kinetic theory

$d_p << \lambda_g$

$f(v)d^3v$: mean number of particles per unit volume with center of mass velocity in the range between v and $v + dv$

$$f(v) = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-mv^2/2kT\right)$$

: Maxwell-Boltzmann distribution

$$\bar{v} = \frac{1}{n}\int v f(v) d^3 v$$

$$d^3 v = dv_x dv_y dv_z$$

in spherical coordinate

$$d^3 v = v^2 dv \sin\theta \, d\theta \, d\vartheta$$

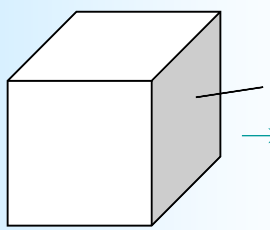
$$\therefore \bar{v} = \frac{1}{n}\int_0^\infty \int_0^\pi \int_0^{2\pi} v^2 dv \sin\theta \, d\theta \, d\vartheta \, f(v) v$$

$$= \frac{4\pi}{n}\int_0^\infty f(v) v^3 dv = \left(\frac{8kT}{\pi m}\right)^{1/2} : \text{mean speed}$$

Collision rate per unit area

#/m^2sec : number of collisions per unit area per unit time

i) crude calculation



number of particle having Z-direction velocity $\approx \dfrac{n}{6}$ per unit volume

particle with $\bar{v}$ moves $\bar{v}dt$ in dt, which makes volume $\bar{v}dtA$

$\therefore$ total number of particles which strike

per unit area per unit time $\approx \dfrac{1}{6}n\bar{v} \approx n\bar{v}$

ii) Exact calculation : consider the distribution of particle velocity

consider particles with velocity v to v+dv its

direction θ to $\theta + d\theta$ and ϑ to $\vartheta + d\vartheta$

$f(v)d^3v \;\; vdt \;\; dA\cos\theta$: number of particles of this type that strike the area dA in time dt

total number of particles that strike a unit area of the wall per unit time

$$\Phi_0 = \int f(v)v\cos\theta d^3v$$

$$0 < v < \infty \qquad 0 < \vartheta < 2\pi \qquad 0 < \theta < \frac{\pi}{2} \qquad \frac{\pi}{2} < \theta < \pi$$

: particles leaving from the wall

$$d^3v = v^2 dv \sin\theta\, d\theta\, d\vartheta$$

$$\therefore \Phi_0 = \int_0^\infty f(v)v^3dv \int_0^{\frac{\pi}{2}} \sin\theta\, \cos\theta\, d\theta \times \int_0^{2\pi} d\vartheta = \pi\int_0^\infty f(v)v^3dv$$

$$\bar{v} = \frac{4\pi}{n}\int_0^\infty f(v)v^3dv \qquad \therefore \Phi_0 = \frac{1}{4}n\bar{v} \;\; \text{Effusion flux}$$

Integration gives

$$n(t) = \frac{d_p kT}{2e^2} \ln\left(1 + \frac{\pi d_p \overline{c_i} e^2 N_i t}{2kT}\right)$$

In the continuum regime, the flux of ions toward the particle is given by

$$J_i = \pi d_p^2 \left(D_i \frac{dN_i}{dr} - Z_i E N_i\right)$$

field induced migration

Diffusion term

→ Lawless (J. of Aerosol Science, 27, 191−215 (1996))

Even with given electric field (not the induced field), diffusion charging is the predominant mechanism for charging particles less than 200 nm in diameter

