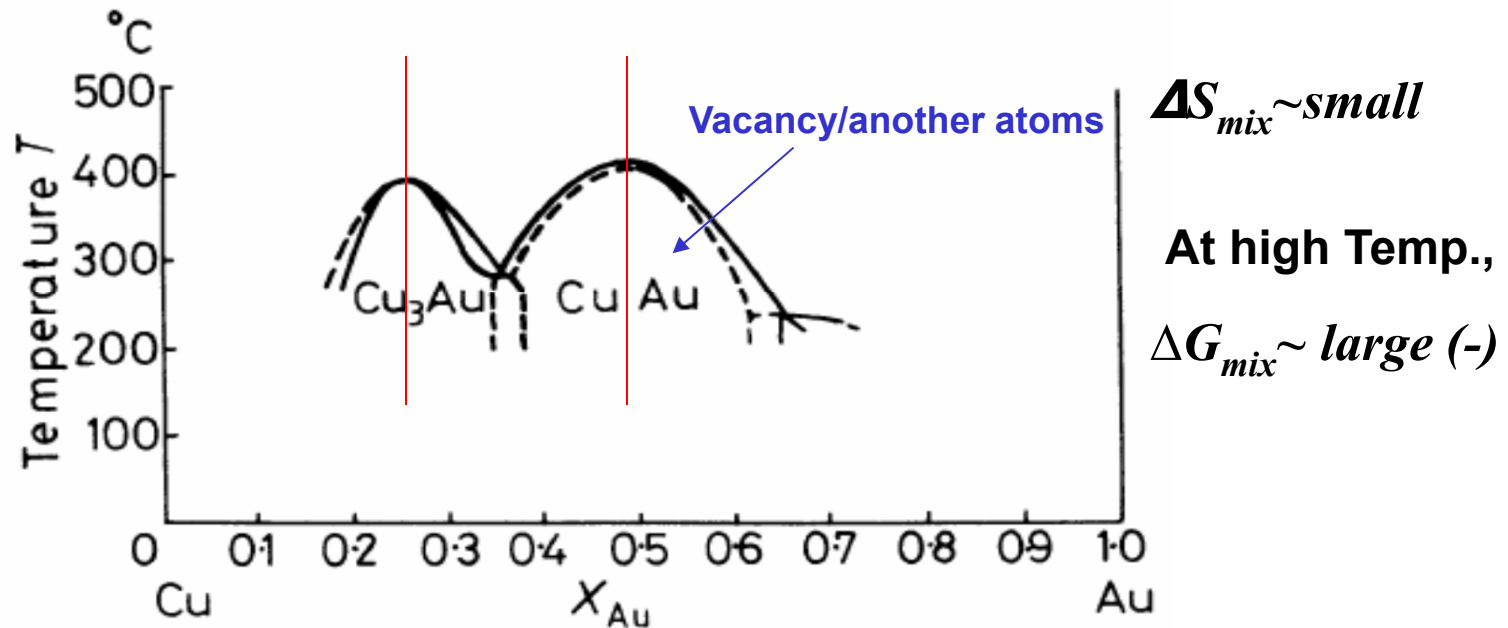
Mg<sub>3</sub>Cd/Cd<sub>3</sub>Mg/Ti<sub>3</sub>Al/Ni<sub>3</sub>Sn/Ag<sub>3</sub>In/  
Co<sub>3</sub>Mo/Co<sub>3</sub>W/Fe<sub>3</sub>Sn/Ni<sub>3</sub>In/Ti<sub>3</sub>Sn

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$

## Ordered phase

$$\varepsilon < 0, \Delta H_{mix} < 0$$

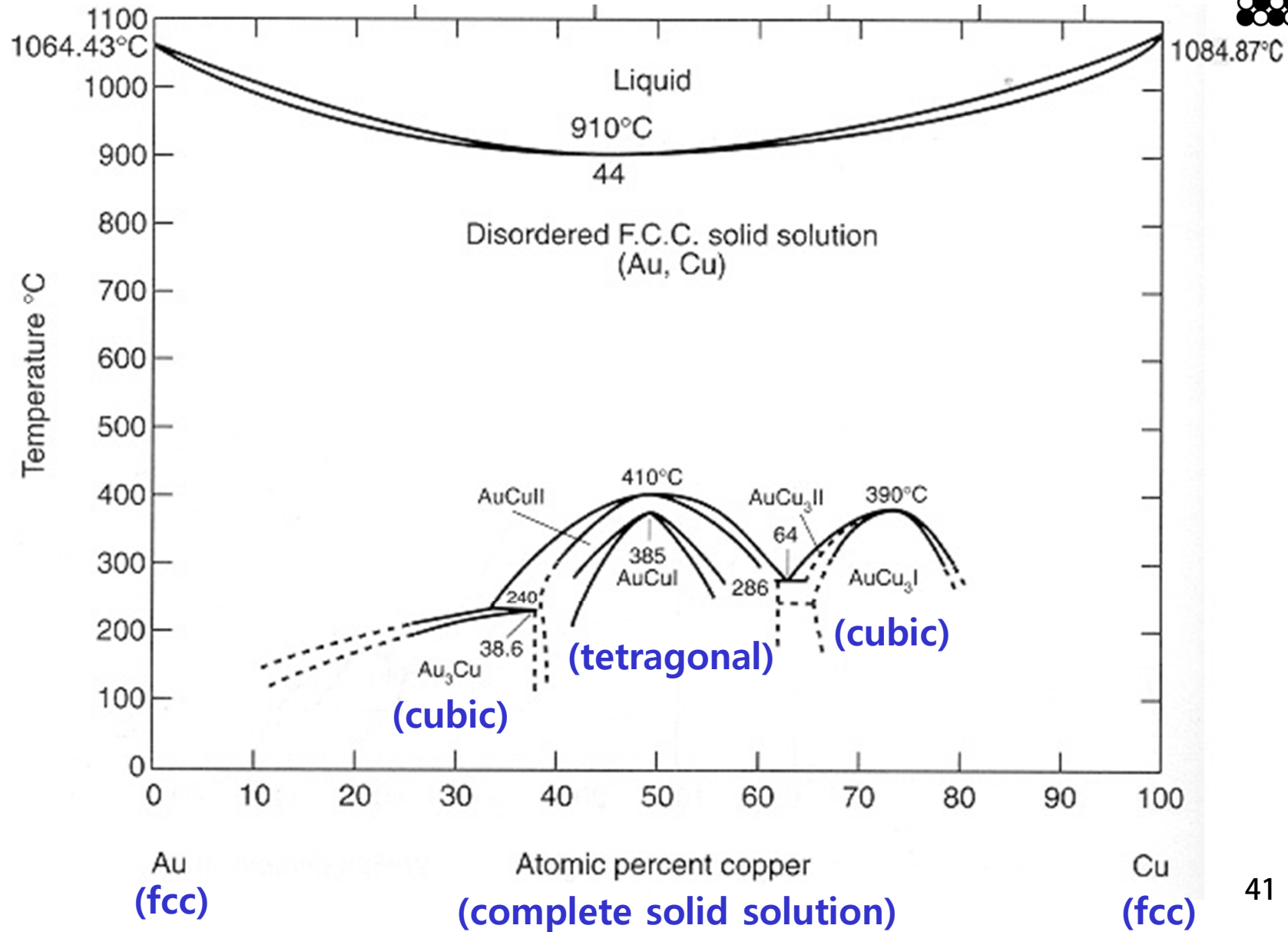
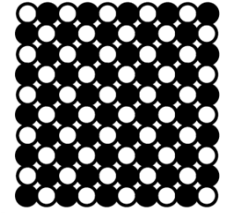
Fig. 1.21. Part of the Cu-Au phase diagram showing the regions where the Cu<sub>3</sub>Au and CuAu superlattices are stable.



- The **entropy** of mixing of structure with LRO is **extremely small** and **the degree of order decrease** with **increasing temperature** until above some **critical temperature** there is no LRO at all.
- This temperature is a maximum when the composition is the ideal required for the superlattice.
- The critical temperature for loss of LRO increases with increasing  $\Omega$  or  $\Delta H_{mix}$  and in many systems the ordered phase is stable up to the melting point.

# Ordered Phase

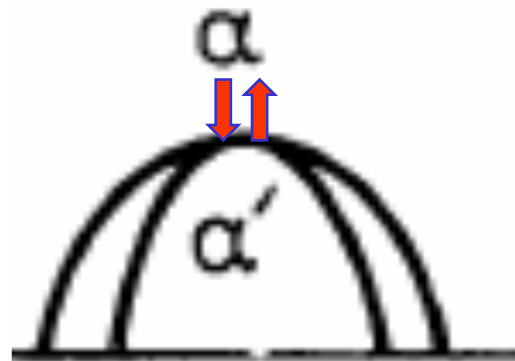
$$\epsilon < 0, \Delta H_{\text{mix}} < 0 / \Delta H_{\text{mix}} \sim -20 \text{ kJ/mol}$$



## **Q9-b: Order-disorder transition**

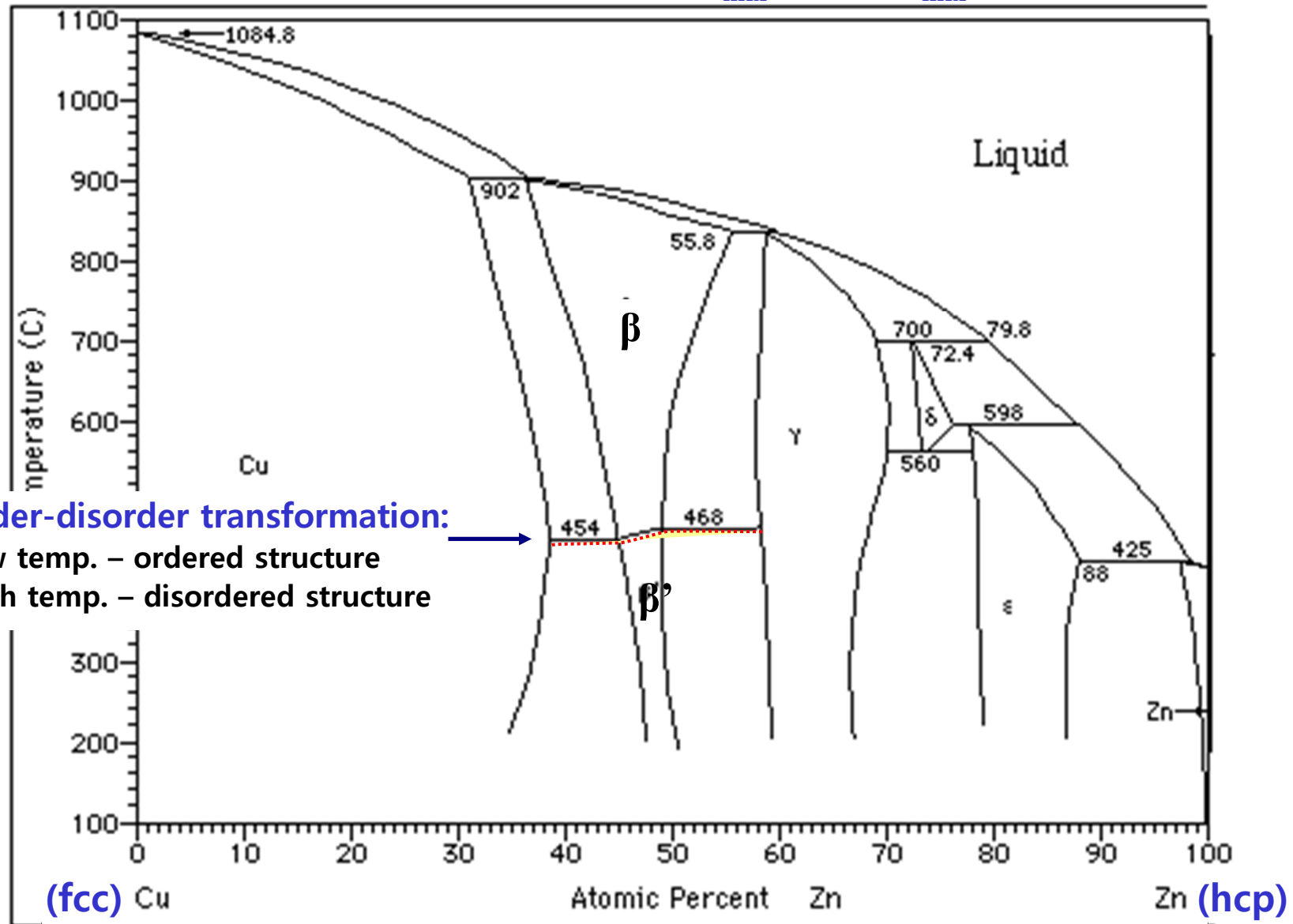
# Order-disorder phase transformation

- Not classical phase change = ~not depend on diffusion process
- **change of temperature allowed a continuous re-arrangement of atoms without changing the phase = "2<sup>nd</sup> order transition"**
- **boundary: ordered lattice & disordered lattice/phase rule could not applied**  
there are cases in which an ordered phase of one composition exists in equilibrium with a disordered phase of a different composition.
- Simple composition of the type AB or AB<sub>3</sub> can the transformation (i.e. at the temperature maximum) be considered diffusionless.



