Deposition of particles on a cylindrical collector by inertial impaction

"Inertial impaction occurs because sufficiently massive particles are unable to follow curvilinear fluid motion and tend to continue along a straight path where the fluid curves around the collector."

Therefore, the basic approach to analyzing inertial impaction is to determine the particle trajectory to see how the particles deviate from the streamline.

– Let us consider the trajectory of a particle initially at a distance y_1 from the centerline. We assume the particles located within y_1 should be collected and the particle located beyond y_1 can escape. Then we call the flow streamline through y_1 is the limiting streamline. Once y_1 has been determined, the collection efficiency is just .

$$\eta = 2y_1 / D_f$$

Also, let us assume this critical particle will be collected at the point $(\theta = \pi / 2)$,

$$y = \frac{d_f}{2} + \frac{d_p}{2}$$ (then, we automatically consider "interception".

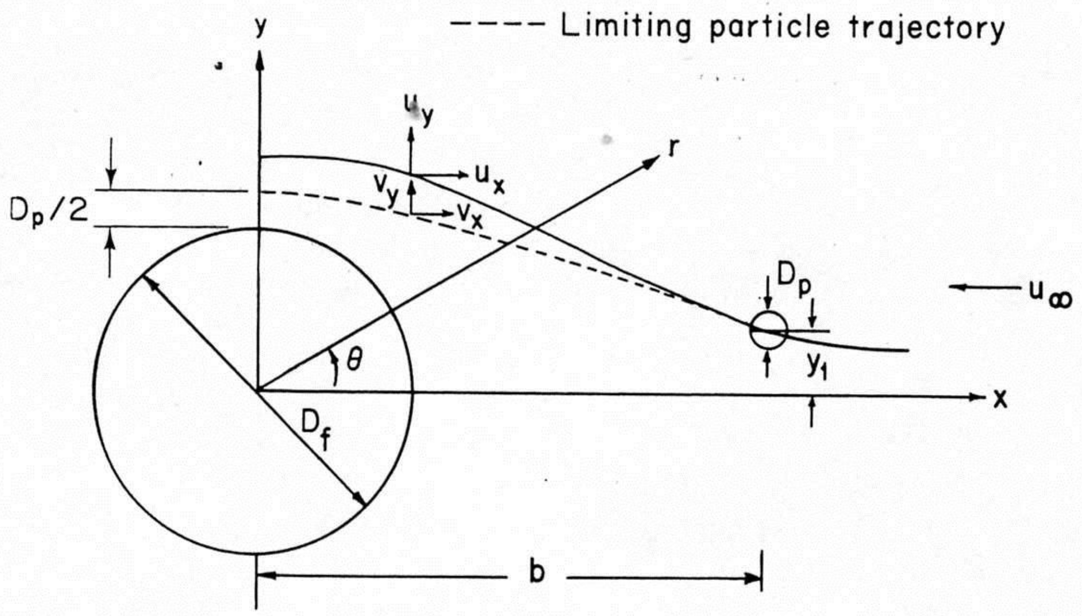


Figure 7.20 Collection of a particle by a cylinder placed transverse to the flow carrying the particles by the mechanisms of inertial impaction and interception.

그림 1.

그림 1. Equation of motion

$$\tau \frac{d\vec{u}}{dt} = \vec{u}_f - \vec{u}$$

$$\tau \frac{d^2 x}{dt^2} + \frac{dx}{dt} = u_x$$

$$\tau \frac{d^2 y}{dt^2} + \frac{dy}{dt} = u_y$$

$$x(0) = b \quad \frac{dx}{dt_{t=0}} = u_x(b, y_1) = -u_\infty$$

$$y(0) = y_1 \quad \frac{dy}{dt_{t=0}} = u_y(b, y_1) = 0$$

Numerical analysis is needed.