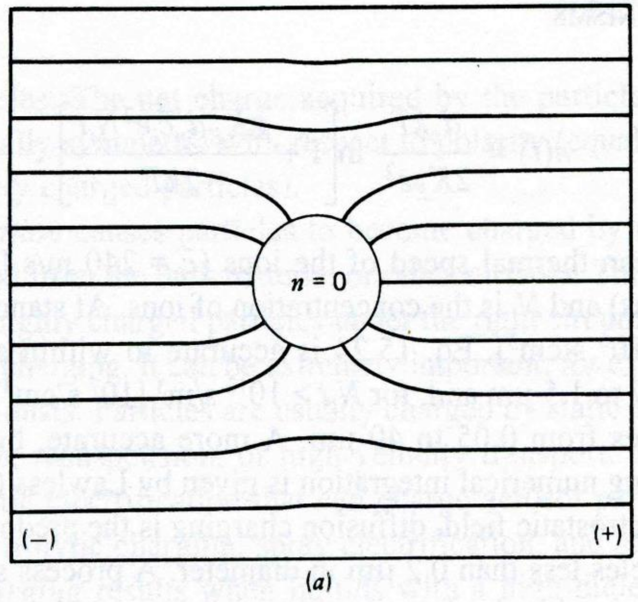
(2) Field Charging

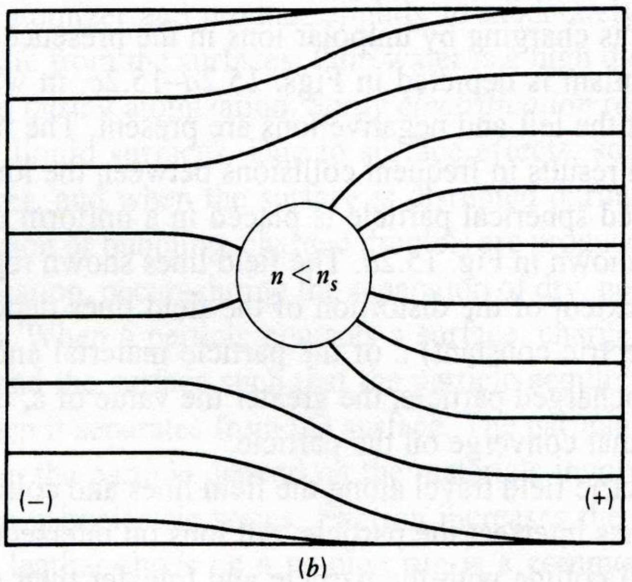
Field charging is the charging by unipolar ions in the presence of a strong electric field.

$\boxed{\text{ppt}}$ for the process of field charging

Ion flux on particles

$$= Z_i E_{at\ r=dp/2} \cdot N_i$$





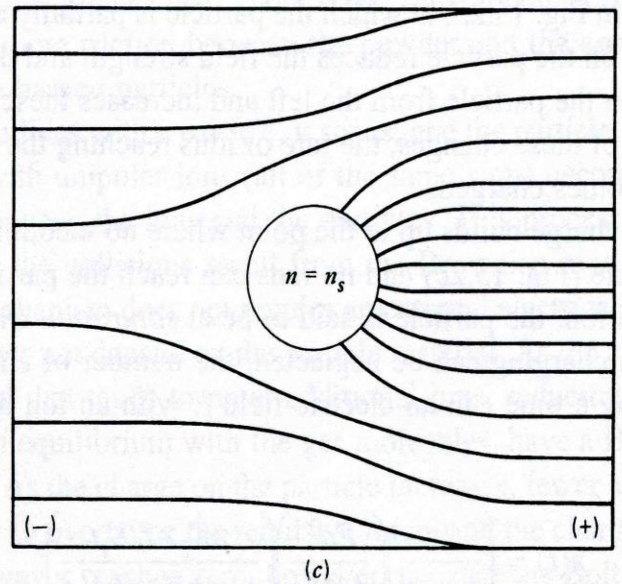


FIGURE 15.2 Electric field lines for a conducting particle in a uniform field (negative ions and negative plate at left). (*a*) An uncharged particle. (*b*) A partially charged particle. (c) A particle at saturation charge.

$$\frac{dq}{dt} = \frac{d(ne)}{dt} = \int Z_i E_{r=dp/2} N_i e dA$$

Equation (15.25)

Number of charges obtained from field charging

$$n(t) = \left(\frac{3\varepsilon}{\varepsilon + 2}\right)\left(\frac{E d_p^{\;2}}{4e}\right)\left(\frac{\pi e Z_i N_i t}{1 + \pi e Z_i N_i t}\right)$$

(15-25)

Where ε is the relative permittivity of the particle (dielectric constant)

Z_i : mobility of ions $\sim 0.00015 m^2 / V \cdot s$

As $t \rightarrow \infty$

$$n(\infty) = n_s = \frac{3\varepsilon}{\varepsilon + 2} \frac{E d_p^2}{4e}$$

: dielectric constant reflects the strength of the electrostatic field produced in different materials by a fixed potential relative to that produced in a vacuum. For most materials, $1 < \varepsilon < 10$

ε is 1.0 for a vacuum

4.3 for quartz

80 for pure water

infinity for conducting particles

$$\frac{n(t)}{n_s} = \frac{t}{t+t_0} \qquad (t_0 = \frac{1}{\pi K_E e Z_i N_i})$$

independent of size but dependent on N_i, Z_i

$$\varepsilon_0 \sim 10^{-11} C/mV, \qquad e \sim 10^{-19} C, \; Z_i \sim 10^{-4} m^2/Vs$$

$$N_i \sim 10^{13}/m^3 \longrightarrow \qquad n(t) \rightarrow n_s$$

when t>0.1 sec

Therefore, if residence time of particle exceed 1 sec, you could assume field charging as a saturated charge