# Intermediate Phase

$$\epsilon < 0, \Delta H_{\text{mix}} < 0 / \Delta H_{\text{mix}} \sim -21 \text{ kJ/mol}$$



**Order-disorder transformation:**  
 Low temp. – ordered structure  
 High temp. – disordered structure

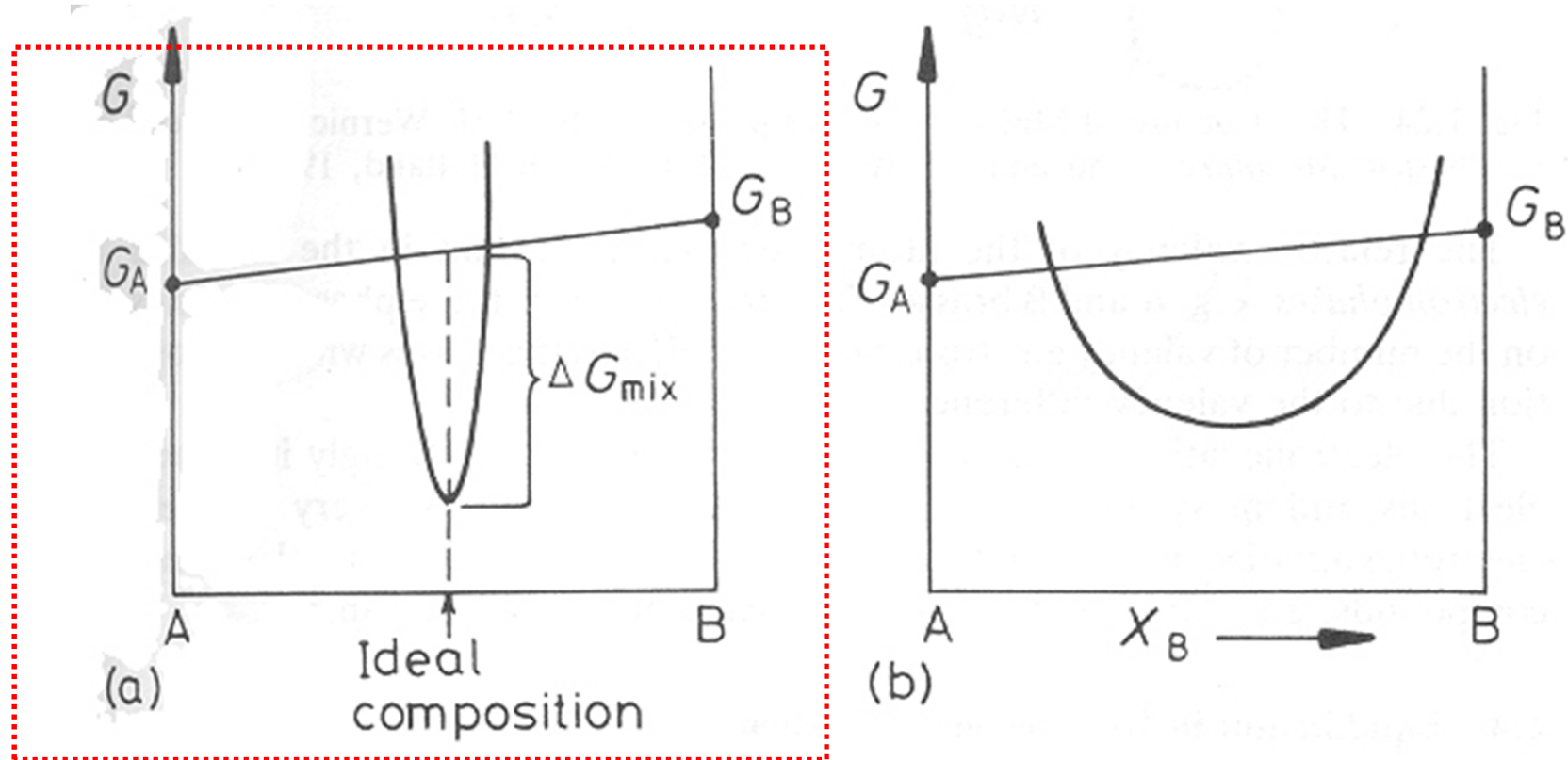
- $\alpha$  and  $\eta$  are terminal solid solutions
- $\beta$ ,  $\beta'$ ,  $\gamma$ ,  $\delta$  and  $\epsilon$  are intermediate solid solutions.



**Q9-c:**

**Intermediate phase vs Intermetallic compound**

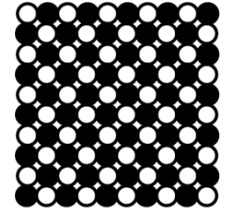
## Intermediate Phase



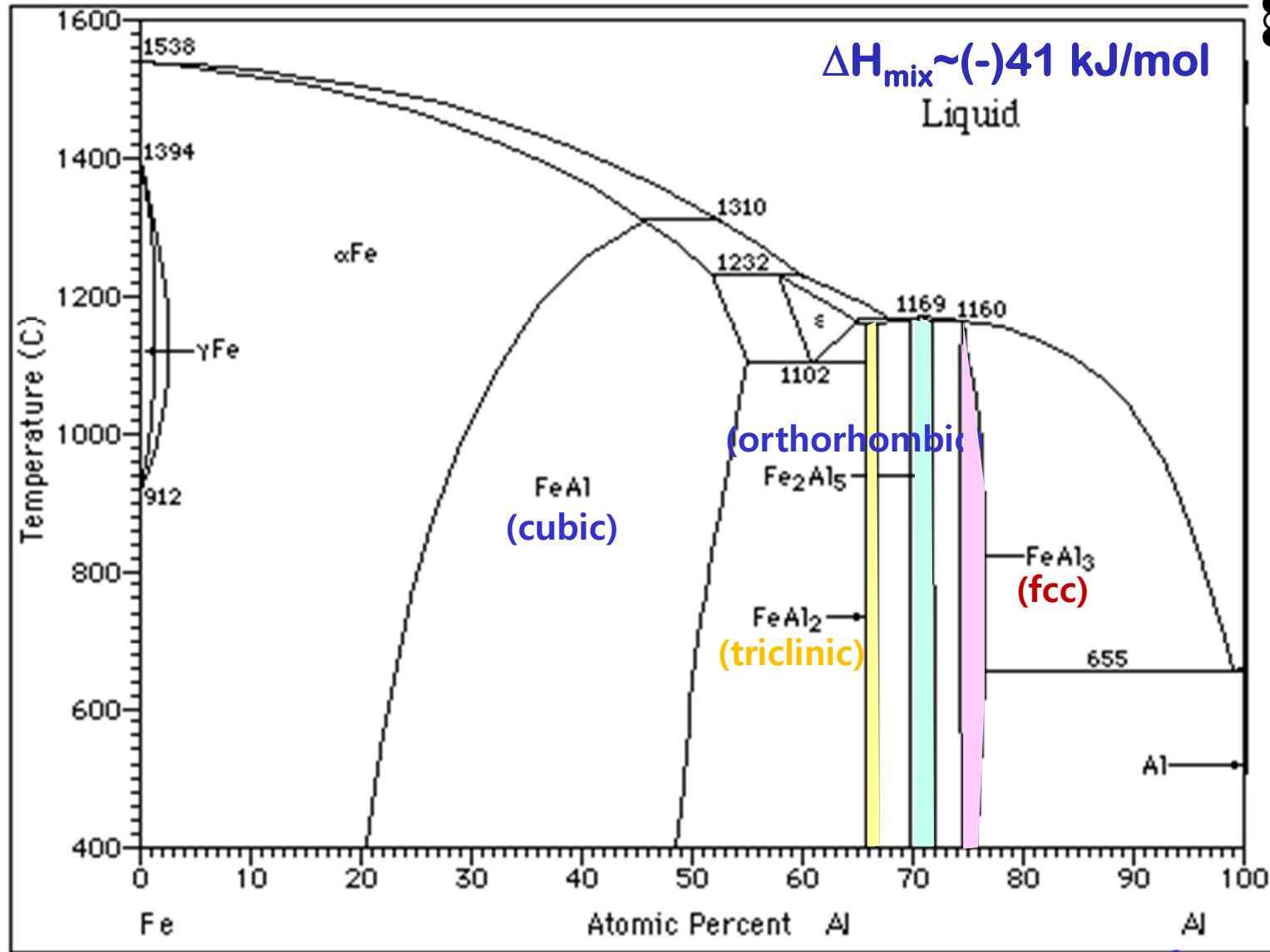
- \* **Many intermetallic compounds** have **stoichiometric composition**  $A_mB_n$  and a characteristic free energy curve as shown in Fig (a).
- \* In other structure, fluctuations in composition can be tolerated by **some atoms occupying 'wrong' position** or by **atom sites being left vacant**, and in these cases the **curvature** of the  $G$  curve is much less, Fig (b).