Approximate Simplified Analysis

$$\overline{u}_x = \frac{1}{2}(u_x\big|_1 + u_x\big|_2)$$

$$\overline{u_x} = -\frac{u_\infty}{2}(1+\frac{y_1}{y_2})$$

from the assumption of

$$-u_\infty y_1 = u_x\big|_2 y_2$$

$$\overline{u_y} = \frac{y_2 + D_f/2 - y_1}{-b/\overline{u}_x}$$

$$= \frac{u_\infty}{2b}\left[(y_2 + \frac{D_f}{2} - y_1)(1+\frac{y_1}{y_2})\right]$$

We know that y_1 and y_2 lie on the same stream line.

$$\Psi\Big|_2 = -\frac{u_\infty [D_f/2 + y_2]}{2Ku} \quad [2\ln\frac{D_f + 2y_2}{D_f} - 1 + \alpha$$

$$+\frac{D_f^2}{4\left[D_f/\ 2 + y_2\right]^2}(1-\frac{\alpha}{2}) - \frac{2\alpha}{D_f^2}(\frac{D_f}{2} + y_2)^2]$$

$$\Psi\Big|_1 = -u_\infty y_1 \quad \textbf{from} \quad u_x = -\frac{\partial\psi}{\partial y}$$

$$\frac{2y_1}{D_f} = \frac{1}{2Ku}(1+\frac{2y_2}{D_f})[2\ln(1+\frac{2y_2}{D_f}) - 1$$

$$+\alpha + \frac{1-\alpha/}{(1+2y_2/\ D_f)^2} - \frac{\alpha}{2}(1+\frac{2y_2}{D_f})^2]$$

→ **This equation gives us the relationship between y_1 and y_2**

We need to find the particular streamline on which a particle starting at y_1 is just captured at $D_f/2+D/_p2$.

$$\tau \frac{d^2x}{dt^2} + \frac{dx}{dt} = \overline{u_x}$$

$$\tau \frac{d^2y}{dt^2} + \frac{dy}{dt} = \overline{u_y}$$

$$x(0) = b \qquad \frac{dx}{dt}_{t=0} = \overline{u_x}$$

$$y(0) = y_1 \qquad \frac{dy}{dt}_{t=0} = 0$$

$$x(t) = b + \overline{u}_x t$$

$$y(t) = y_1 - \overline{u}_y \tau (1 - e^{-t/\tau}) + \overline{u}_y t$$

$$y_1 = \frac{D_f + D_p}{2} + \overline{u}_y \tau (1 - e^{b/\overline{u}_x \tau}) + \frac{\overline{u}_y b}{\overline{u}_x}$$

since $y = \dfrac{(D_f + D_p)}{2}$ **at *x*=0.**

$$t = -b/\overline{u}_x$$

$$b = \frac{D_f}{2\sqrt{a}}$$ **from** $$\alpha = \frac{D_f^2}{4b^2}$$

$$\frac{\bar{u}_x}{u_\infty} = -\frac{1}{2}\left(1 + \frac{2y_1/D_f}{2y_2/D_f}\right)$$

$$\frac{\bar{u}_y}{u_\infty} = \frac{\sqrt{\alpha}}{2}\left[\left(1 + \frac{2y_1/D_f}{2y_2/D_f}\right)\left(1 + \frac{2y_2}{D_f} - \frac{2y_1}{D_f}\right)\right]$$

$$\frac{2y_1}{D_f} = \left(1 + \frac{D_p}{D_f}\right) + St\sqrt{\alpha}\left[\left(1 + \frac{2y_1/D_f}{2y_2/D_f}\right)\left(1 + \frac{2y_2}{D_f} - \frac{2y_1}{D_f}\right)\right]$$

$$\times\left\{1 - \exp\left[-\frac{1}{St\sqrt{\alpha}}\left(1 + \frac{2y_1/D_f}{2y_2/D_f}\right)^{-1}\right]\right\} - \left(1 + \frac{2y_2}{D_f} - \frac{2y_1}{D_f}\right)$$

Function of stk no., D_p/D_f, α

$$\eta = \frac{2y_1}{D_f}$$

Of course, this is not a rigorous analysis. Since this critical particle can be collected at the different angular position.

Fig. 7.21

This figure clearly shows the effect of α. For the larger value of α, the streamline lie closer to the cylinder than at smaller α. Thus, at fixed Stokes number, increasing α leads to increasing collection efficiency.

-We see that at a fixed value of α the collection efficiency increase with increasing Stokes no., eventually reaching a value of unity. Physically, a convenient way to think of increasing *Stk* is to imagine the particle density increasing at fixed size D_p. Thus, as *Stk* increases the particles becomes heavier and heavier and is less able to follow the streamline. A point is

eventually reached as *Stk* increases where all the particles contained in the upstream projected area of the cylinder are collected.

In fact, we see that η becomes slightly larger than 1.0 reflecting the interception contribution.