$n_s \sim d_p^2$ while diffusion charging $n \sim d_p$

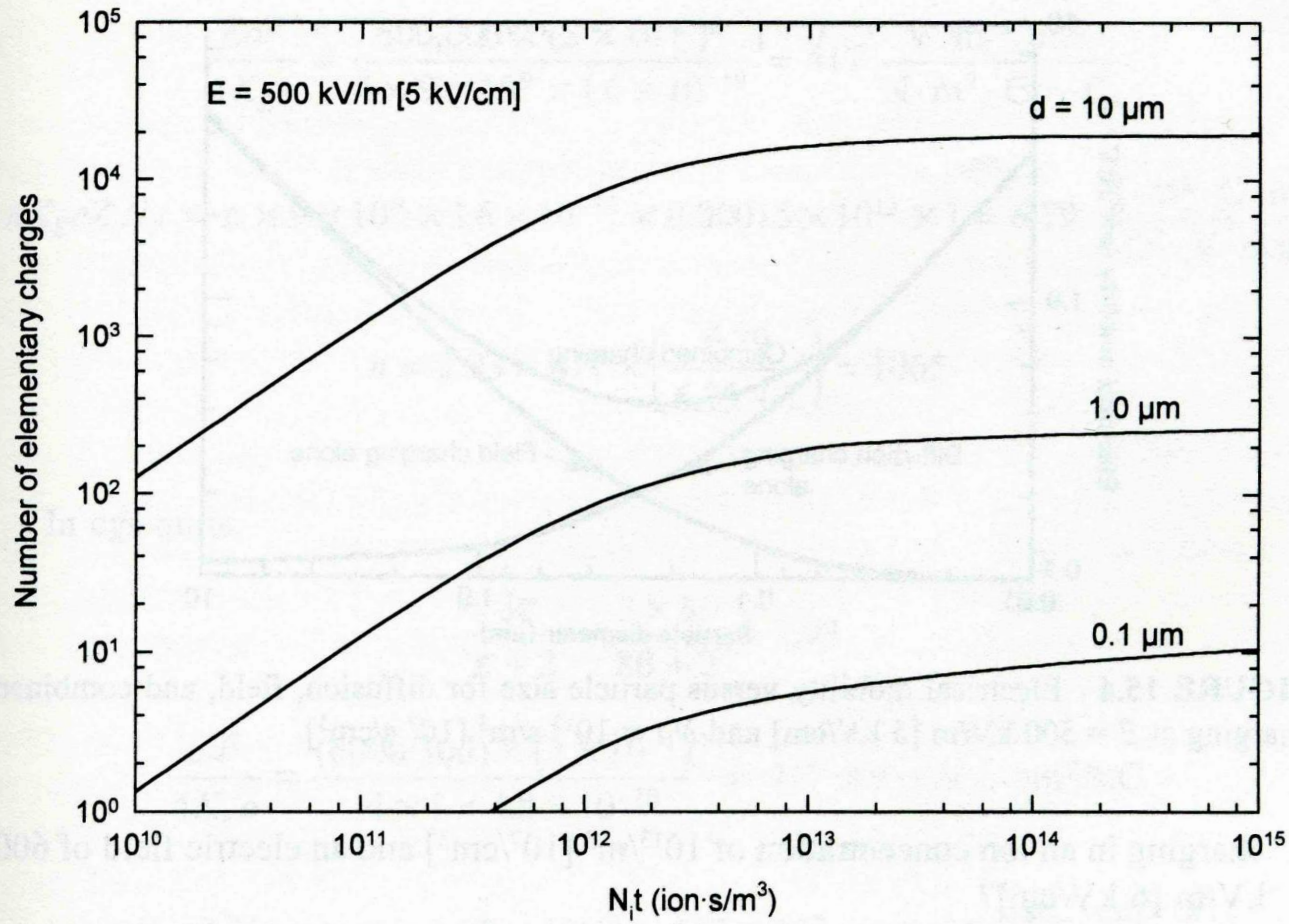
, therefore

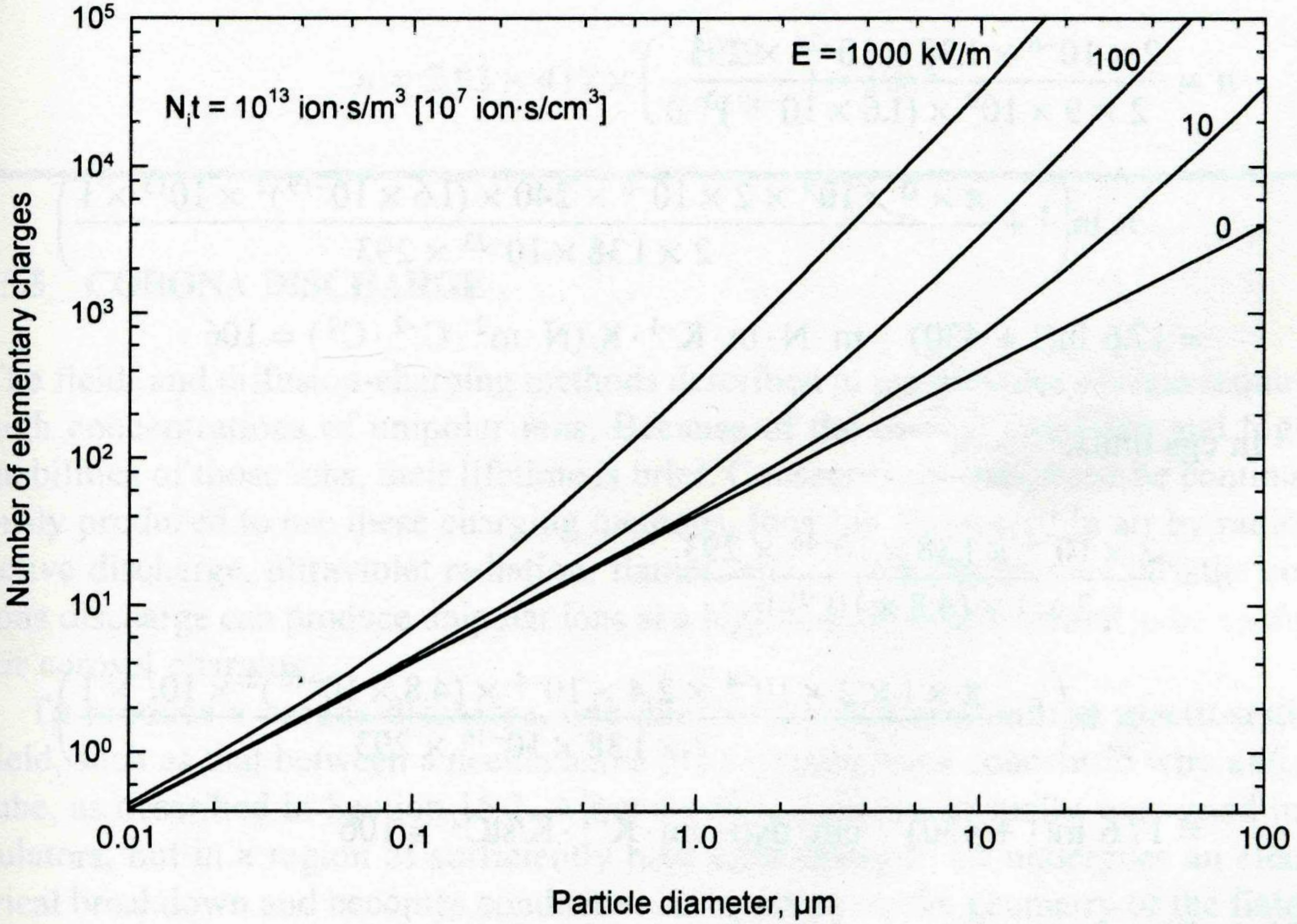
so, field charging is the dominant mechanism for particles larger than 1.0 ㎛ and diffusion charging is the dominant mechanism for particles less than 100 nm. (even in the presence of an electric field)

TABLE 15.3 Comparison of Calculation Methods for Charging by Field, Diffusion, and Combined Charging at $N_i t = 10^{13}$ s/m^3 [10^7 s/cm^3]. $\varepsilon = 5.1$.

Particle Diameter (μm)	Number of Elementary Units of Charged Acquired			
	Diffusion Charging		Field Charging E = 500 kV/m [5 kV/cm]	Combined Charging E = 500 kV/m [5 kV/cm]
	Eq. 15.24	Numerical Solution[a]	Eq. 15.25	Numerical Solution[a]
0.01	0.10	0.41	0.02	0.42
0.04	0.79	1.6	0.26	1.9
0.1	2.7	4.1	1.6	5.6
0.4	15.7	16.3	25.9	40
1.0	47	41	162	162
4.0	237	163	2580	2680
10	673	407	16,200	16,540
40	3180	1630	259,000	264,000

[a]Lawless (1996).


E = 500 kV/m [5 kV/cm]
d = 10 μm
1.0 μm
0.1 μm
Number of elementary charges
10^0
10^1
10^2
10^3
10^4
10^5
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
$N_i t$ (ion·s/m^3)
(a)



(*b*)

FIGURE 15.3 Field and diffusion charging. (*a*) Number of charges acquired versus $N_i t$ for particle diameters of 0.1, 1, and 10 μm at a field strength of 500 kV/m [5 kV/cm]. (*b*) Number of charges acquired versus particle diameter for field strengths of 0, 100, 1000, and 10,000 V/cm at $N_i t = 10^{13}$ s/m³ [10^7 s/cm³]. ε = 5.1.

