## PATTERN RECOGNITION

JIN YOUNG CHOI
ECE, SEOUL NATIONAL UNIVERSITY

## Notice

Lecture Notes: Slides
References:
Pattern Classification by Richard O. Duda, et al.
Pattern Recognition and Machine Learning by Cristopher M. Bishop

Assistant: 박슬기, 133(ASRI)-412, seulki.park@snu.ac.kr
Evaluation: Quiz 40\%, Midterm 30\%, Final 30\%, Video: Every week two videos are uploaded.

Video 1: upload: Sun. 09:00, Quiz Due: Tue. 24:00
Video 2: upload: Wed. 09:00, Quiz Due: Fri. 24:00
Class web: etl.snu.ac.kr
Office: 133(ASRI)-406, jychoi@snu.ac.kr

# INTRODUCTION TO AI: ARTIFICIAL INTELLIGENCE ML: MACHINE LEARNING DL: DEEP LEARNING 

JIN YOUNG CHOI
ECE, SEOUL NATIONAL UNIVERSITY

## Artificial Intelligence

## Filtering <br> Estimation <br> Optimization

## Bayesian Theory

Statistical Machine Learning

Data base
Search, Inference
Decision Tree
Symbolism
Cognitive science
Minsky

Backpropagation Rule
Deep Learning Neural Networks
Connectionism
Neuroscience
Rosenblatt

## Artificial Intelligence

Learning from Experience (Observations, Examples)
?

Inference(Reasoning) for a Question

## Artificial Intelligence

Learning from Experience (Observations, Examples)
If birds are given, then we can learn their features such as \# of legs, shape of mouth, etc. If cancer patients are given, then we can observe their symptoms via diagnosis

Inference(Reasoning) for a Question
If features of something are given, then we can recognize what is it.
If symptoms of a patient are given, then we can infer what is his decease.

## Artificial Intelligence

Learning from Experience (Observations, Examples)
If birds are given, then we can learn their features such as \# of legs, shape of mouth, etc. If cancer patients are given, then we can record their symptoms via diagnosis If $y=y_{1}$, then $x=x_{1}$
If $y=y_{2}$, then $x=x_{2}$

Inference(Reasoning) for a Question

| $y$ |  |  | $x$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ |  |
| $y_{2}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{26}$ | DB |
| Decision Tree |  |  |  |  |  |  |  |

If features of something are given, then we can recognize what is it.
If symptoms of a patient are given, then we can infer what is his decease.
If $x=x_{1}$, then $y=y_{1}$
If $x=x_{2}$, then $y=y_{2}$

| $y$ |  |  | $x$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ |
| $y_{2}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{26}$ |

## Artificial Intelligence

Learning from Experience (Observations, Examples)
If $y=y_{1}$, then $x=x_{1}$
If $y=y_{2}$, then $x=x_{2}$

$$
p\left(x=x_{i} \mid y=y_{j}\right), p\left(y=y_{j}\right)
$$

## Density Estimation



Inference(Reasoning) for a Question

$$
\begin{aligned}
& \text { If } x=x_{1} \text {, then } y=y_{1} \\
& \text { If } x=x_{2} \text {, then } y=y_{2} \\
& \qquad \begin{aligned}
p\left(y=y_{j} \mid x=x_{i}\right) & =\frac{\left(p\left(x=x_{i} \mid y=y_{j}\right) p\left(y=y_{j}\right)\right)}{p(x)} \\
p(x) & =\sum_{i} p\left(x=x_{i} \mid y=y_{j}\right) p\left(y=y_{j}\right)
\end{aligned}
\end{aligned}
$$



Bayesian Theory

## Artificial Intelligence

Deep Neural Networks for Learning and Inference

$$
o=f(W, x), \text { ex, } o_{j}=p\left(y=y_{j} \mid x=x_{i}\right)
$$

## Training (learning)

Find $W$ to minimize the errors between $o_{j}$ and $d_{j}$ for given training data $\left\{\left(x_{p}, l_{p}\right)\right\}$