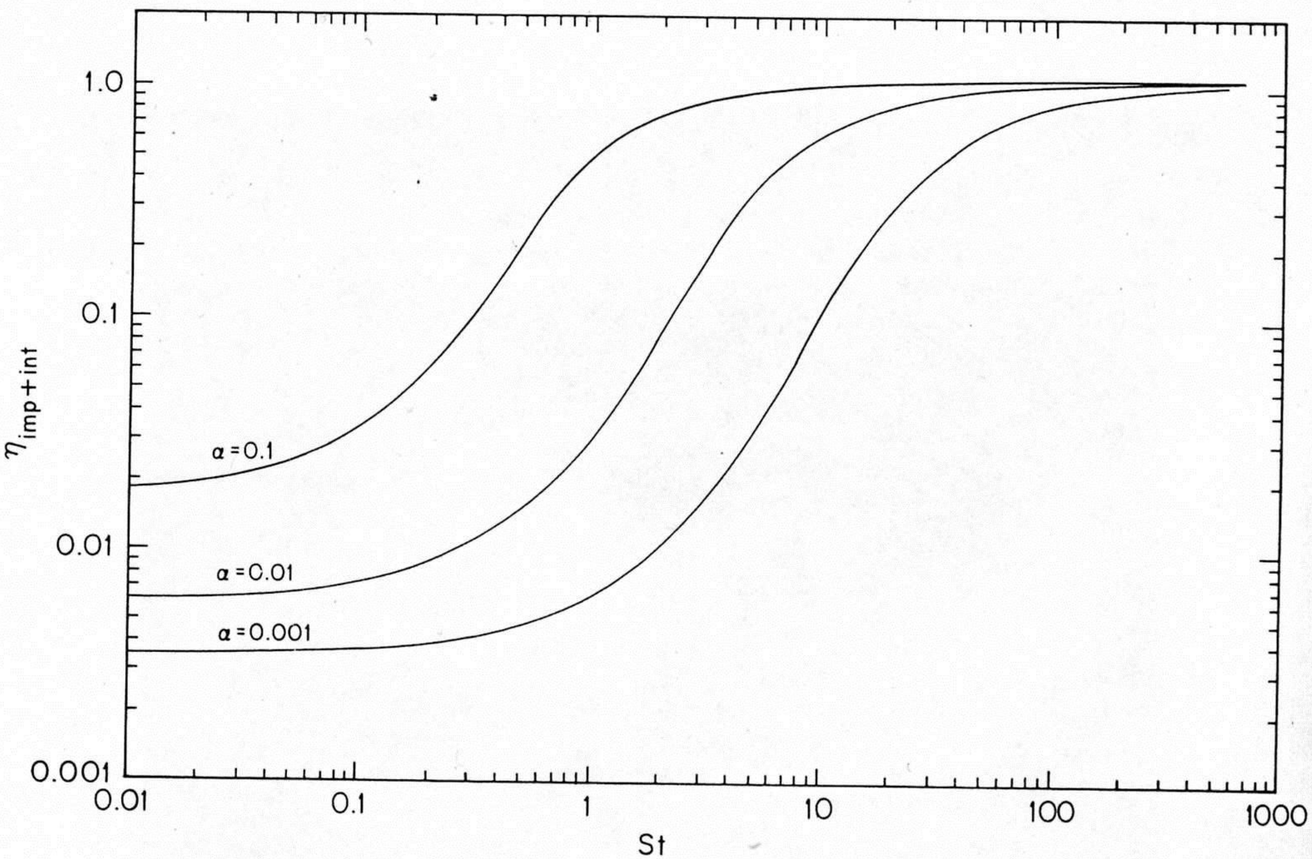


Figure 7.21 Collection efficiency for combined impaction and interception for a cylinder placed transverse to the flow as a function of Stokes number for $D_p/D_f = 0.1$.