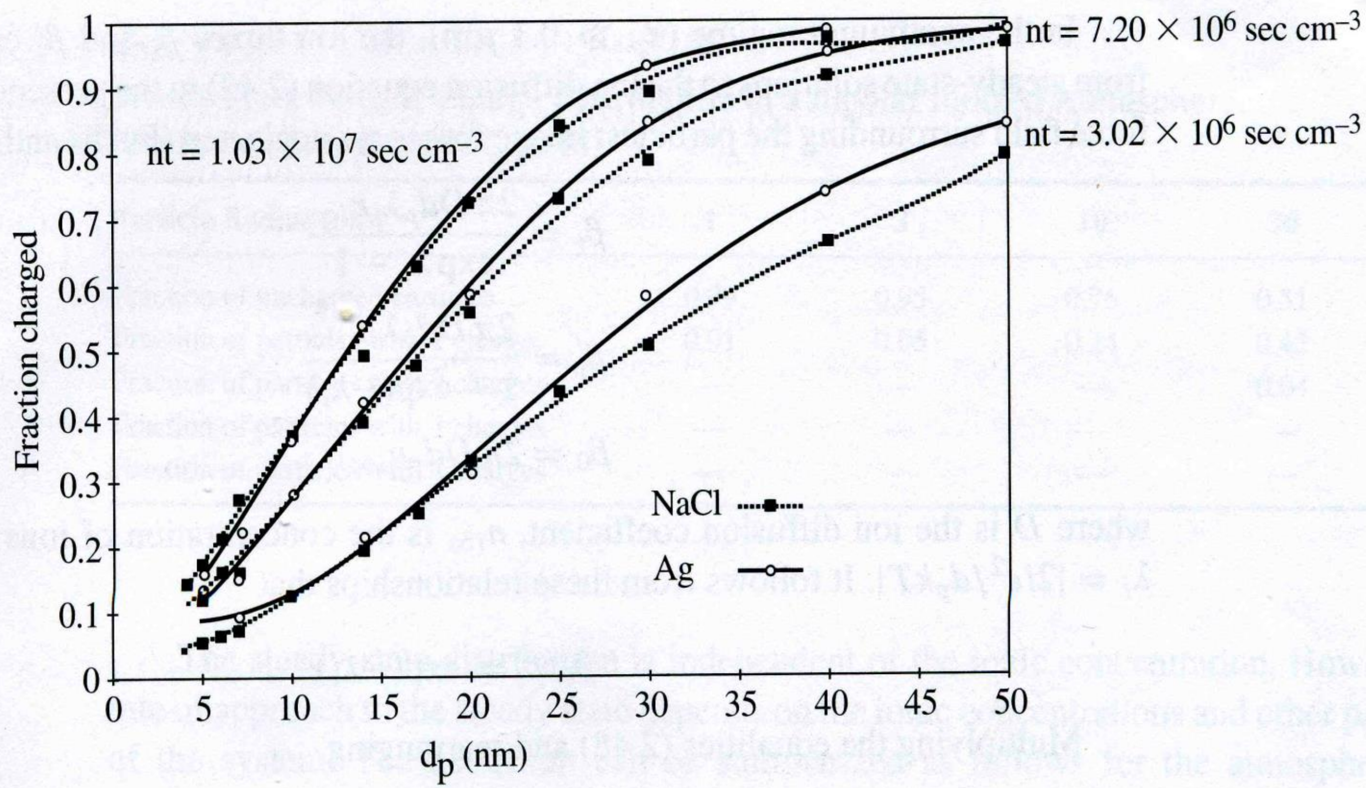


Figure 2.7 Fraction of particles with unit charge after exposure to unipolar air ion sources for various *nt* products. (Based on Pui et al., 1988.)

The calculated number of charges per particle decreases to a value less than unity (physically unacceptable). This means that only a fraction of such particles acquire unit charge.

(Friedlander, Smoke, dust and haze, 2nd Ed.)

