Introduction to Photonics

Coherence and Interference (1)

Yoonchan Jeong
School of Electrical Engineering, Seoul National University
Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Principle of Linear Superposition

Linear superposition of fields:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$
$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$

$$\rightarrow \mathbf{E} = \mathbf{E}_{(1)} + \mathbf{E}_{(2)} + \mathbf{E}_{(3)} + \cdots$$

Consider two plane harmonic linearly polarized waves of the same freq.:

$$\mathbf{E}_{(1)} = \mathbf{E}_1 \exp[i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \phi_1)]$$
$$\mathbf{E}_{(2)} = \mathbf{E}_2 \exp[i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \phi_2)]$$

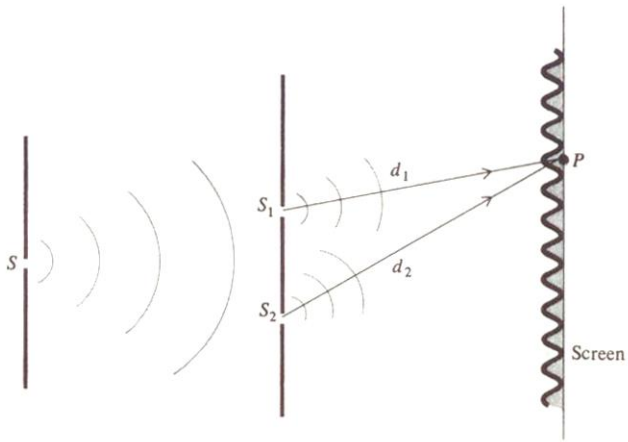
Irradiance at a point:

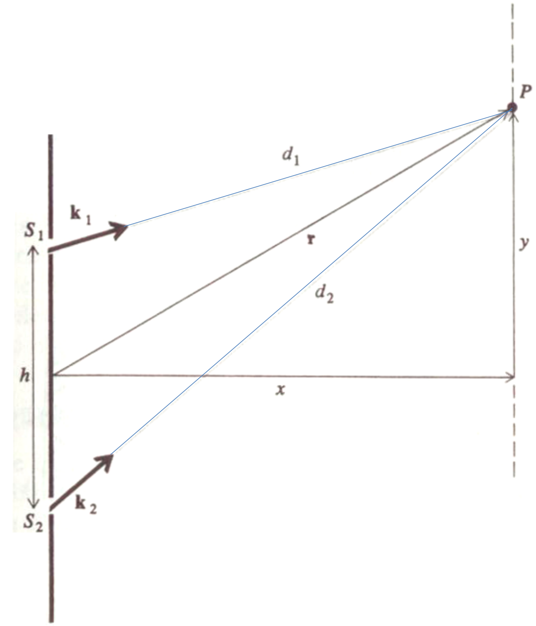
$$I = |\mathbf{E}|^2 = \mathbf{E} \cdot \mathbf{E}^* = (\mathbf{E}_{(1)} + \mathbf{E}_{(2)}) \cdot (\mathbf{E}^*_{(1)} + \mathbf{E}^*_{(2)})$$
$$= |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \cos\theta$$

Interference term

$$\leftarrow \theta = \mathbf{k}_1 \cdot \mathbf{r} - \mathbf{k}_2 \cdot \mathbf{r} + \phi_1 - \phi_2$$

Young's Experiment





G. R. Fowles, Introduction to Modern Optics, 1975.

Phase difference:

$$\theta = \mathbf{k}_1 \cdot \mathbf{r} - \mathbf{k}_2 \cdot \mathbf{r} + \phi_1 - \phi_2 = k(d_1 - d_2)$$

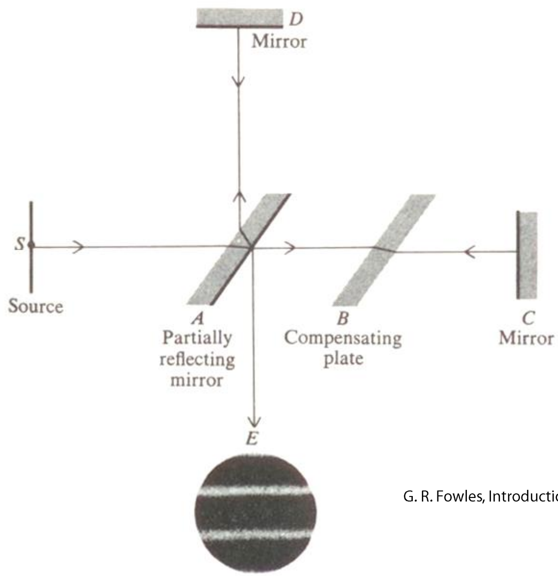
$$\rightarrow \ \theta = k(d_1 - d_2) = \pm 2m\pi$$ ← For constructive interference

$$\rightarrow \ |d_1 - d_2| = m\lambda$$

Fringe pattern:

$$\left[x^2 + \left(y + \frac{h}{2}\right)^2\right]^{1/2} - \left[x^2 + \left(y - \frac{h}{2}\right)^2\right]^{1/2} = m\lambda \quad \rightarrow \quad \frac{yh}{x} = m\lambda$$

Michelson Interferometer



G. R. Fowles, Introduction to Modern Optics, 1975.

Path length difference

Irradiance at a point:

$$I \propto 1+\cos\theta = 1+\cos kd = 1+\cos\frac{2\pi d}{\lambda} \leftarrow \Delta = \frac{d}{2}$$