# Intermediate Phase

$$\epsilon < 0, \Delta H_{\text{mix}} < 0$$



$\Delta H_{\text{mix}} \sim (-)41 \text{ kJ/mol}$   
Liquid

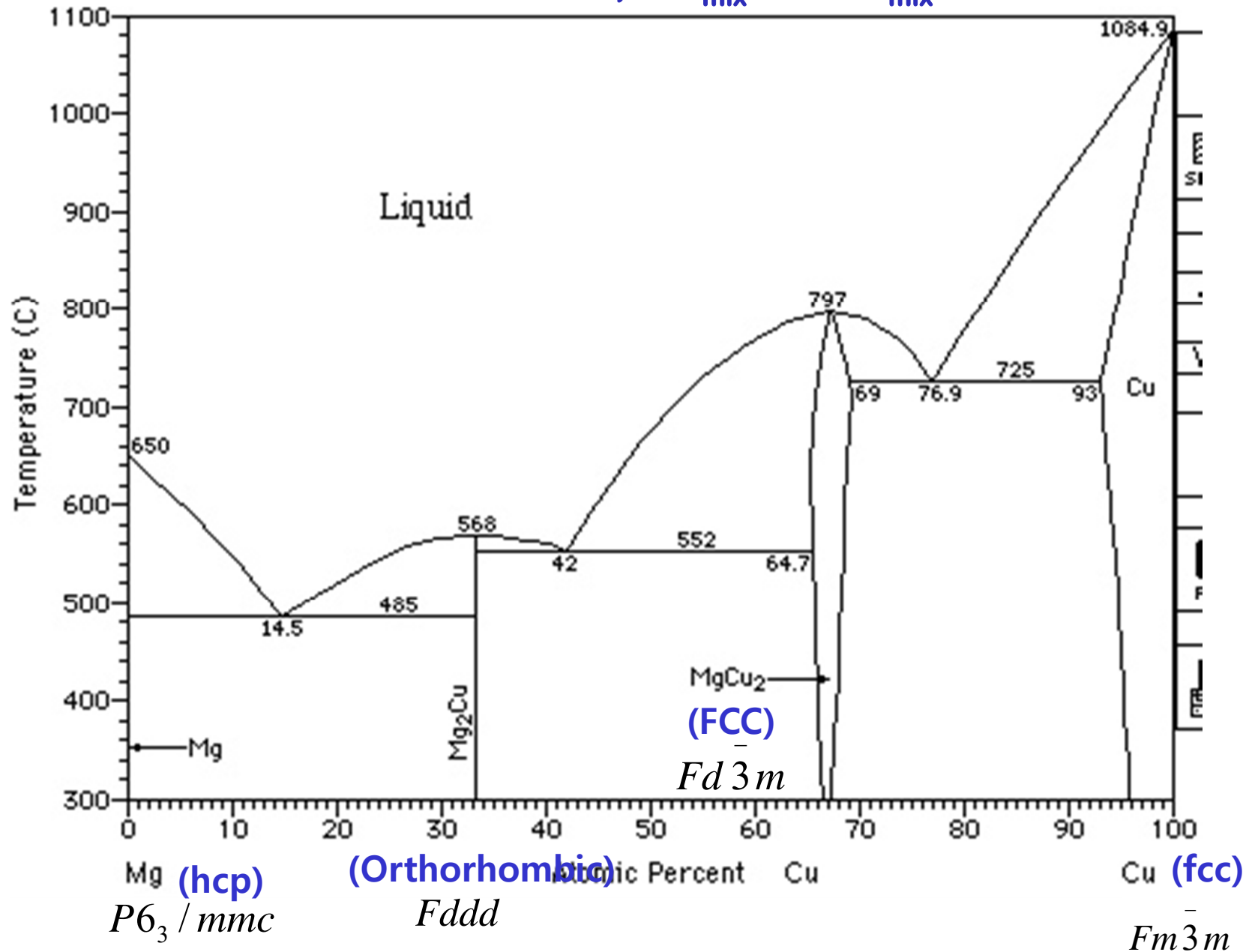


(bcc)

(fcc)

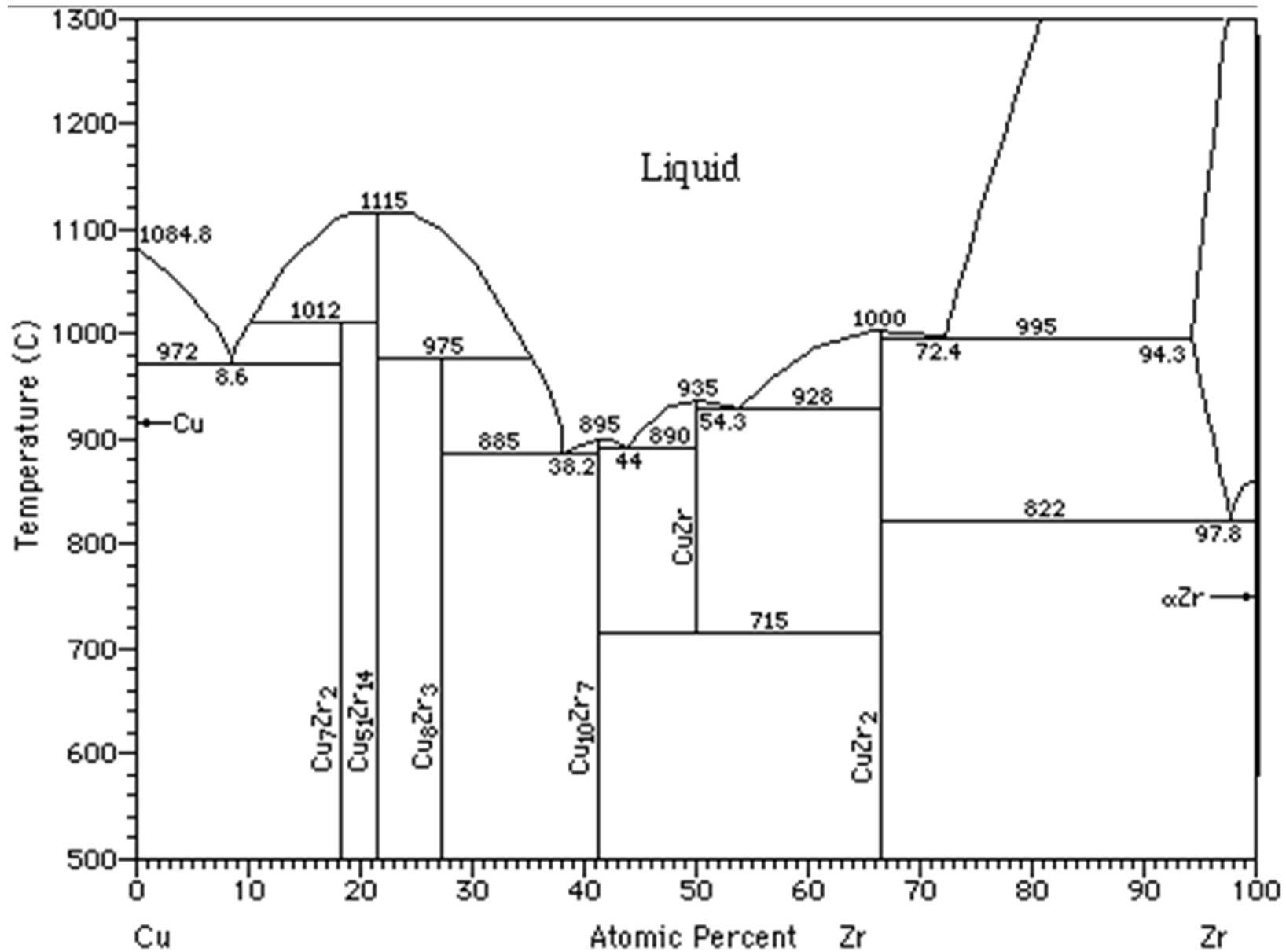
# Intermediate Phase

$$\epsilon < 0, \Delta H_{\text{mix}} < 0 / \Delta H_{\text{mix}} \sim -38 \text{ kJ/mol}$$



# Intermediate Phase

$$\epsilon \ll 0, \Delta H_{\text{mix}} \ll 0 / \Delta H_{\text{mix}} \sim -142 \text{ kJ/mol}$$



## 1.3 Binary Solutions

# Intermediate Phase

\_"different crystal structure as either of the pure component"

## 3 main factors

determining the structure of Intermediate phase ?

### 1) Relative atomic size

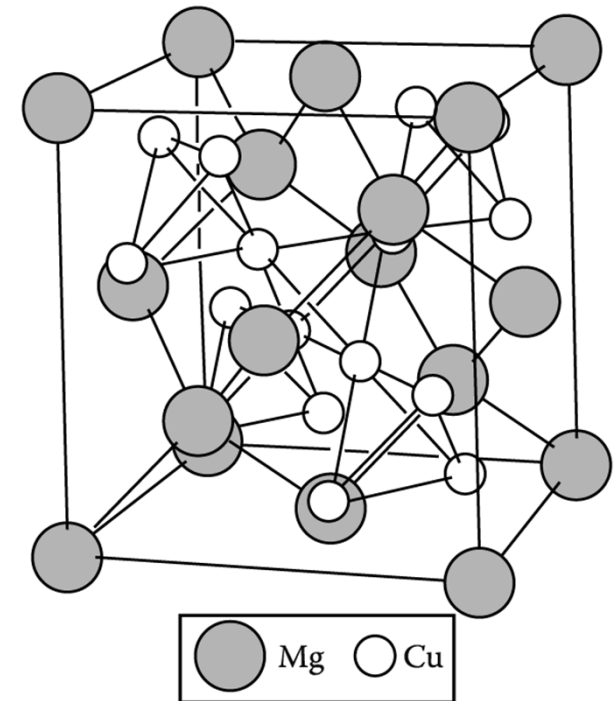
- **Laves phase** (size difference: 1.1~1.6 ex:  $\text{MgCu}_2$ )  
fill space most efficiently ~ stable
- **Interstitial compound**:  $\text{MX}$ ,  $\text{M}_2\text{X}$ ,  $\text{MX}_2$ ,  $\text{M}_6\text{X}$   
M= Cubic or HCP ex: Zr, Ti, V, Cr, etc, X= H, B, C, and N

### 2) Relative valency electron

- **electron phases** ex\_  $\alpha$  &  $\beta$  brass  
# of valency electrons per unit cell  
→ depending on compositional change

### 3) Electronegativity

- very different electronegativities → **ionic bond\_ normal valency compounds**  
ex  $\text{Mg}_2\text{Sn}$



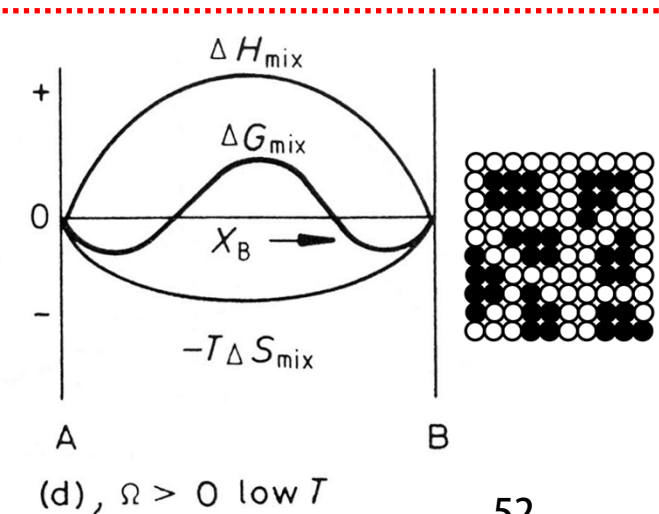
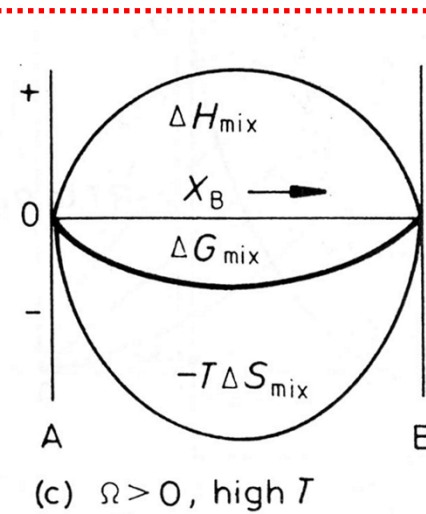
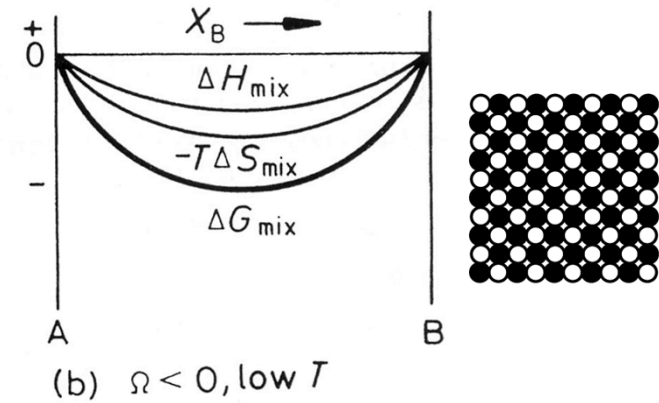
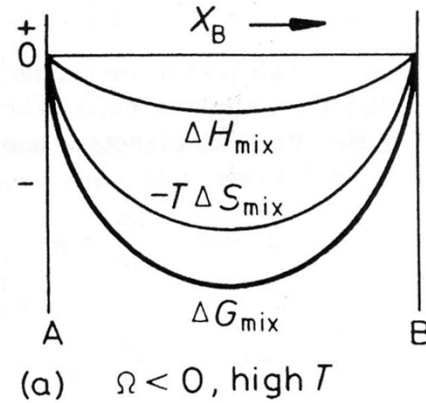
**$\text{MgCu}_2$  (A Laves phase)**

**Q10: “Clustering”? → Phase separation**  
**Metastable vs Stable miscibility gap**

\* The degree of **ordering or clustering** will decrease as temp. **increases** due to the increasing importance of **entropy**.