- migration velocity of charged particle due to electric field

$$c_e = Z_p E$$

$$qE = fC_e = \frac{3\pi\mu d_p}{C_c} C_e$$

$$\therefore C_e = \frac{qEC_c}{3\pi\mu d_p} = Z_p E$$

$$Z_p = \frac{qC_c}{3\pi\mu d_p}$$

field charging $\quad q \propto d_p^2 \qquad C_c \to 1$

$$Z_p \sim d_p$$

diffusion charging $q \propto d_p$ $C_c \sim \frac{1}{d_p}$

$$Z \sim \frac{1}{d_p}$$

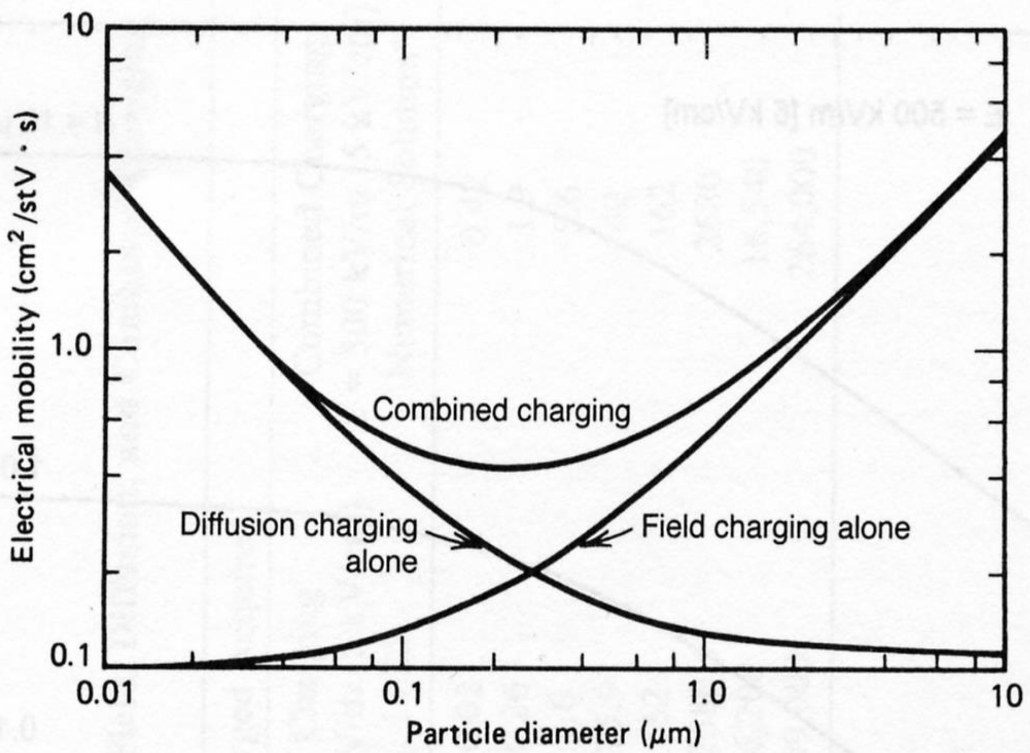


FIGURE 15.4 Electrical mobility versus particle size for diffusion, field, and combined charging at E = 500 kV/m [5 kV/cm] and $N_i t = 10^{13}$ s/m^3 [10^7 s/cm^3].

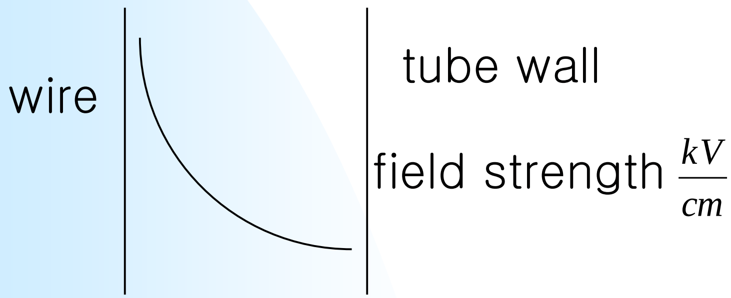
- Both field charging and diffusion charging need high concentration of unipolar ions. Lifetime of these ions is short due to high mobility and repulsion → they deposits on the surface and particles.
So, in order to keep the charging on the particles that come in the charging section, we need to continue to provide unipolar ions. How?

Ions can be created by radioactive discharge, UV radiation, flames and corona discharge.

-Corona discharge

--use non-uniform electrical field (a wire and a tube) (or a needle and a plate)

--Only corona discharge can produce unipolar ions at a high enough concentration to be useful for aerosol charging.



Even though air is insulator, air can become conductive for sufficiently high electric field

Breakdown electric field

$$E_b = 30 + 12.7 d_w^{-1/2} \ \ kV/cm$$

Breakdown can occur only near the wire where high E field exists locally.

(If the whole region is above E_b, then arc occurs direct current will flow).

This is why we need to use "non-uniform" electric field corona regime

(wire and tube) (can not use parallel plate)