Inference(Reasoning)
Calculate $o_{j}$ via the deep network

Connectionism

## Network Training

Inference (feedforward)


## Learning and Inference



## Learning and Inference



A general tree structure


Symbolism

## Learning and Inference



Bayesian Theory

## Learning and Inference



$$
g_{i}(\mathbf{x})=\mathbf{w}_{i}^{t} \mathbf{x}+w_{i 0}
$$

## Connectionism

Network Training
Inference (feedforward)


## Convolutional Neural Networks



## Supervised/Unsupervised Learning,



## Generative/Discriminative Model

- Generating images

$$
p\left(z_{1}, z_{2}, z_{3} \mid x\right)
$$

Generative approach:
Model $p(z, x)=p(x \mid z) p(z)$
Use Bayes' theorem

$$
p(z \mid x)=\frac{p(x \mid z) p(z)}{p(x)}
$$

Discriminative approach:
Model $p(z \mid x)$ directly

## Unsupervised Learning,

Clustering: K-means, etc.
Variational Auto-Encoder


## Statistical Learning

$L_{2}$ Loss

$$
L(d, f(W, x))=\|d-f(W, x)\|_{2}^{2}
$$

Total Loss

$$
\mathcal{L}(W)=\int L(d, f(W, x)) d p(x, d)
$$

where $p(x, d)$ is a joint PDF of $x$ and $d$, but unknown

Empirical Total Loss

$$
\mathcal{L}_{i}(W)=\frac{1}{N} \sum_{n=1}^{N} L\left(d_{n}, f\left(W, x_{n}\right)\right)
$$

## Statistical Learning

$I\left(x_{k}\right)=\log \left(\frac{1}{p_{k}}\right) \rightarrow\left\{\begin{array}{l}\text { base } 2 \rightarrow \text { bits } \\ \text { base } e \rightarrow \text { nats }\end{array}\right.$
32 bits $\rightarrow p_{k}=1 / 2^{32}$ for uniform distribution $\rightarrow I_{k}=32$ bits
(1) $I\left(x_{k}\right)=0$ for $p_{k}=1$
(2) $I\left(x_{k}\right) \geq 0$ for $0 \leq p_{k} \leq 1$
(3) $I\left(x_{k}\right) \geq I\left(x_{j}\right)$ for $p_{k} \leq p_{j}$

Entropy : a measure of the average amount of information conveyed per message, i.e., expectation of Information

$$
H(x)=E\left[I\left(x_{k}\right)\right]=\sum_{k} p_{k} I\left(x_{k}\right)=-\sum_{k} p_{k} \log p_{k}
$$

## Statistical Learning

Entropy becomes maximum when $p_{k}$ is equiprobable 32 bits $\rightarrow p_{k}=1 / 2^{32}$ for uniform distribution $\rightarrow I_{k}=32$ bits

$$
\begin{aligned}
& \rightarrow 0 \leq H(X) \leq-\sum_{k=1}^{2^{32}} \frac{1}{2^{32}} \log \frac{1}{2^{32}}=32 \\
& \rightarrow H(X)=0 \text { for an event } p_{k}=1 \text { and } p_{j \neq k}=0
\end{aligned}
$$

Cross Entropy Loss:

$$
\begin{aligned}
& \mathcal{L}(\mathrm{W})=-\sum_{k}^{K} t_{k} \log f_{k}(W, x) \\
& \quad f_{k}(W, x)=\frac{e^{a_{k}}}{\sum_{j} e^{a_{j}}}(\text { softmax })
\end{aligned}
$$

$t_{k}:$ target label (one hot: 0000100)