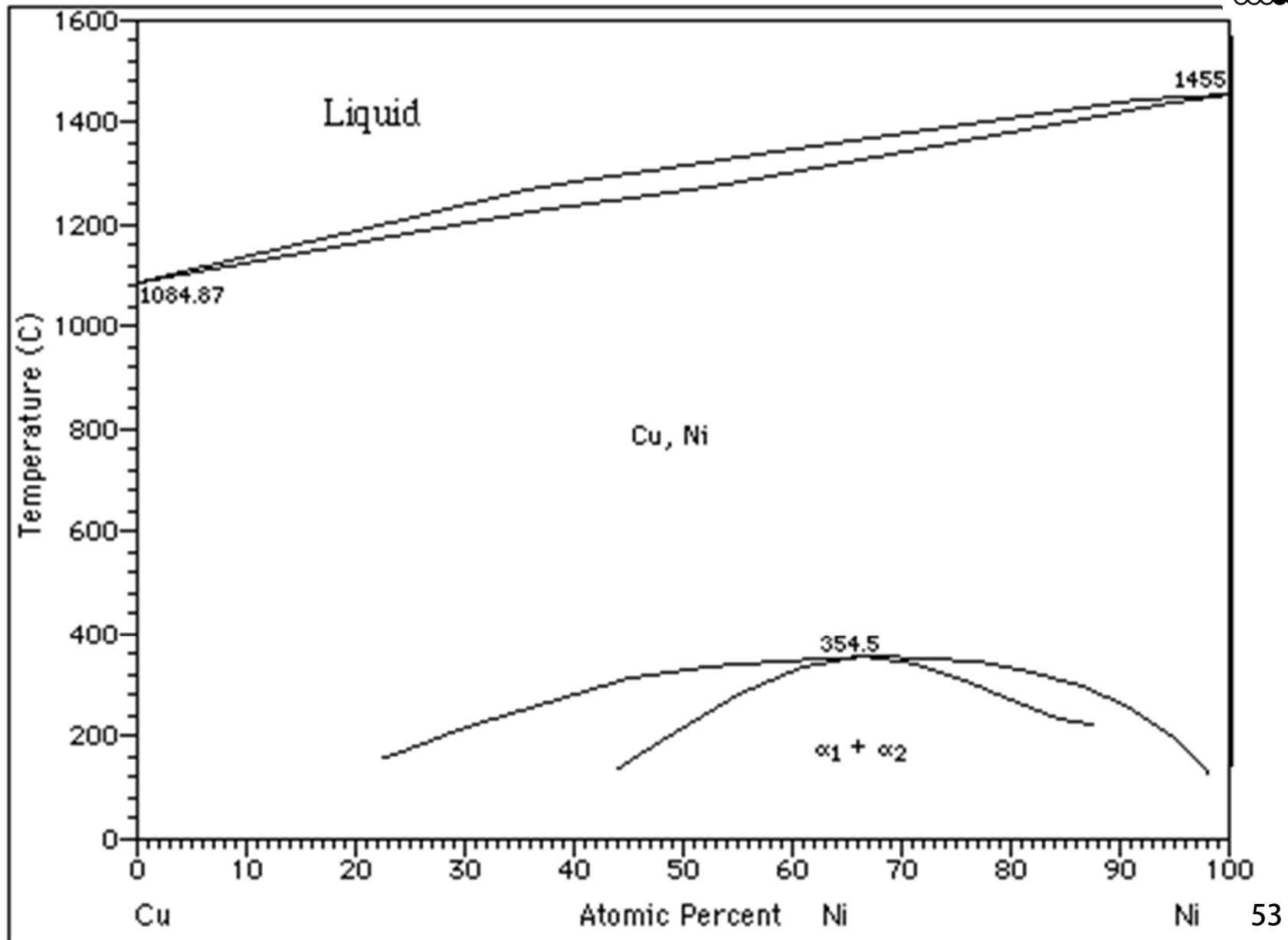
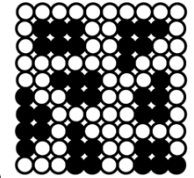
High temp.  $\longrightarrow$  Entropy effect  $\uparrow$   $\longrightarrow$  Solution stability  $\uparrow$

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$



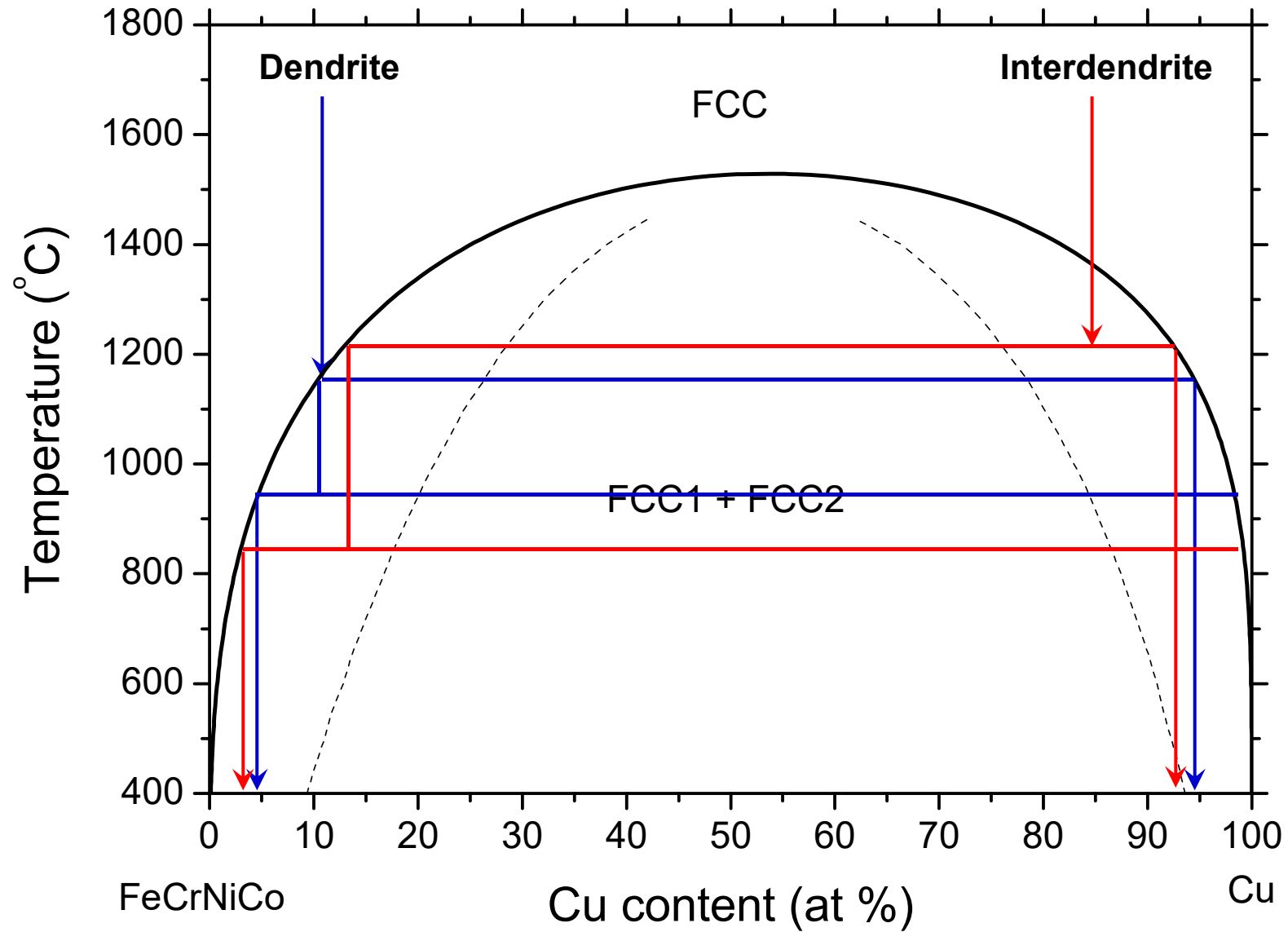


$\epsilon > 0, \Delta H_{\text{mix}} > 0 / \Delta H_{\text{mix}} \sim +26 \text{ kJ/mol}$

