Introduction to Photonics

The Propagation of Light

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: yoonchan@snu.ac.kr

Elementary Optical Phenomena & the Nature of Light

What is light?

- Rays of light: Isaac Newton
- Wave motion: Christiaan Huygens
- Electromagnetic waves: James Clerk Maxwell
  - $\rightarrow$ Propagation of light
- Quantum theory of light (Photons): Plank, Einstein, and Bohr
  - $\rightarrow$ Interaction of light and matter (absorption & emission)
- Quantum electrodynamics

Dual nature of light:

- Wave character: e.g., Interference
- Particle aspect: e.g., Photoelectric effect

Maxwell's Equations

Findings of 19th century:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's law

"Displacement current"

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Ampère's law

$$\nabla \cdot \mathbf{D} = \rho$$

Gauss's law

$$\nabla \cdot \mathbf{B} = 0$$

Constitutive relations:

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$ $\rightarrow$ Permittivity

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$ $\rightarrow$ Permeability

The two divergence equations can be derived from the two curl equations! $\rightarrow$ *12 unknowns & 12 equations!*

Electromagnetic Boundary Conditions

Continuity conditions:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \;\rightarrow\; \oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \cancel{\frac{\partial \mathbf{B}}{\partial t}} \cdot d\mathbf{s}$$

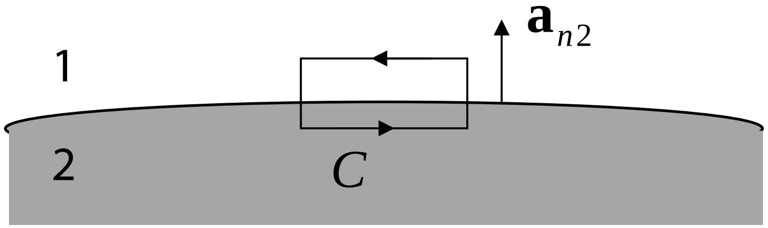
$$\rightarrow \boxed{E_{1t} = E_{2t} \quad \text{(V/m)}}$$

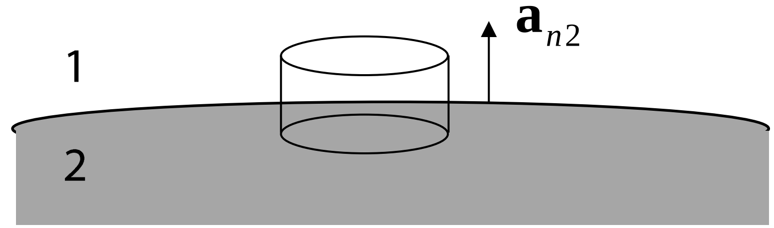
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \;\rightarrow\; \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \cancel{\frac{\partial \mathbf{D}}{\partial t}} \right) \cdot d\mathbf{s}$$

$$\rightarrow \boxed{\mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s \quad \text{(A/m)}}$$

$$\nabla \cdot \mathbf{D} = \rho \;\rightarrow\; \boxed{\mathbf{a}_{n2} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \quad (\text{C/m}^2)}$$

$$\nabla \cdot \mathbf{B} = 0 \;\rightarrow\; \boxed{B_{1n} = B_{2n} \quad \text{(T)}}$$





Electromagnetic Waves

Wave equations:

$$\nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad \nabla^2 \mathbf{H} - \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$ (Homogeneous and no source)

$$e.g.\ \ f(x,t) = f(x - \delta x, t - \delta t)$$

Plane waves:

$$\psi = e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}, \quad |\mathbf{k}| = \omega\sqrt{\mu\varepsilon}$$

Phase velocity:

$$\omega t - \mathbf{k}\cdot\mathbf{r} = \text{constant}, \quad u = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}},$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 299{,}792{,}456.2 \pm 1.1\ \text{m/s}$$

Speed of Light in a Medium

Relative permittivity: $K = \dfrac{\varepsilon}{\varepsilon_0}$

Relative permeability: $K_m = \dfrac{\mu}{\mu_0}$

Speed of light:

$$u = (\mu\varepsilon)^{-1/2} = (K_m\mu_0 K\varepsilon_0)^{-1/2} = c(KK_m)^{-1/2}$$

Index of refraction:

$$n \equiv \frac{c}{u} = (KK_m)^{1/2} \quad \rightarrow \quad n = \sqrt{K}$$ ← Nonmagnetic media

Table 1.2. INDEX OF REFRACTION VERSUS THE SQUARE ROOT OF THE STATIC PERMITTIVITY [14]

Substance	*n (Yellow Light)*	$\sqrt{K}$
Air (1 atm)	1.0002926	1.000295
CO_2 (1 atm)	1.00045	1.0005
Polystyrene	1.59	1.60
Glass*	1.5–1.7	2.0–3.0
Fused quartz	1.46	1.94
Water	1.33	9.0
Ethyl alcohol	1.36	5.0