Partial Coherence (1)

Irradiance as a time average:

$$I = \left\langle \mathbf{E} \cdot \mathbf{E}^* \right\rangle = \left\langle (\mathbf{E}_{(1)} + \mathbf{E}_{(2)}) \cdot (\mathbf{E}^*_{(1)} + \mathbf{E}^*_{(2)}) \right\rangle$$

$$= \left\langle \left|\mathbf{E}_1\right|^2 + \left|\mathbf{E}_2\right|^2 + 2\,\mathrm{Re}(\mathbf{E}_1 \cdot \mathbf{E}^*_2) \right\rangle$$

$$\leftarrow \left\langle f \right\rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)\, dt$$

$$\rightarrow I = I_1 + I_2 + 2\,\mathrm{Re}\left\langle E_1 E_2^* \right\rangle$$

Mutual coherence function or correlation function:

Time delay

$$\rightarrow 2\,\mathrm{Re}\,\Gamma_{12}(\tau) \quad \leftarrow \Gamma_{12}(\tau) = \left\langle E_1(t) E_2^*(t+\tau) \right\rangle$$

Self-coherence function or autocorrelation function:

$$\Gamma_{11}(\tau) = \left\langle E_1(t) E_1^*(t+\tau) \right\rangle$$

Degree of partial coherence:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}} = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}}$$

Partial Coherence (2)

Irradiance:

$$\rightarrow\ I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}\,\gamma_{12}(\tau)$$

$$|\gamma_{12}| = 1 \quad \leftarrow \text{Complete coherence}$$

$$0 < |\gamma_{12}| < 1 \quad \leftarrow \text{Partial coherence}$$

$$|\gamma_{12}| = 0 \quad \leftarrow \text{Complete incoherence}$$

Interference fringes:

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma_{12}|$$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}\,|\gamma_{12}|$$

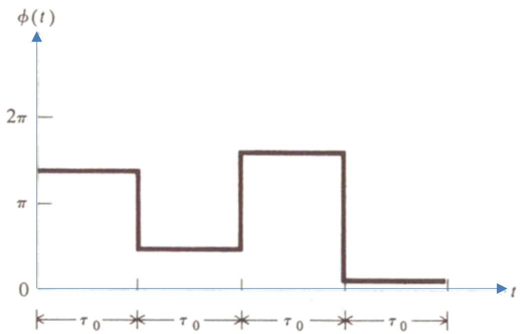
Fringe visibility:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I_1 I_2}\,|\gamma_{12}|}{I_1 + I_2}$$

$$\rightarrow\ V = |\gamma_{12}| \quad \leftarrow \quad I_1 = I_2$$

Coherence Time (1)

Hypothetical quasi-monochromatic source:



G. R. Fowles, Introduction to Modern Optics, 1975.

$$E(t) = E_0 e^{-i\omega t} e^{i\phi(t)}$$

$\rightarrow \ \tau_0$: Coherence time

Autocorrelation:

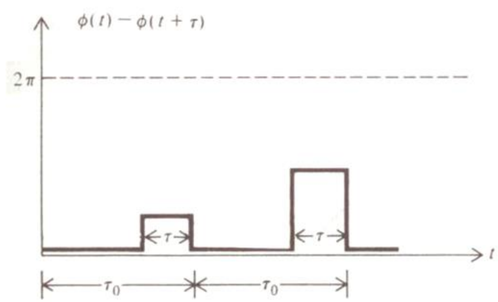
$$|E_1| = |E_2| = |E|$$

$$\rightarrow \ \gamma(\tau) = \frac{\langle E(t)E^*(t+\tau)\rangle}{\langle |E|^2 \rangle}$$

$$= \left\langle e^{i\omega\tau} e^{i[\phi(t)-\phi(t+\tau)]} \right\rangle \quad = e^{i\omega\tau} \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i[\phi(t)-\phi(t+\tau)]} dt$$

Coherence Time (2)

Self-coherence function:



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$$\frac{1}{\tau_0}\int_0^{\tau_0} e^{i[\phi(t)-\phi(t+\tau)]}dt$$

$$= \frac{1}{\tau_0}\int_0^{\tau_0-\tau} dt + \frac{1}{\tau_0}\int_{\tau_0-\tau}^{\tau_0} e^{i\Delta}dt$$

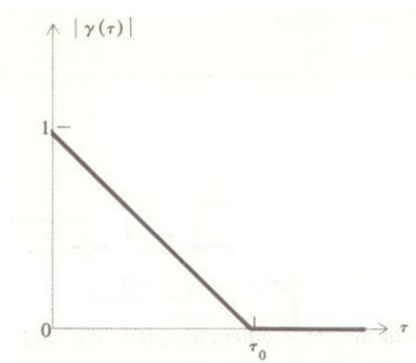
Random phase difference

$$= \frac{\tau_0-\tau}{\tau_0} + \frac{\tau}{\tau_0}e^{i\Delta}$$

$$\rightarrow \gamma(\tau) = \begin{cases} \left(1-\frac{\tau}{\tau_0}\right)e^{i\omega\tau} & \tau < \tau_0 \\ 0 & \tau \ge \tau_0 \end{cases}$$

$$\rightarrow |\gamma(\tau)| = \begin{cases} 1-\frac{\tau}{\tau_0} & \tau < \tau_0 \\ 0 & \tau \ge \tau_0 \end{cases}$$

$\leftarrow$ Visibility



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Coherence length:

$$l_c = c\tau_0$$