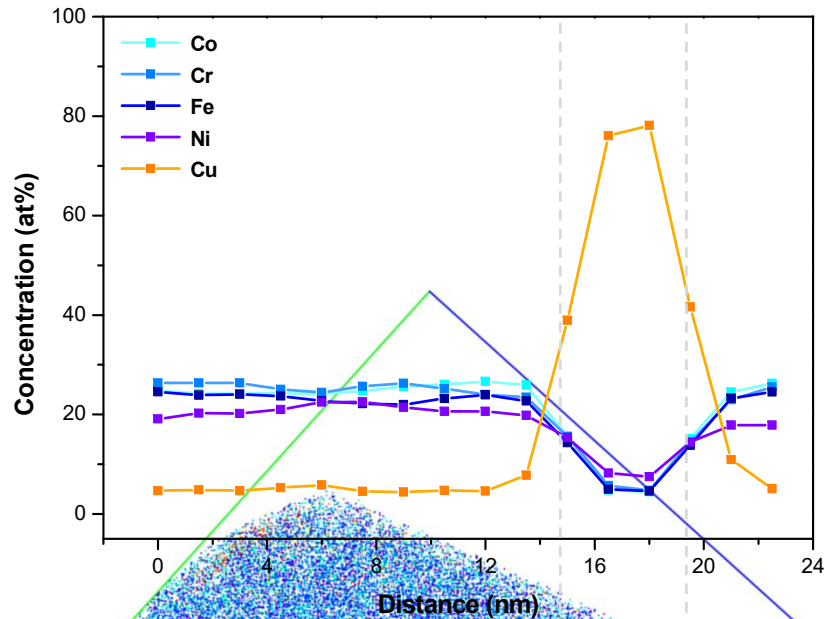


# Cooling process in the miscibility gap

## Dendrite / interdendrite formation



# Compositional analysis of as-cast CoCrFeNi/Cu HEA (dendrite)



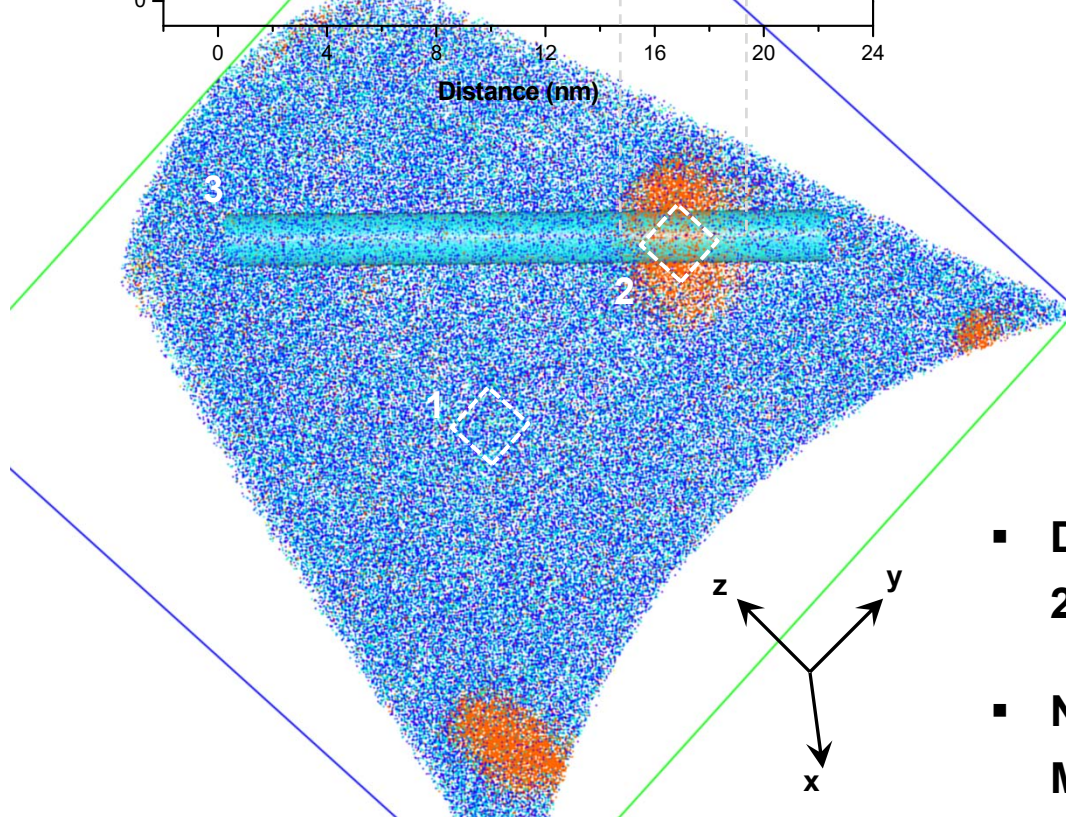
ROI 1, 2 : 1.4 nm x 2 nm x 2 nm

ROI 3 : 1.2 nm x 2 nm x 23 nm

(1D concentration profile)

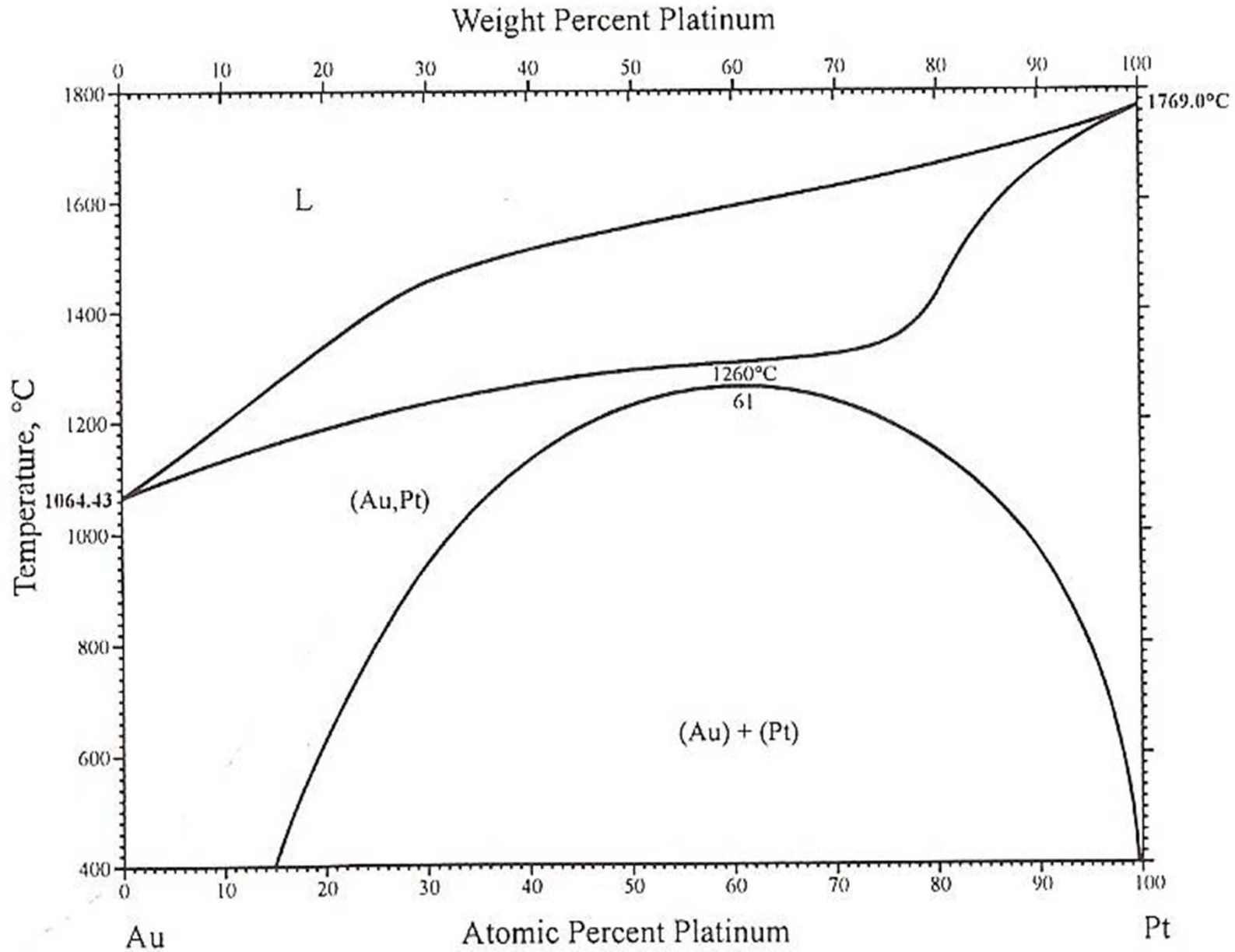
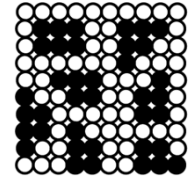
1	at%	2	at%
Co	26.19	Co	0.33
Cr	24.15	Cr	0.46
Fe	24.59	Fe	0.39
Ni	19.59	Ni	5.00
Cu	4.74	Cu	93.56

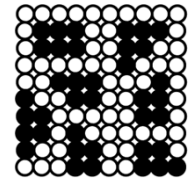
3-1'	at%	3-2'	at%
Co	25.29	Co	2.01
Cr	25.63	Cr	3.35
Fe	23.63	Fe	2.56
Ni	20.66	Ni	6.90
Cu	4.42	Cu	84.92



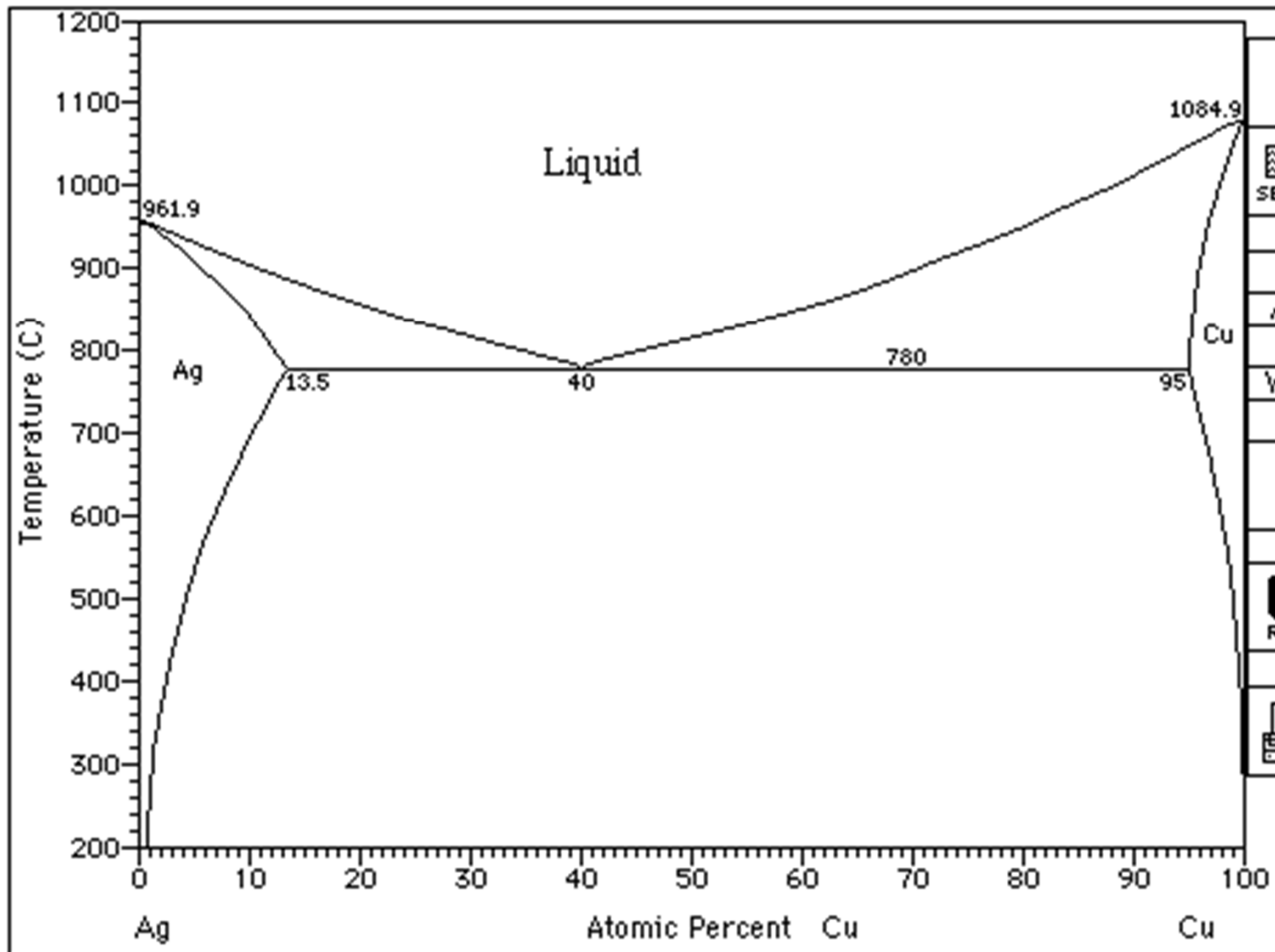
- Dendrite region: matrix (4.74 at%Cu) + 2nd phase (93.56 at%Cu)
- No segregation at the interface between Matrix and 2nd phase

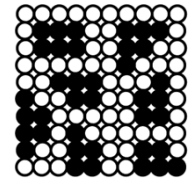
$\epsilon > 0, \Delta H_{\text{mix}} > 0 / \Delta H_{\text{mix}} \sim +17 \text{ kJ/mol}$