G. R. Fowles, Introduction to Modern Optics, 1975.

← Static polarizability & dispersion

Plane Harmonic Waves (1)

General scalar wave equation:

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}=\frac{1}{u}\frac{\partial^2 U}{\partial t^2}$$

Waves in one dimension:

$$\frac{\partial^2 U}{\partial z^2}=\frac{1}{u}\frac{\partial^2 U}{\partial t^2} \quad \rightarrow U(z,t)=U_0\cos(kz-\omega t) \quad \leftarrow \frac{\omega}{k}=u$$

"Plane harmonic wave"

Phase velocity: $\Delta z = u\Delta t$

Angular frequency & angular wavenumber: ω, k

Wavelength: λ ← Distance of wave for one complete cycle

Spectroscopic wavenumber: σ ← Reciprocal of the wavelength

Period: T ← Time for one complete cycle

Frequency: ν ← Number of cycles per unit time

$$\lambda = uT = \frac{2\pi}{k}=\frac{1}{\sigma}$$

$$\nu=\frac{u}{\lambda}=\frac{\omega}{2\pi}=\frac{1}{T}$$

Plane Harmonic Waves (2)

Waves in three dimension:

$$U(x, y, z, t) = U_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$

$$\leftarrow \quad \mathbf{r} = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z$$

$$\leftarrow \quad \mathbf{k} = \hat{\mathbf{i}}k_x + \hat{\mathbf{j}}k_y + \hat{\mathbf{k}}k_z$$

$$|\mathbf{k}| = k = (k_x^2 + k_y^2 + k_z^2)^{1/2}$$

Surface of constant phase:

$$\mathbf{k} \cdot \mathbf{r} - \omega t = k_x x + k_y y + k_z z - \omega t = constant$$

$\rightarrow$ **k**: normal to the wave surfaces

$$\rightarrow \quad u = \frac{\omega}{k} = \frac{\omega}{\sqrt{k_x^2 + k_y^2 + k_z^2}}$$

Complex wave function:

$$U = U_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

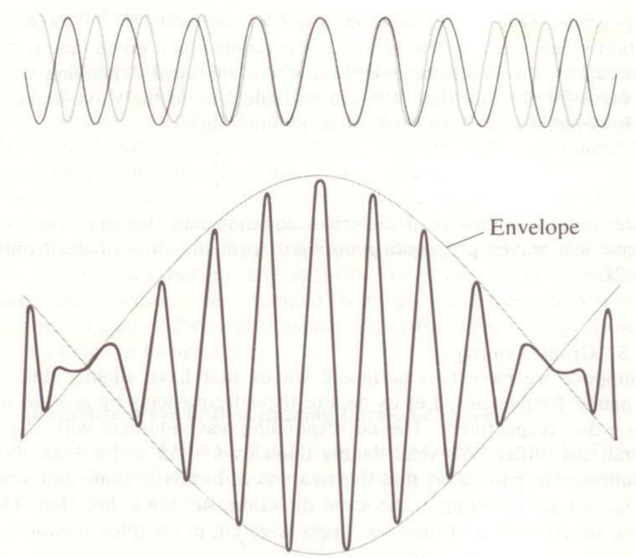
Group Velocity

Superposition of two harmonic waves

$$U = U_0 e^{i[(k+\Delta k)z-(\omega+\Delta\omega)t]} + U_0 e^{i[(k-\Delta k)z-(\omega-\Delta\omega)t]}$$

$$\rightarrow\ U = U_0 e^{i(kz-\omega t)}[e^{i(z\Delta k - t\Delta\omega)} + e^{-i(z\Delta k - t\Delta\omega)}]$$