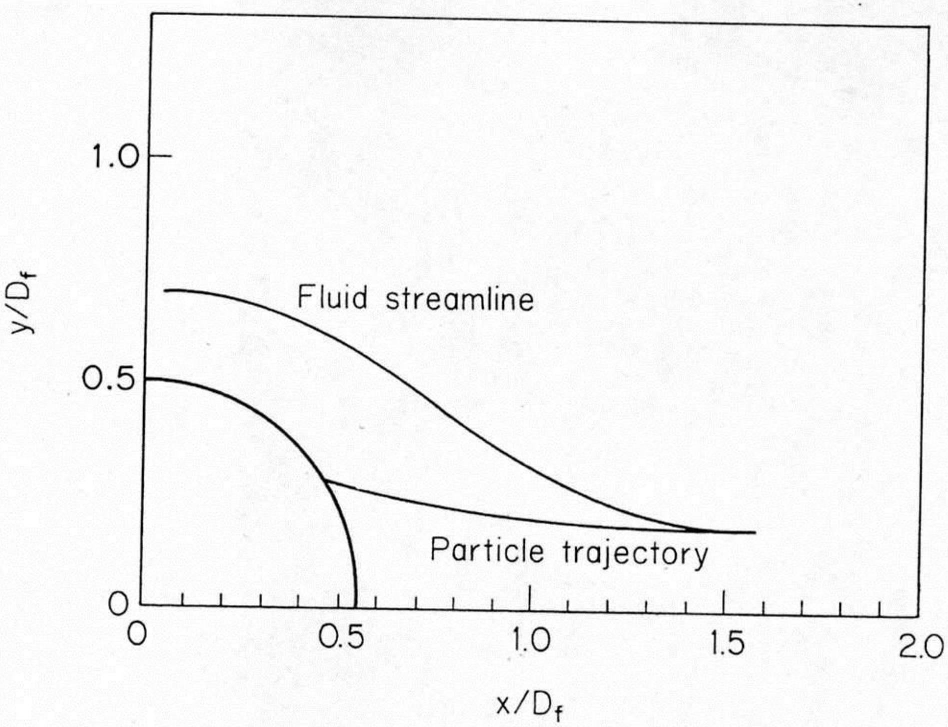


Figure 7.22 Particle trajectory approaching a cylinder in the Kuwabara flow field for St = 1, $\alpha = 0.1$, $D_p/D_f = 0.1$, and $y_1/D_f = 0.2$.

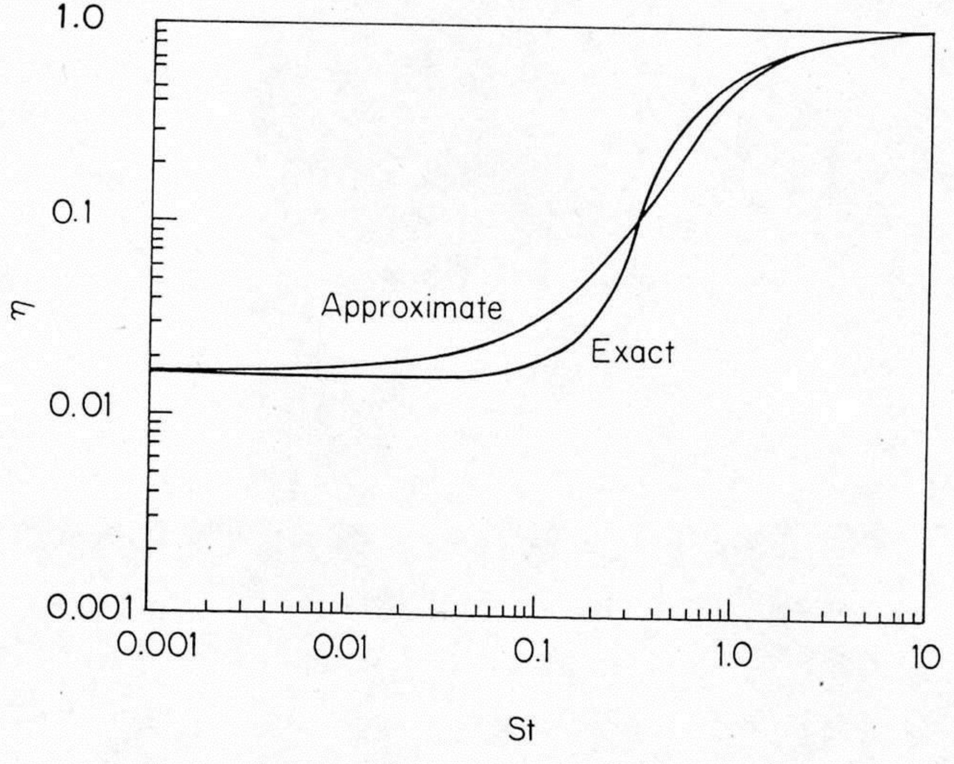


Figure 7.23 Approximate and exact collection efficiencies for inertial impaction and interception by a cylinder. The Kuwabara flow field is assumed with a filter solid fraction $\alpha = 0.1$. The approximate efficiency is that already given in Figure 7.21; the exact is that determined from numerical solution of the particle trajectories.