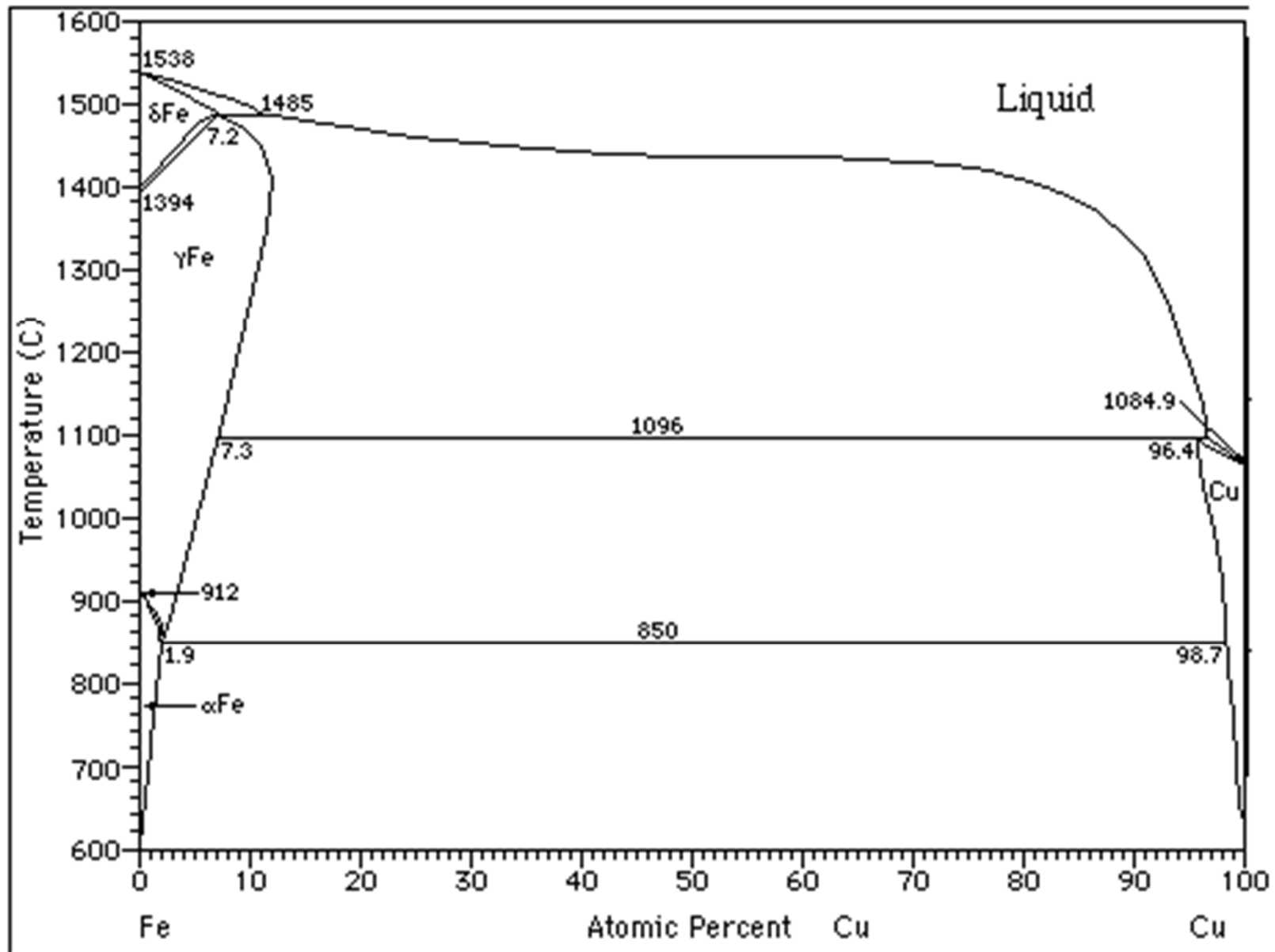


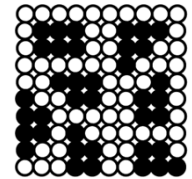
$\epsilon > 0, \Delta H_{\text{mix}} > 0 / \Delta H_{\text{mix}} \sim +5 \text{ kJ/mol}$



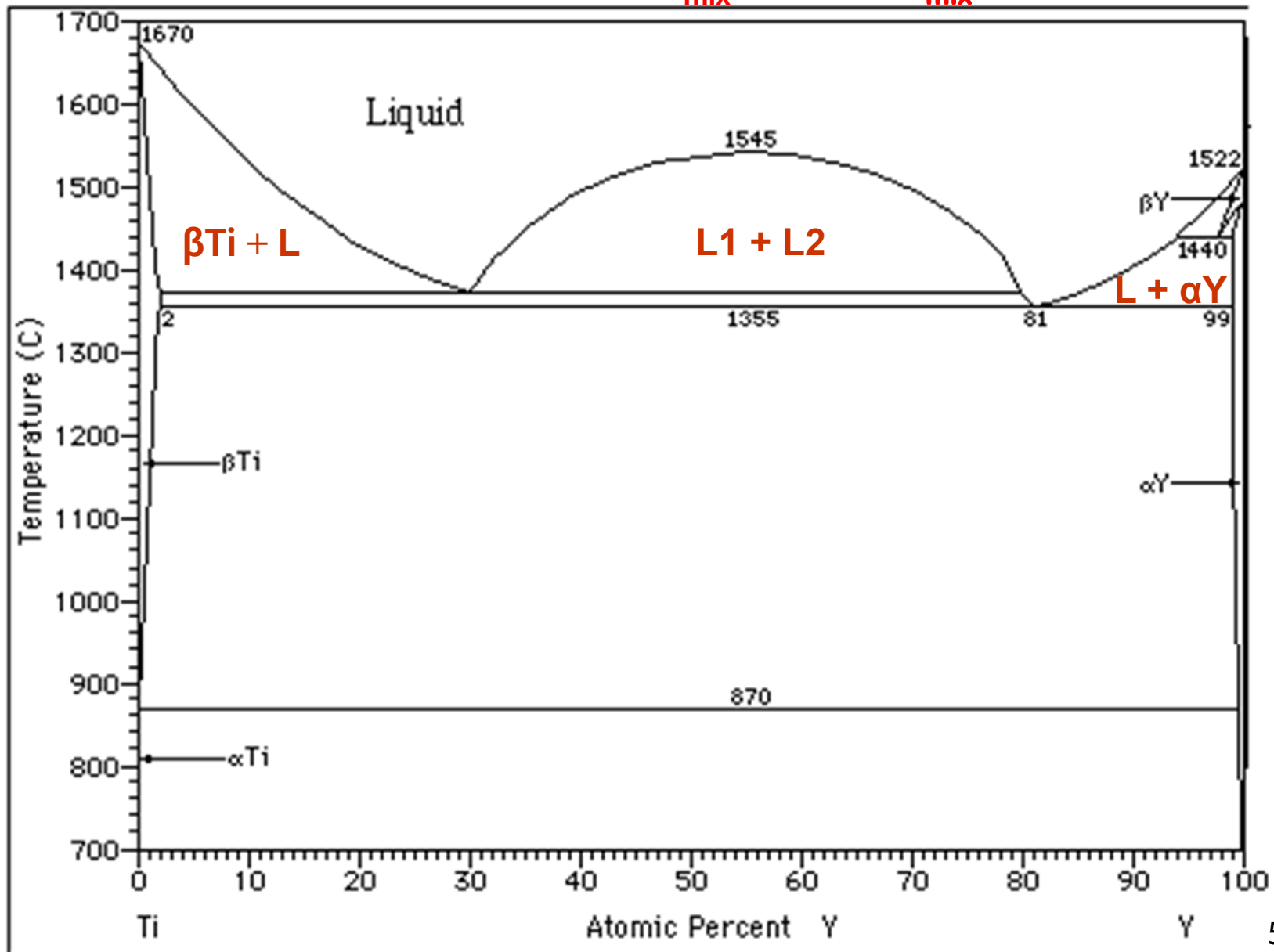


$\epsilon \gg 0, \Delta H_{\text{mix}} \gg 0 / \Delta H_{\text{mix}} \sim +60 \text{ kJ/mol}$





$\epsilon \gg 0, \Delta H_{mix} \gg 0 / \Delta H_{mix} \sim +58 \text{ kJ/mol}$



# Positive heat of mixing relation among constituent elements

- ▶ Alloy design considering heat of mixing relation among constituent elements

$$\Delta H_{\text{mix}} \gg 0 \text{ between A \& B}$$

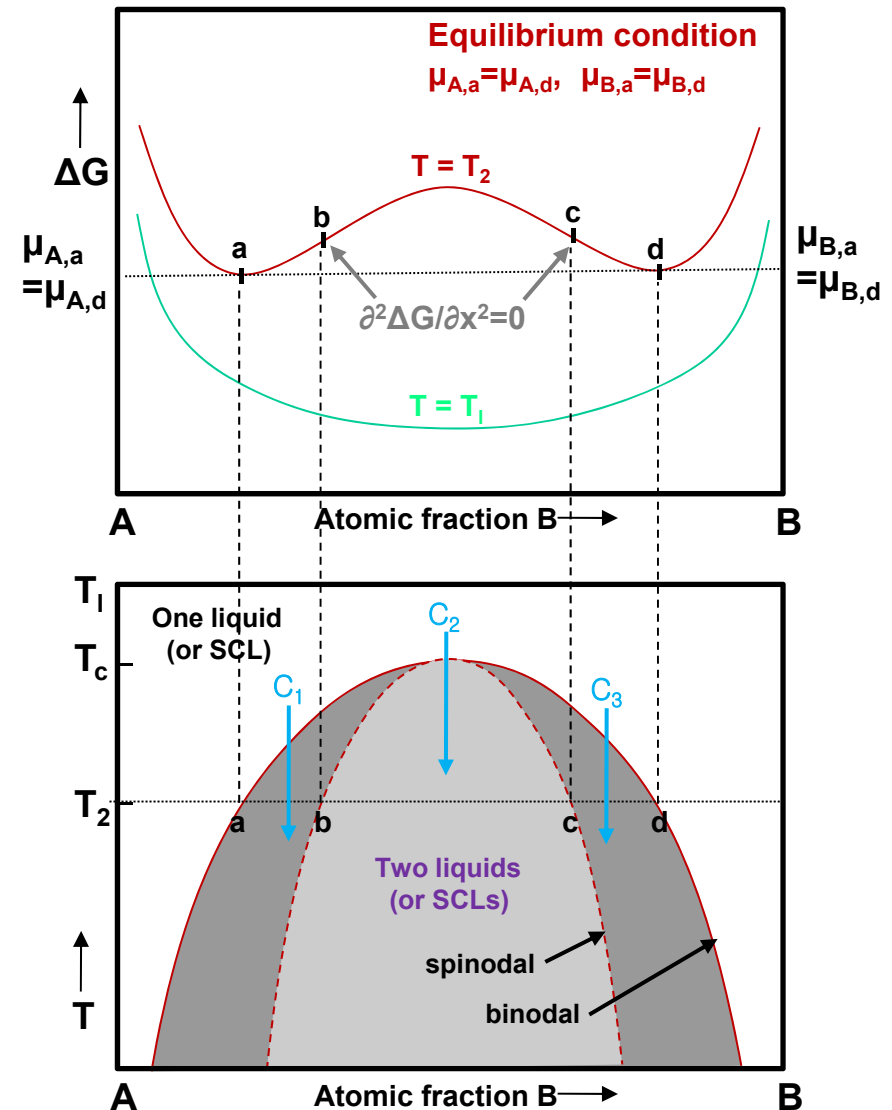
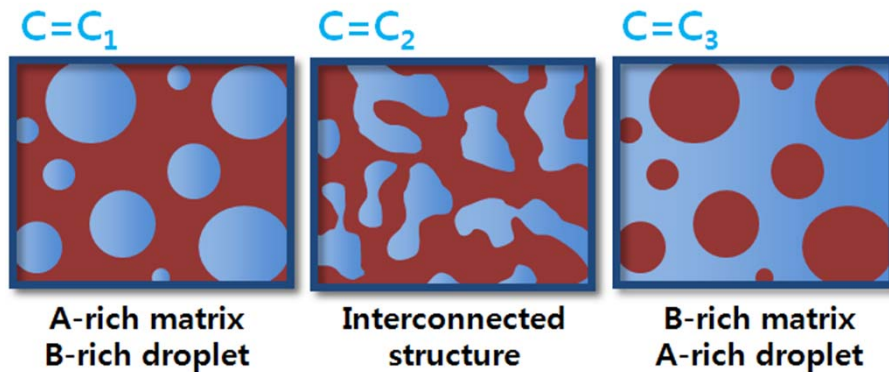