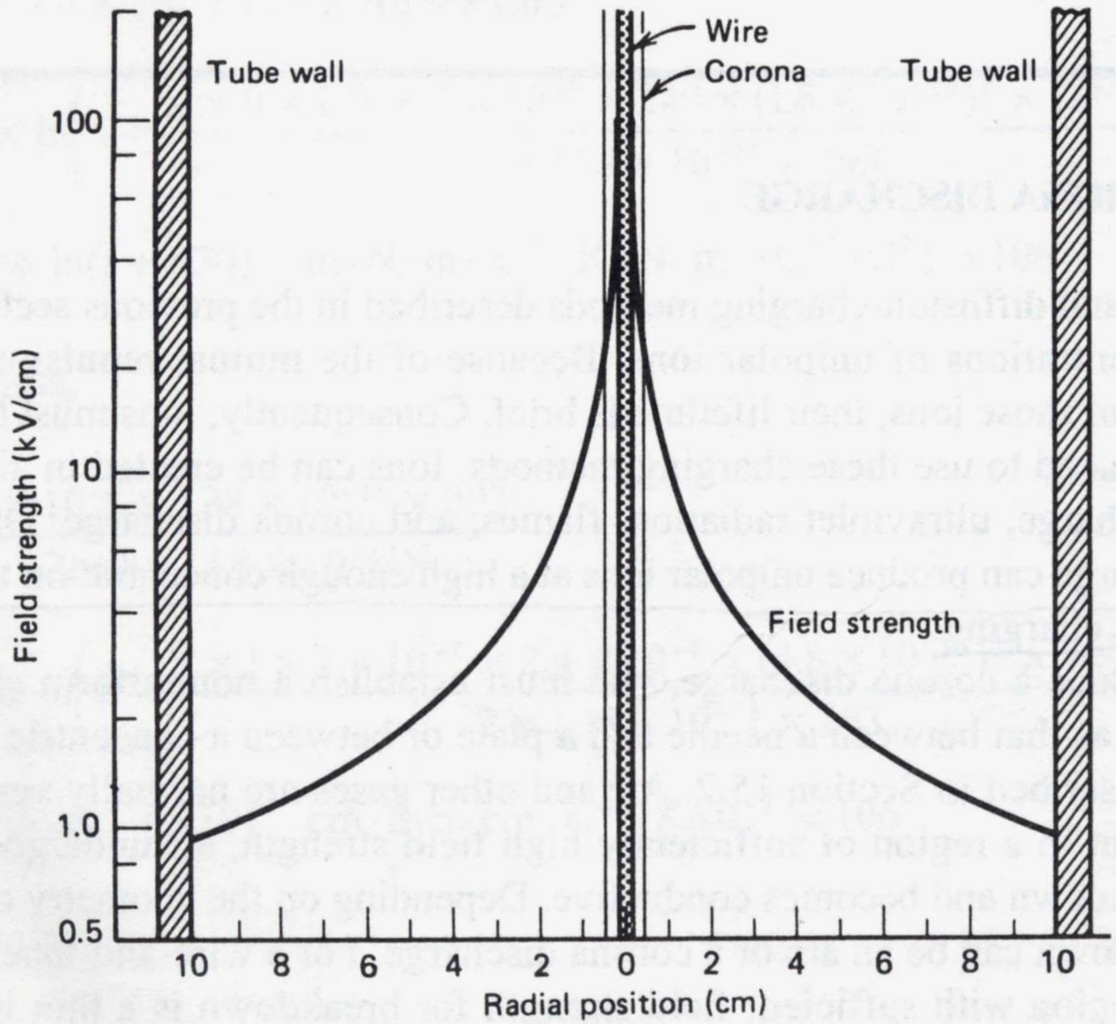
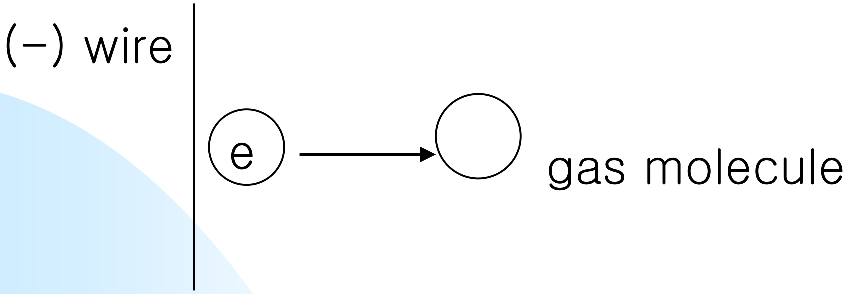
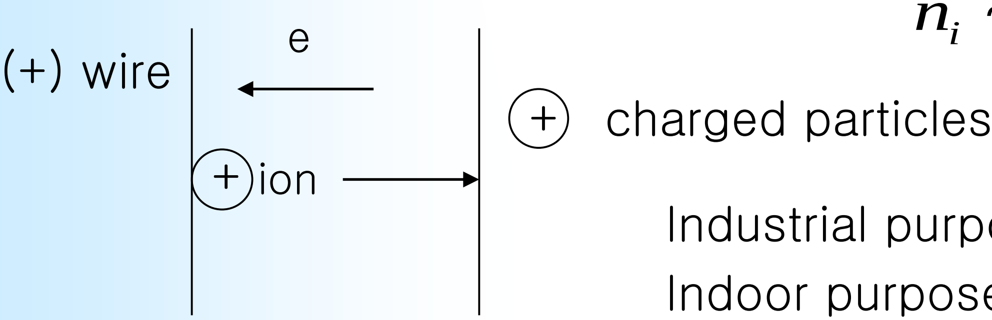


FIGURE 15.5 Field strength in a tube of diameter 0.2 m [20 cm] with a concentric wire 1 mm in diameter at 50 kV. Corona region not to scale.



electrons are accelerated and bombard gas molecules and cause gas molecules to eject electron and become positively charged ion, charged ions now are accelerated towards the wire and collide with gas molecules and produce their positive ions and electrons. In this way, a lot of electrons are generated and move towards the tube wall and decelerate and finally attach to gas molecules that become negative ions.
Negative ions move towards the tube surface.

$$n_i \sim 10^7 \sim 10^9 / cm^3$$



velocity ~70~80 m/sec

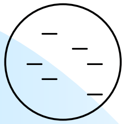
Industrial purpose: negative corona

Indoor purpose: positive corona

Positive and negative coronas have quite different properties and appearances. With positive corona, the entire region around the wire has a stable, glowing sheath with a characteristic bluish-green color. With negative corona, the corona glow exists in tufts or blushes that appear to be a dancing motion over the surface of the wire. These tufts may be several mm in length. There is sufficient energy in the corona region to produce ozone from oxygen. A negative corona produces about 10 times as much ozone as a positive corona.