$$\rightarrow\ U = 2U_0 e^{i(kz-\omega t)}\cos(z\Delta k - t\Delta\omega)$$



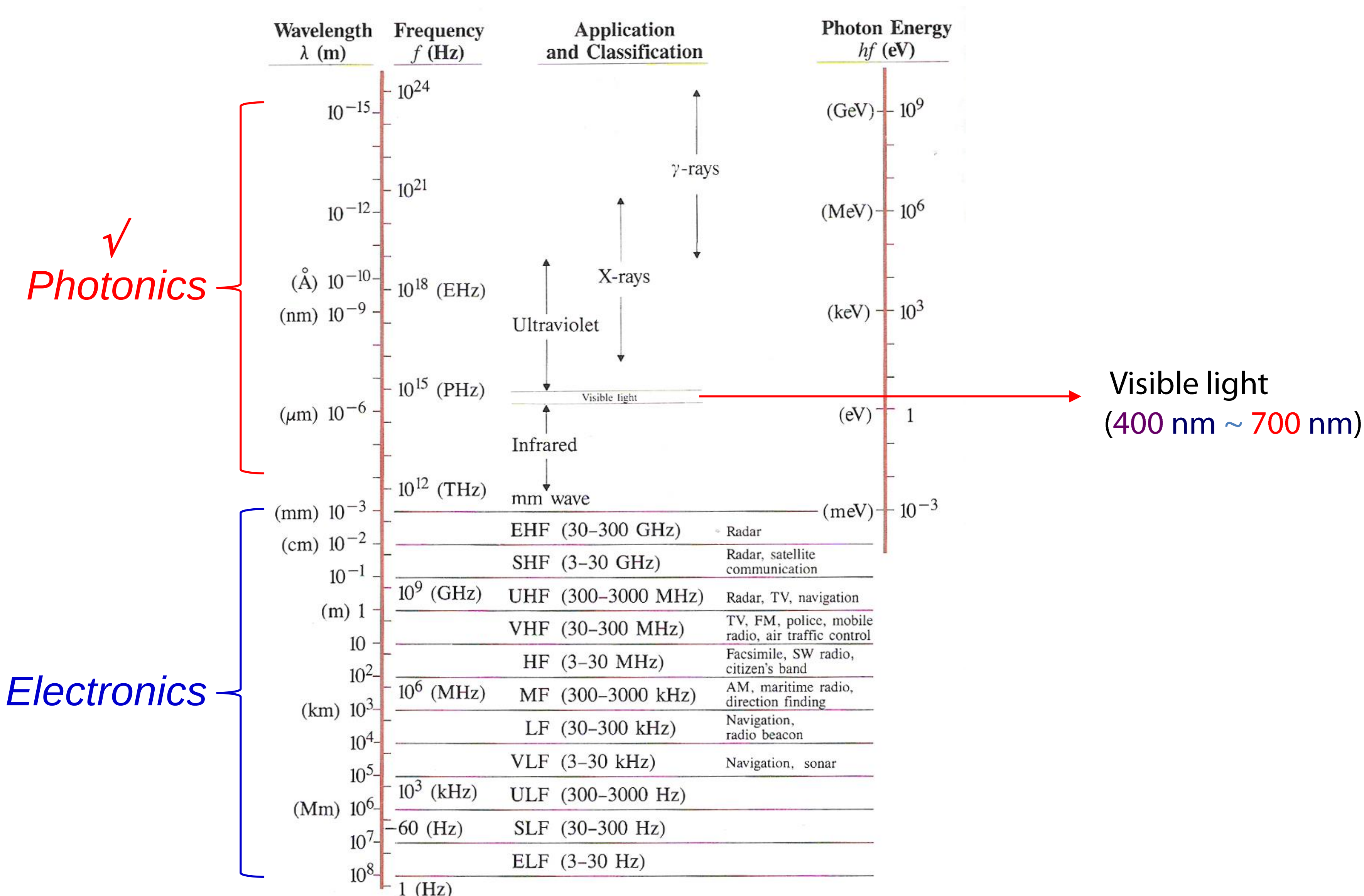
G. R. Fowles, Introduction to Modern Optics, 1975.

Group velocity:

$$u_g = \frac{\Delta\omega}{\Delta k} \quad \rightarrow\ u_g = \frac{d\omega}{dk} = u\left(1 - \frac{k}{n}\frac{dn}{dk}\right)$$

H.W.#1: Derive eqs. (1.36) and (1.37).

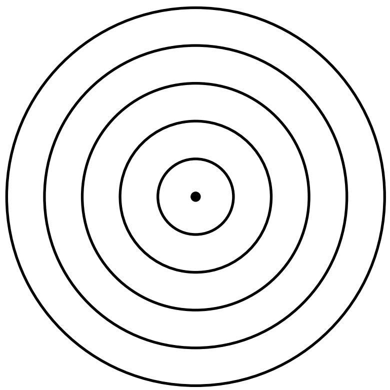
Electromagnetic Spectrum



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Doppler Effect

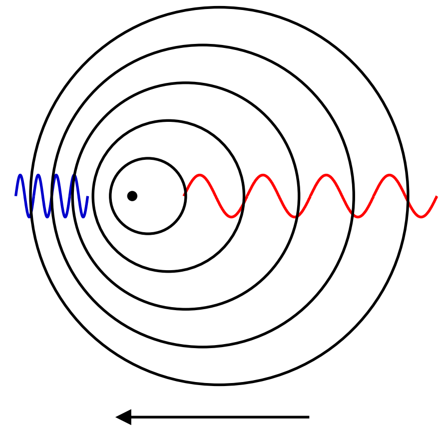
Stationary source:



$v = 0$

Moving source:

Blue-shift



Red-shift

$v \neq 0$

Recall: Speed of light

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}792{,}456.2 \pm 1.1\,\mathrm{m/s}$$

$\leftarrow$ Is this frequency dependent?

$\rightarrow$ *Relative to ether?*

Incoming wave (blue-shifted): $c'_{in} = c + v$

Outgoing wave (red shifted): $c'_{out} = c - v$

Is this true?

Special & general relativity presented by A. Einstein!!

H.W.#2: Derive eq. (1.43).