## Statistical Learning

Cross Entropy Loss:

$$
\begin{aligned}
& \mathcal{L}(\mathrm{W})=-\sum_{k}^{K}\left[t_{k} \log f_{k}(W, x)+\left(1-t_{k}\right) \log \left(1-f_{k}(W, x)\right)\right] \\
& \quad f_{k}(W, x)=\frac{1}{1+e^{-a_{k}}}(\text { sigmoid }) \\
& \left.t_{k}: \text { target label (multi-hot: } 00110100\right)
\end{aligned}
$$



## Statistical Learning

Theorem (Gray 1990)

$$
\sum_{k} p_{k} \log \frac{p_{k}}{q_{k}} \geq 0
$$

Relative entropy (or Kullback - Leibler divergence)

$$
\begin{aligned}
& D_{K L}(p \| q)=\sum_{k} p_{k} \log \frac{p_{k}}{q_{k}} \\
& D_{K L}(p \| q)=0 \text { for } p \equiv q
\end{aligned}
$$

$p_{k}$ probability mass function
$q_{k}$ reference probability mass function

## Scene and Object Generation



## $\mathcal{L}(\theta, \phi)=\mathcal{L}_{\text {ref }}+\mathcal{L}_{\text {pose }}+\mathcal{L}_{i d}$.

$$
\begin{aligned}
& \mathcal{L}_{\text {ref }}=\mathbb{E}_{q_{\phi}\left(z \mid x_{a}^{k}, \varphi_{a}^{k}\right)} {\left[\log p_{\theta}\left(x_{a}^{k} \mid z\right)\right] } \\
& \quad D_{K L}\left(q_{\phi}\left(z \mid x_{a}^{k}, \varphi_{a}^{k}\right) \| p_{\theta}(z)\right) . \\
& \mathcal{L}_{\text {pose }}=\mathbb{E}_{q_{\phi}\left(z|x| x_{a}^{k}, \varphi_{2}^{k}\right)} {\left[\log p_{\theta}\left(x_{a}^{l} \mid z\right)\right] } \\
& \quad D_{K L}\left(q_{\phi}\left(z \mid x_{a}^{k}, \varphi_{a}^{l}\right) \| p_{\theta}(z)\right) \\
& \quad- \lambda_{u} \cdot D_{K L}\left(q_{\phi}\left(u \mid x_{a}^{l}\right) \| q_{\phi}\left(u \mid x_{a}^{k}\right)\right) . \\
&\left.\mathcal{L}_{i d}=\mathbb{E}_{q_{\phi}\left(z \mid z x_{b}^{\prime}, \varphi_{a}^{k}\right)}\right)\left.\log p_{\theta}\left(x_{b}^{k^{\prime}} \mid z\right)\right] \\
& \quad D_{K L}\left(q_{\phi}\left(z \mid x_{b}^{k^{\prime}}, \varphi_{a}^{k}\right) \| p_{\theta}(z)\right) \\
& \quad-\lambda_{c} \cdot D_{K L}\left(q_{\phi}\left(c \mid \varphi_{b}^{k^{\prime}}\right) \| q_{\phi}\left(c \mid \varphi_{a}^{k}\right)\right)
\end{aligned}
$$

## Motion Retargeting



## Outline of ML techniques



## Course Outline

Intro. AI, ML, and DL
Intro. Linear Algebra
Intro. Prob. \& Information
Bayesian Decision Theory
Dim reduction PCA \& LDA
Learning Rules
Support Vector Machine
Deep Convolutional Networks
Bayesian Networks
Parametric pdf Estimation
Non-Parametric pdf Estimation

Boltzman Machine
Markov Chain Monte Carlo
Inference of Bayesian Net, MCMC
Inference of Bayesian Net, VI
Traffic Pattern Analysis, VI
Recent Papers

- Active Learning
- Imbalanced Data Learning
- Out of Distribution
- Weakly Supervised Learning
- Etc.


## Questions

1. Describe the common things and differences among symbolism, connectionism, and Bayesian approach.
2. Explain supervised/weekly-supervised/ unsupervised learning.
3. What is the difference between discriminative and generative model?