Fig. 7.25

η for combined impaction and interception

The limiting cases as *Stk* becomes smaller will represent the effect of pure interception

$$\eta \sim (D_p / D_f)^2$$

At large enough *Stk* numbers, the efficiency curves for different values of D_p/D_f converge as impaction become dominant mechanism with negligible interception.

Fig. 7.26

It is interesting to compare the three mechanisms of collection: Brownian Diffusion, impaction, interception

Fig. 5.27

The overall collection efficiency versus particle diameter

→ exhibits a minimum in the efficiency between 0.1 ㎛ and 1.0 ㎛ in diameter. In this range, the particle is large enough so that its Brownian diffusivity is too small to lead to a substantial efficiency by that mechanism, and at the same time, it is too small for its inertial to be large and inertial impaction can be also small.

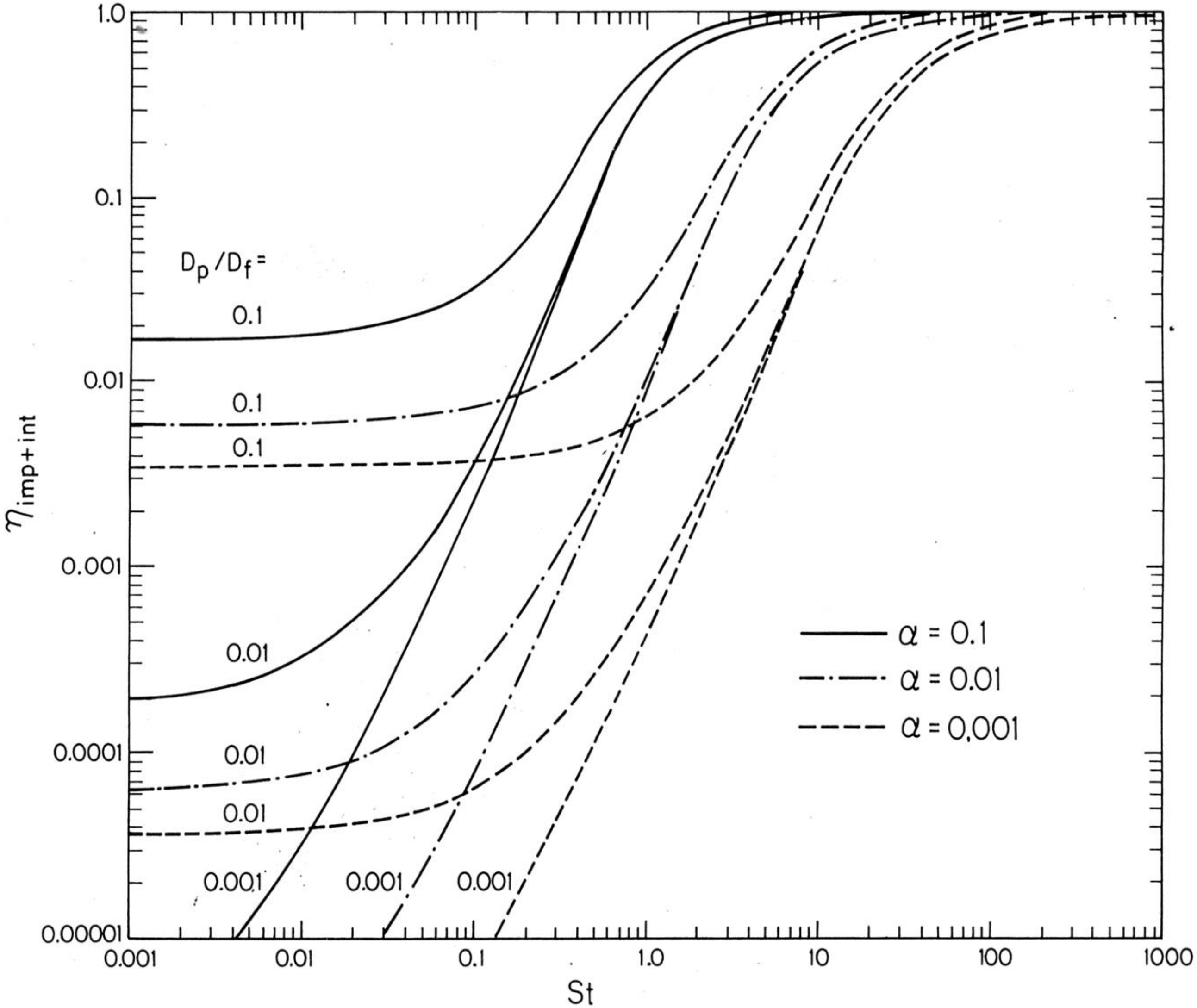


Figure 7.25 Collection efficiency for combined impaction and interception for a cylinder placed transverse to the flow as a function of Stokes number for D_p/D_f = 0.001, 0.01, and 0.1 and α = 0.001, 0.01, and 0.1.

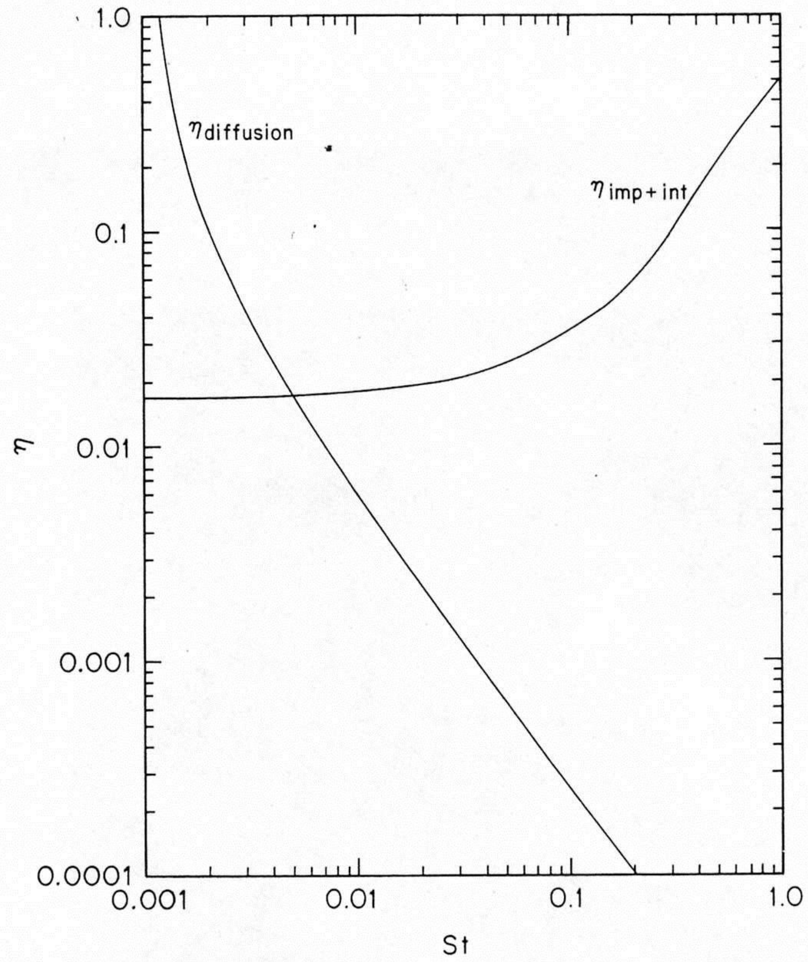


Figure 7.26 Collection efficiencies by Brownian diffusion and impaction/interception for a cylinder placed transverse to the flow as a function of Stokes number for $\alpha = 0.1$, $D_p/D_f = 0.1$, and $u_\infty = 1.0 \text{ cm s}^{-1}$.