creates (meta)stable miscibility gap in limited composition range



Phase separation to A-rich & B-rich phase

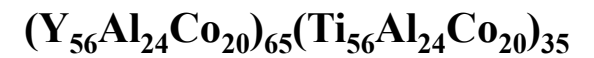
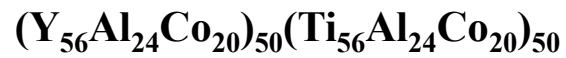
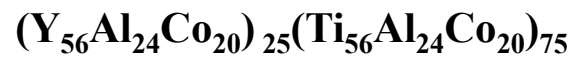
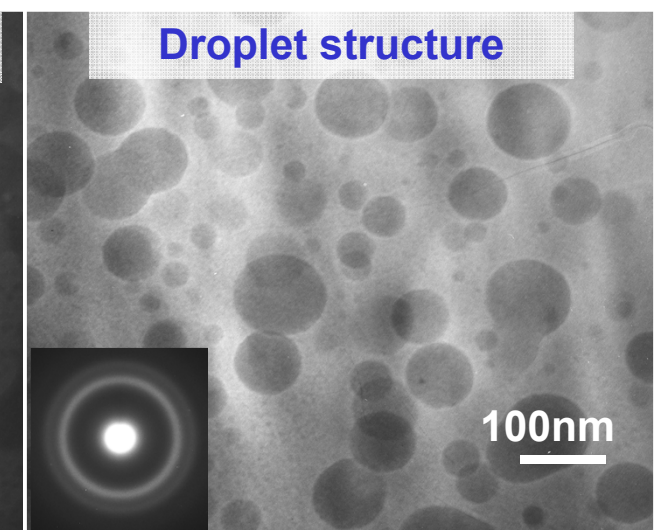
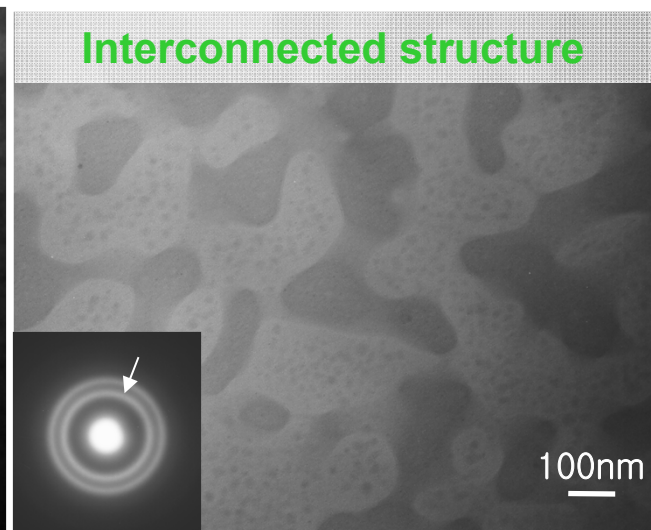
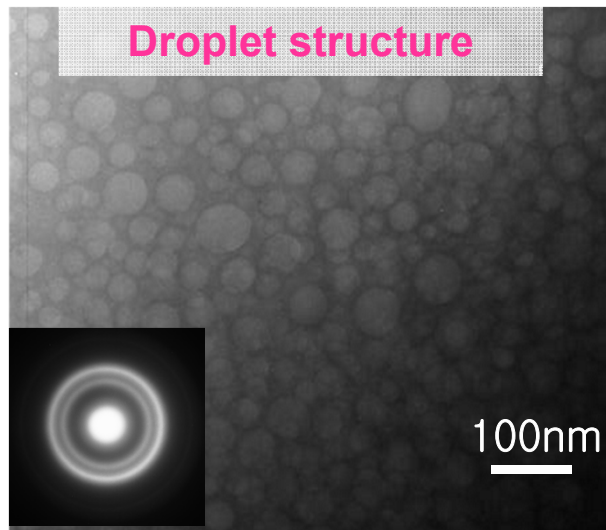
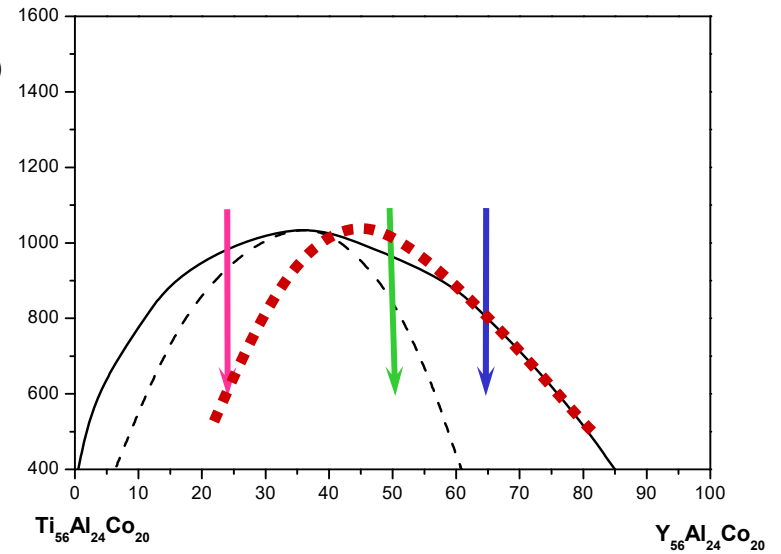
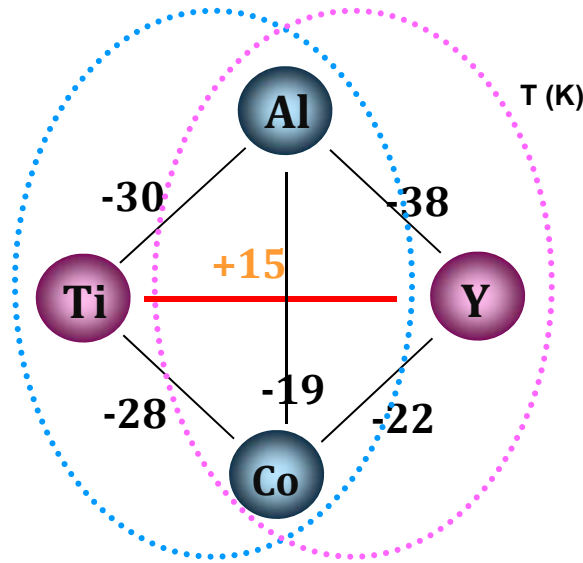
- ▶ Different two-phase structure by initial composition before phase separation



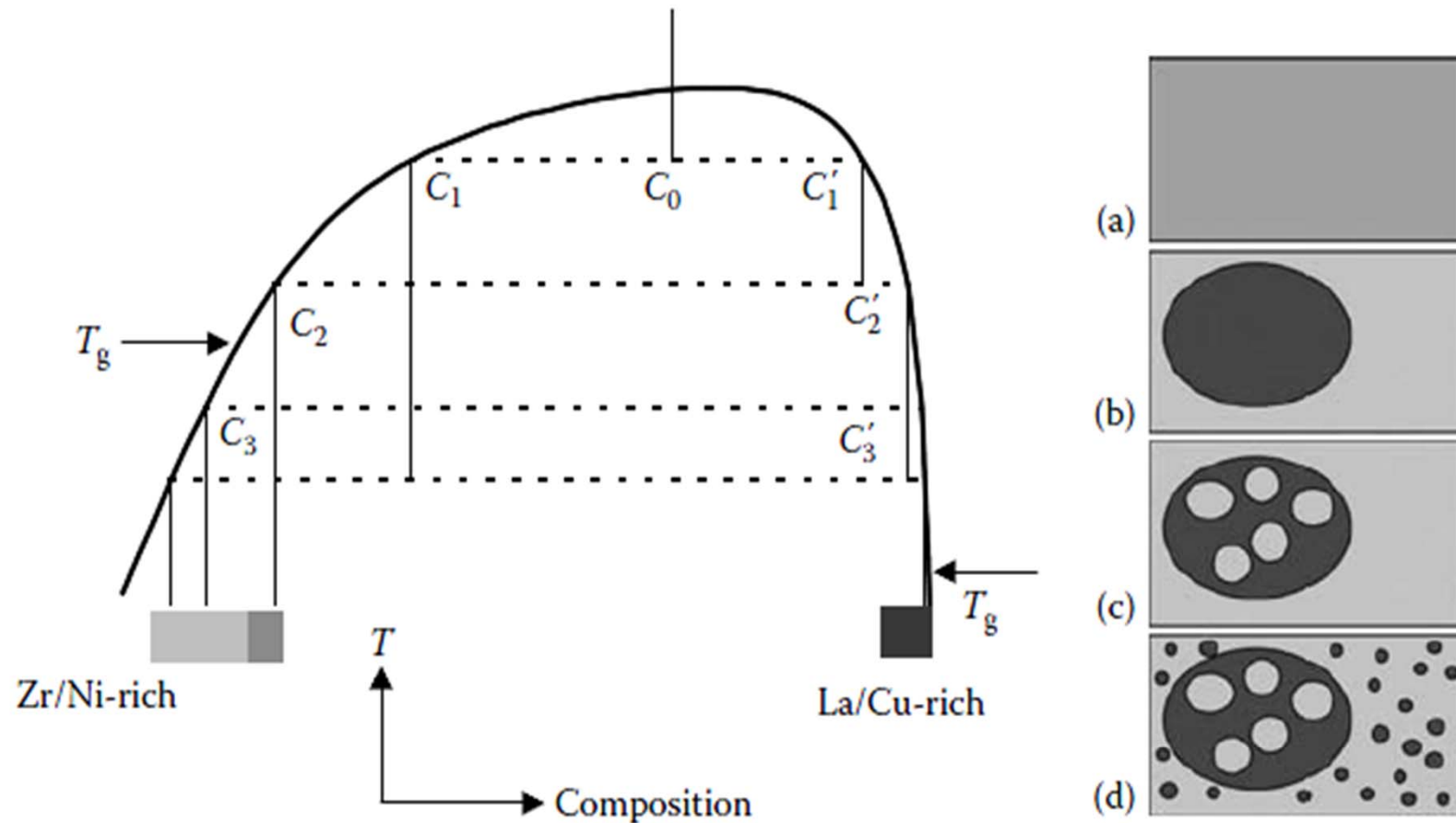
**Nucleation and growth ↔ Spinodal decomposition without any barrier to the nucleation process**