The introduction of aerosol particles into the space bewteen the wire and the tube will result in field charging of the particles to the same polarity as the wire. If clean air is blown through the tube at high velocity, it will carry the unipolar ions out of the field region where they may be mixed with aerosol particles for diffusion charging.

- Charge Limit



negatively charged particle

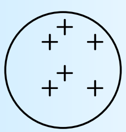
: the maximum charge is reached when they self-generated field at the surface of a particle reaches the value required for spontaneous emission of electrons

$$n_L = \frac{d_p^2 E_L}{4eK_E}$$

E_L : surface field strength for spontaneous emission of electrons

$$9.0 \times 10^8 V/m$$

positively charged particles



$$E_r \sim 2.1 \times 10^{10} V/m$$

-Rayleigh Limit for liquid droplets

When the mutual repulsion of electric charges within a droplet exceed the confinement force of surface tension, the droplet shatters into smaller droplets.

$$n_L = \frac{(2\pi\gamma d_p^3)^{1/2}}{(K_E e^2)^{1/2}}$$

ppt file for Fig. 15-6 (Hinds)

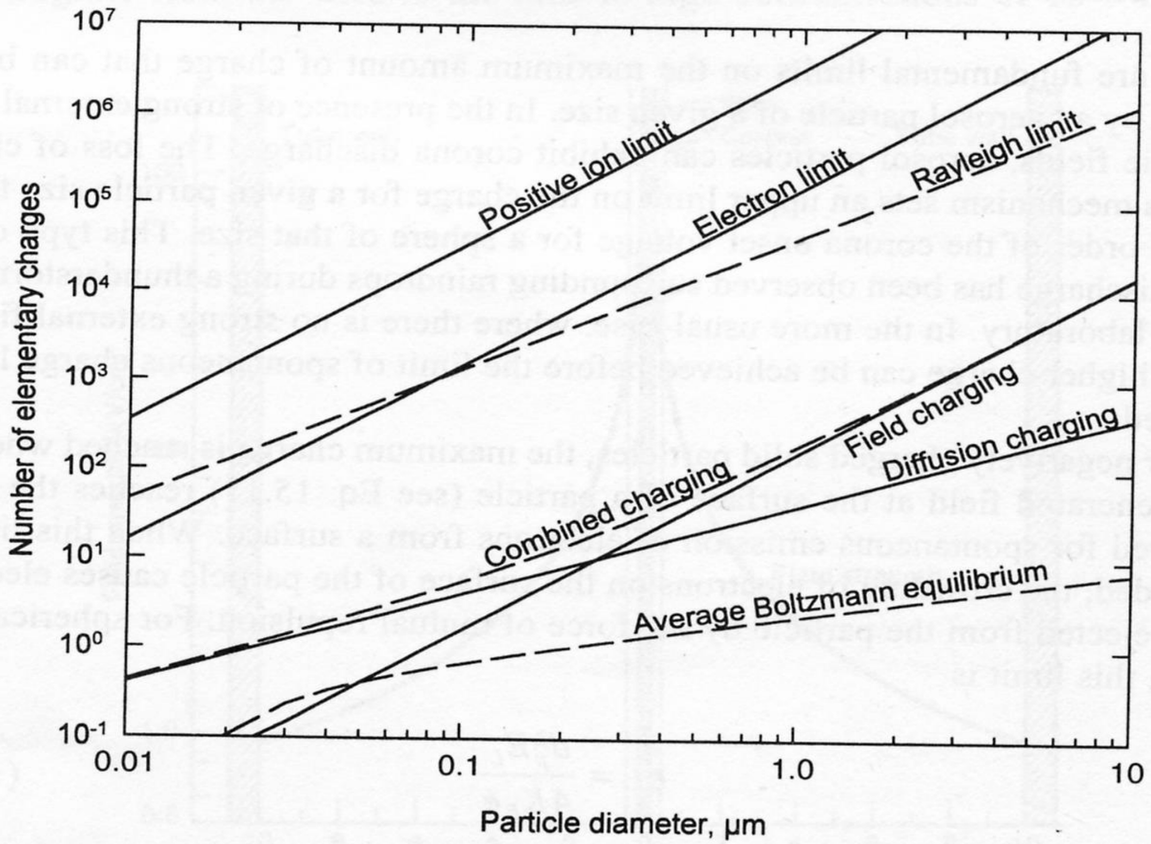


FIGURE 15.6 Particle charge limits. Rayleigh limit for water ($\gamma = 0.073$ N/m [73 dyn/cm]), diffusion charging at $N_i t = 10^{13}$ s/m^3 [10^7 s/cm^3] and field charging at $E = 500$ kV/m [5 kV/cm] and $N_i t = 10^{13}$ s/m^3 [10^7 s/cm^3]. $\varepsilon = 5.1$.