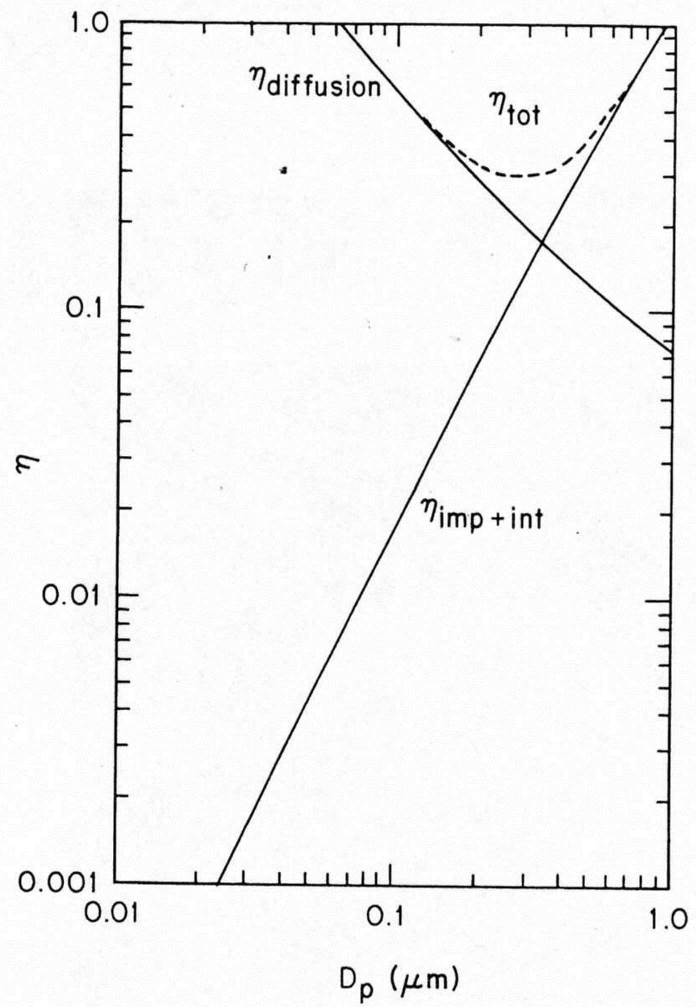


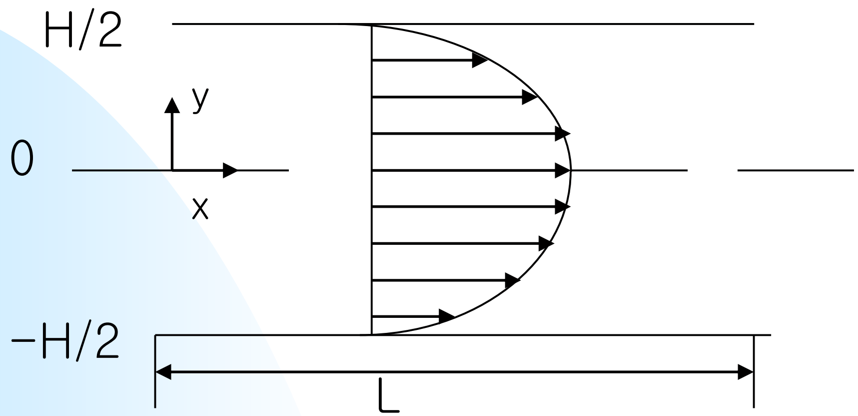
Figure 7.27 Individual collection efficiencies due to Brownian diffusion and impaction/interception, together with total collection efficiency as a function of particle diameter. The other parameters are $\alpha = 0.1$, $u_\infty = 1.0$ cm s^{-1}, and $D_f = 1.0$ μm.

Settling Chamber ← gravitation effect

: Gravitational settling is perhaps the most obvious means of separating particles from a flowing gas. A settling chamber is simply a large box or diffuser in which the flow velocity becomes low so that particles have sufficient time to settle down.

Settling chamber is used to remove usually large particles greater than 50 ㎛. As a precleaner removing large and possibly (harmful) particles prior to other more precise collection devices or measuring instrumentation.

(1) Laminar flow. Settling chamber : Channel type



$$u_x = \frac{3}{2}\bar{u}\left[1-\left(\frac{2y}{H}\right)^2\right]$$

We assume that particles are introduced uniformly across the entrance to the channel at concentration No.

There will be a critical height y* such that a particle of d_p located initially at x=0, y= y* will be just settled down the plate at x=L, y=−H/2 .

This particle will be the last particle of d_p collected in this chamber. Particles that entered the chamber above y= y* will not be collected.

