Introduction to Photonics

Multi-Beam Interference (1)

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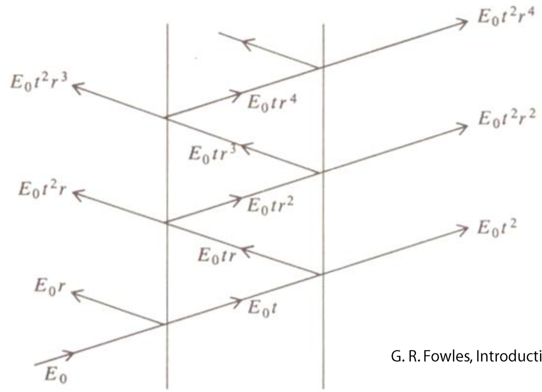
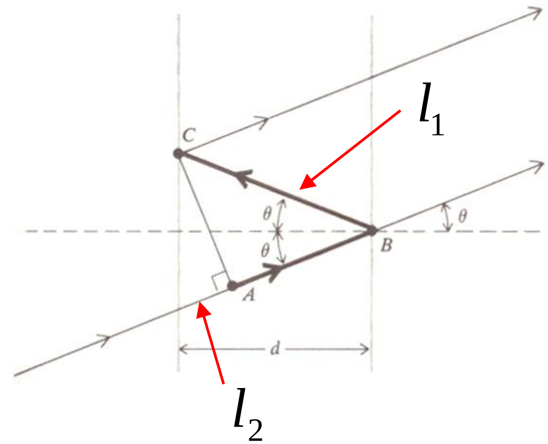
Interference with Multiple Beams (1)



G. R. Fowles, Introduction to Modern Optics, 1975.



Phase difference:

$$l_1 = \frac{d}{\cos\theta}, \quad l_2 = 2(l_1 \sin\theta)\sin\theta$$

$$\rightarrow \; \delta = k(2l_1 - l_2) = 2kd\cos\theta = \frac{4\pi}{\lambda} d\cos\theta \; = \frac{4\pi}{\lambda_0} nd\cos\theta$$

Transmission:

$$E_T = E_0t^2 + E_0t^2r^2e^{i\delta} + E_0t^2r^4e^{2i\delta} + \; \cdots$$

$$\rightarrow \; E_T = \frac{E_0t^2}{1-r^2e^{i\delta}} \qquad \rightarrow \; I_T = I_0 \frac{|t|^4}{|1-r^2e^{i\delta}|^2}$$

Interference with Multiple Beams (2)

Reflectance and transmittance:

$$r = |r| e^{i\delta_r/2}$$

$$\rightarrow \; R = |r|^2 = rr^*$$

$$\rightarrow \; T = |t|^2 = tt^*$$

$$\rightarrow \; I_T = I_0 \frac{T^2}{\left|1 - Re^{i\Delta}\right|^2} \quad \leftarrow \; \Delta = \delta + \delta_r$$

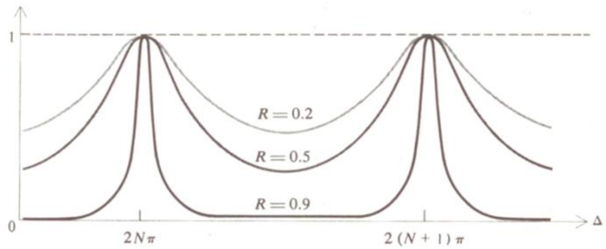
$$\rightarrow \; \left|1 - Re^{i\Delta}\right|^2 = (1 - Re^{i\Delta})(1 - Re^{-i\Delta}) = 1 - R(e^{i\Delta} + e^{-i\Delta}) + R^2$$

$$= 1 - 2R\cos\Delta + R^2$$

$$= (1-R)^2 \left[1 + \frac{4R}{(1-R)^2} \sin^2\frac{\Delta}{2}\right]$$

Coefficient of finesse:

$$\rightarrow \; I_T = I_0 \frac{T^2}{(1-R)^2} \frac{1}{1 + F\sin^2\frac{\Delta}{2}} \quad \leftarrow \text{Airy function}$$



G. R. Fowles, Introduction to Modern Optics, 1975.

$$F = \frac{4R}{(1-R)^2} \quad \leftarrow \text{Finesse}$$

Interference with Multiple Beams (3)

Condition for a fringe maximum:

$$\frac{\Delta}{2} = N\pi \;\rightarrow\; 2N\pi = \frac{4\pi}{\lambda_0} nd\cos\theta + \delta_r$$