# \* Ti-Y-Al-Co system



## \* La-Zr-Al-Cu-Ni system

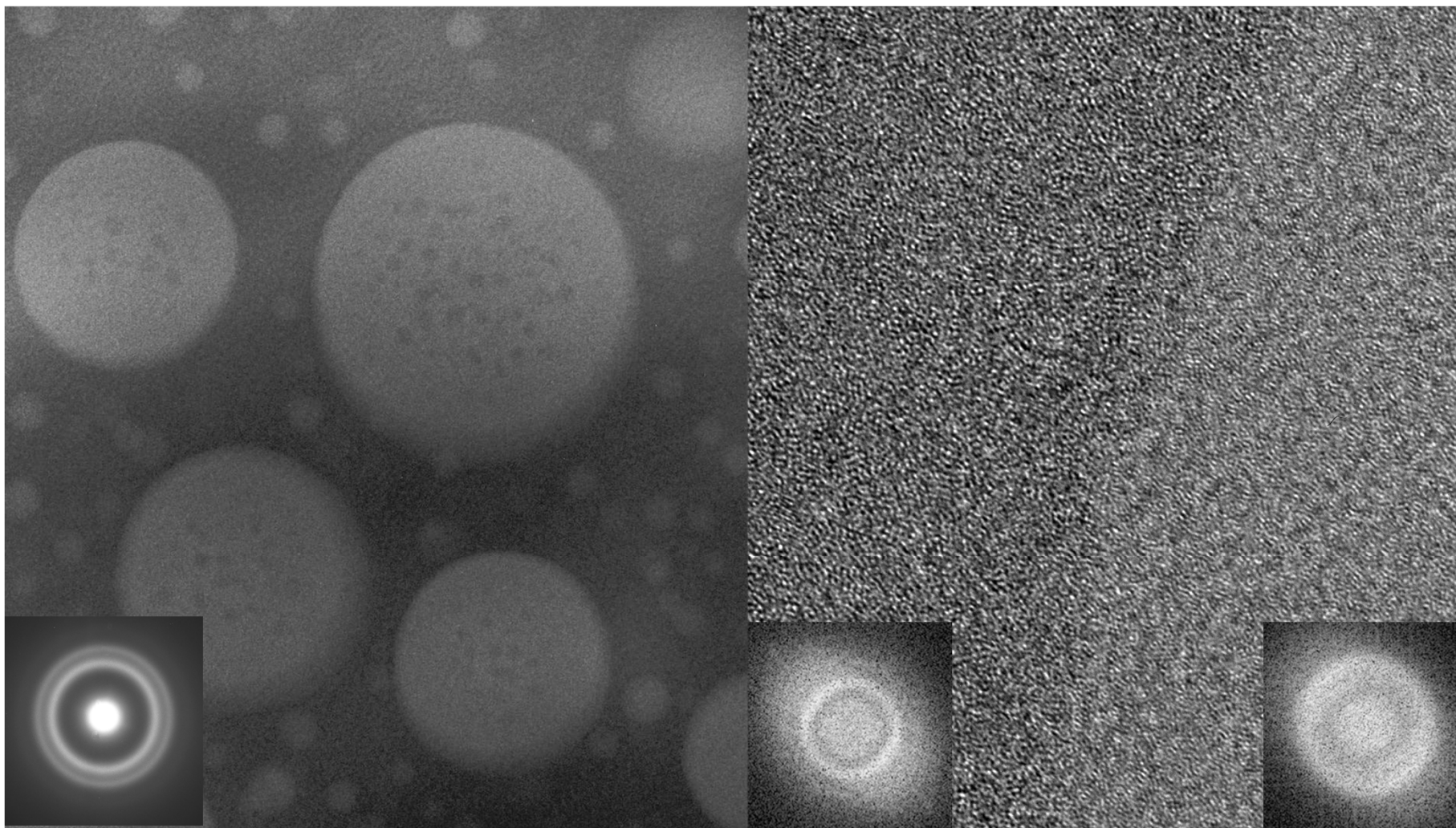
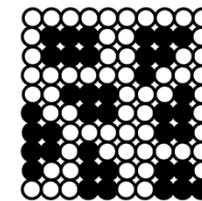


**FIGURE 5.17**

Schematic of the miscibility gap and the sequence of phase formation during cooling in the La-Zr-Al-Cu-Ni system. The positions of letters (a) to (d) in the diagram on the left correspond to the schematic microstructures (a) to (d) on the right. (Reprinted from Kündig, A.A. et al., *Acta Mater.*, 52, 2441, 2004. With permission.)



# Phase separation in metallic glasses



- **Binary System** mixture/ solution / compound

**Ideal solution** ( $\Delta H_{\text{mix}} = 0$ )

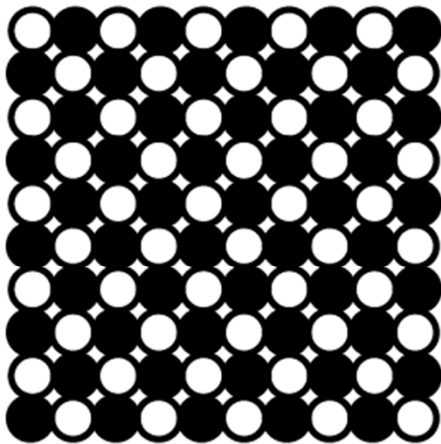
**Random distribution**

**Regular solution**  $\Delta H_{\text{mix}} = P_{AB}\epsilon$  where  $\epsilon = \epsilon_{AB} - \frac{1}{2}(\epsilon_{AA} + \epsilon_{BB})$   $\epsilon \approx 0$

$\Delta H_{\text{mix}} > 0$  or  $\Delta H_{\text{mix}} < 0$

**Real solution**

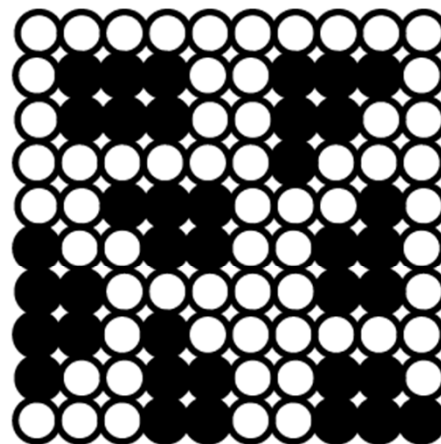
**Ordered structure**



(a)  $\epsilon < 0, \Delta H_{\text{mix}} < 0$

**Ordered alloys**

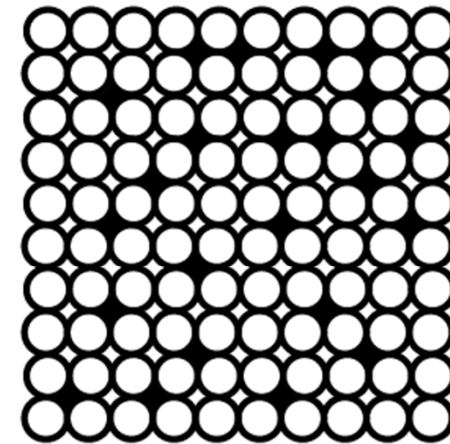
$P_{AB} \uparrow \longrightarrow \text{Internal } E \downarrow$



(b)  $\epsilon > 0, \Delta H_{\text{mix}} > 0$

**Clustering**

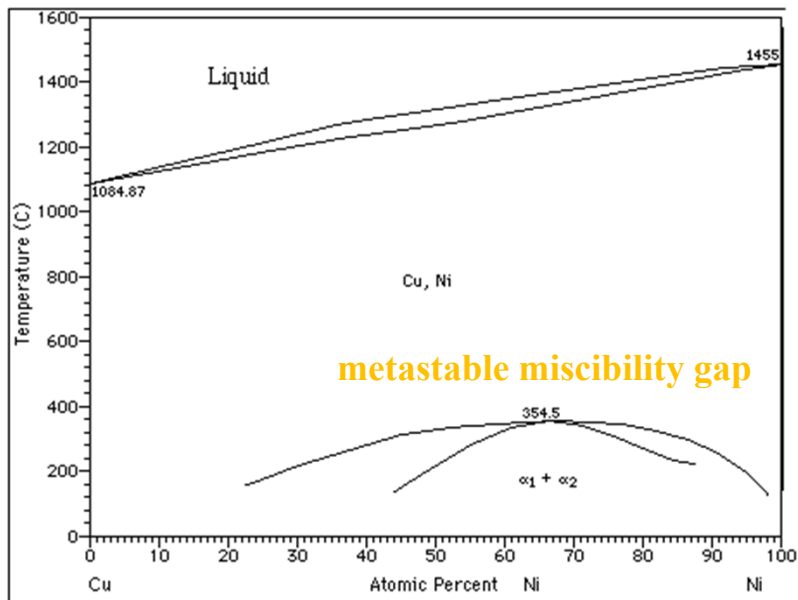
$P_{AA}, P_{BB} \uparrow$



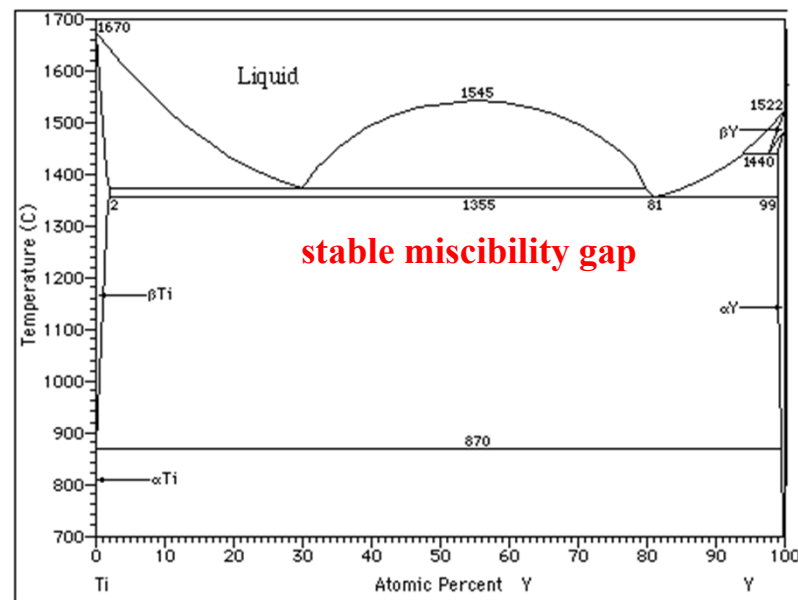
(c) *when the size difference is large*  
strain effect

**Interstitial solution**

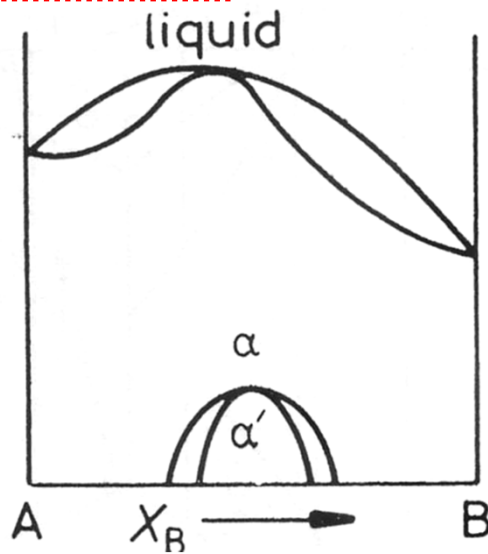
$\Delta H_{mix}^S > 0$  : Solid solution  $\rightarrow$  solid state phase separation (two solid solutions)



$\Delta H_{mix}^S \gg 0$  : liquid state phase separation (up to two liquid solutions)



$\Delta H_{mix}^S < 0$  : Solid solution  $\rightarrow$  ordered phase



$\Delta H_{mix}^S \ll 0$  : Compound : AB, A<sub>2</sub>B...