The time needed for this last particle to settle down should be

$$t_f = \frac{y^* + H/2}{v_t}$$

Let us determine the trajectory of this last particle.

$$y = y^* - v_t t$$

At time=t , the particle has x-direction velocity

$$v_x = \frac{3}{2}\bar{u}\left[1 - \left(\frac{y^* - v_t t}{H/2}\right)^2\right] = \frac{dx}{dt}$$

$$\therefore \int_0^{t_f} \frac{dx}{dt} dt = L$$

$$\frac{LH^2}{6\bar{u}} = t_f\left(H^2/4 - y^{*2}\right) + v_t y^* t_f^{\,2} - \frac{v_t^{\,2}}{3} t_f^{\,3}$$

$$t_f = \frac{y^* + H/2}{v_t}$$

We can finally obtain y*.

$$\boxed{\begin{gathered} \frac{2v_t}{3\bar{u}} \Big/ H/L = 2Z^2 - \frac{4}{3}Z^3 \\ z = \frac{1}{2} + y^*/H \end{gathered}}$$

implicit form

Collection efficiency

$$\eta = \frac{N_0 \int_{-H/2}^{y^*} u_x(y)dy}{N_0 \bar{u} H} = \frac{1}{2} + \frac{3}{2}\frac{y^*}{H} - 2\left(\frac{y^*}{H}\right)^3$$

$$= 3Z^2 - 2Z^3$$

$$\eta(d_p) = \frac{v_t L}{\bar{u} H} \qquad v_t \sim d_p^{\,2}$$

$$\therefore \eta(d_p) \sim d_p^{\,2}$$

(2) Turbulent flow settling chamber (for rectangular chamber)

$$\mathrm{Re}_c > 4000 \qquad \mathrm{Re}_c = \frac{4 r_H \bar{u}}{\nu}$$

r_H: hydraulic radius $= \dfrac{A_c}{P}$

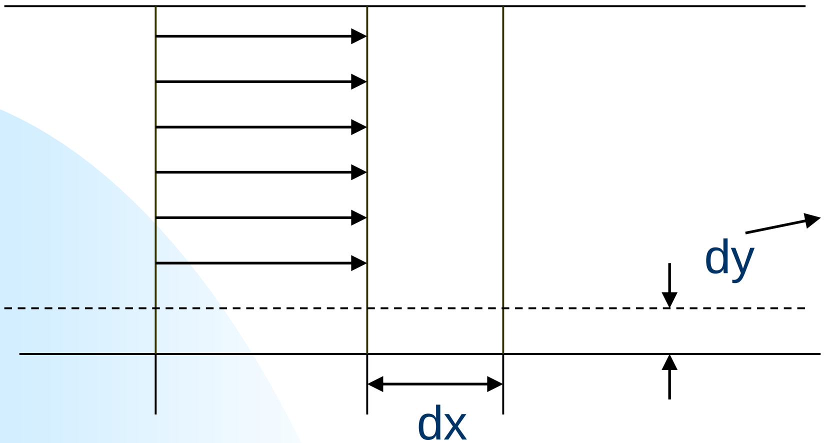
Detailed particle trajectory

calculation can be done numerically, but here, we want to make simplified analysis to obtain some functional form of collection efficiency. This kind of simple analysis is helpful to find which parameters govern the collection efficiency.

So we assume in the bulk flow or in the core region, turbulent mixing is vigorous enough so that particles can not settle. Also we can assume uniform concentration of particles.

Only particles very near the surface will be deposited.

So we assume there exists a thin layer where once a particle enters, it settles to the surface.



$\frac{dy}{H}$: fraction of particle deposition

$$v_t dt = dy \qquad dy = v_t \frac{dx}{\bar{u}}$$

$$\therefore \frac{dy}{H} = \frac{v_t dx}{\bar{u} H}$$

$$\bar{u} A_c \left(n_p \Big|_x - n_p \Big|_{x+dx} \right) = \frac{v_t dx}{\bar{u} H} \bar{u} A_c n_p \Big|_x$$

$$\frac{dn_p}{dx} = -\frac{v_t}{\bar{u}H} n_p$$

$$\eta(d_p) = 1 - \exp\left(-\frac{v_t L}{\bar{u}H}\right) \qquad n_p(x) = N_0 \exp\left(-\frac{v_t x}{\bar{u}H}\right)$$

$$= 1 - \exp\left(-\frac{LW\rho_p g d_p{}^2}{18\mu Q}\right) \qquad v_t = v_t(d_p)$$

→ design equation

$$Q = \bar{u}HW$$