Order of interference

Maximum and minimum transmission:

$$\mathscr{T}_{\max} = \frac{I_{T(\max)}}{I_0} = \frac{T^2}{(1-R)^2}$$

$$\mathscr{T}_{\min} = \frac{I_{T(\min)}}{I_0} = \frac{T^2}{(1+R)^2}$$

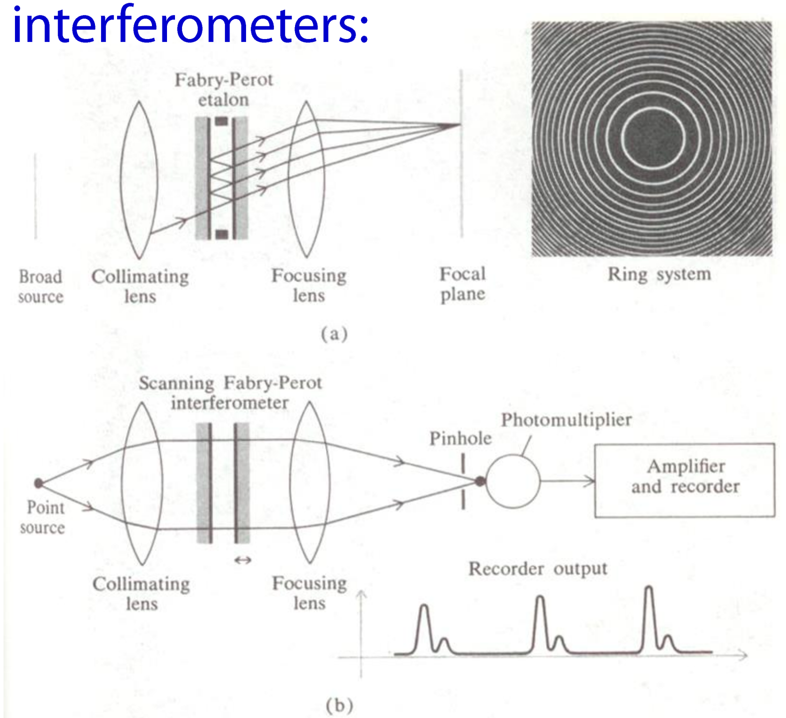
With absorption:

$$A + R + T = 1$$

$$\rightarrow\; \mathscr{T}_{\max} = \left(\frac{1-A-R}{1-R}\right)^2$$

Fabry-Perot Interferometer (1)

Arrangement for FP interferometers:



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Free spectral range:

"Separation between adjacent orders of interference"

$$\Delta_{N+1} - \Delta_N = 2\pi$$

$$\rightarrow \omega_{N+1} - \omega_N = \frac{\pi c}{nd\cos\theta}$$

$$\rightarrow \nu_{N+1} - \nu_N = \frac{c}{2nd\cos\theta} \approx \frac{c}{2nd} \quad (\theta << 1)$$

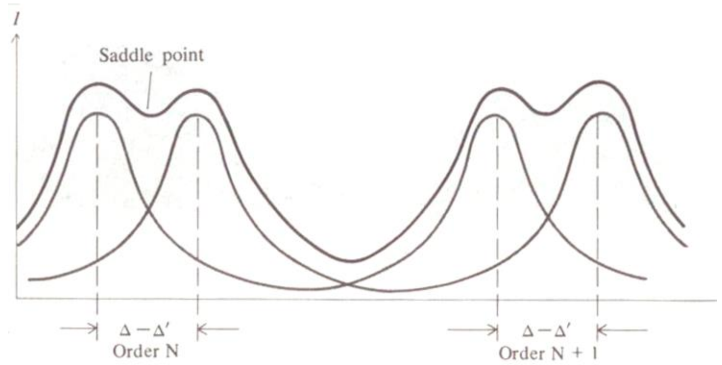
Fabry-Perot Interferometer (2)

Resolution of Fabry-Perot instruments:

$$I_T = I_0\left(1 + F\sin^2\frac{\Delta}{2}\right)^{-1} + I_0\left(1 + F\sin^2\frac{\Delta'}{2}\right)^{-1} \quad for \quad \omega \,\&\, \omega'$$

$$\leftarrow \ \Delta \approx \delta_r + 2kd = \delta_r + \frac{2\omega d}{c}$$

$$\leftarrow \ \Delta' \approx \delta_r + 2k'd = \delta_r + \frac{2\omega' d}{c}$$



G. R. Fowles, Introduction to Modern Optics, 1975.

Taylor criterion:

$$\rightarrow \ I = 2I_0\left[1 + F\sin^2\left(\frac{\Delta - \Delta'}{4}\right)\right]^{-1} = I_0 \quad \rightarrow F\sin^2\left(\frac{\Delta - \Delta'}{4}\right) = 1$$

$$\rightarrow |\Delta - \Delta'| \approx 4F^{-1/2} = 2\left(\frac{1-R}{\sqrt{R}}\right)$$

$$\rightarrow \delta\omega = |\omega - \omega'| \approx \frac{2c}{d}F^{-1/2} = \frac{c}{d}\left(\frac{1-R}{\sqrt{R}}\right)$$

Fabry-Perot Interferometer (3)

Reflecting finesse:

$$\mathscr{F} = \frac{\Delta_{N+1} - \Delta_N}{\left|\Delta - \Delta'\right|} = \frac{\pi}{2}\sqrt{F} = \pi\left(\frac{\sqrt{R}}{1-R}\right)$$

Resolving power:

$$\rightarrow \mathrm{RP} = \frac{\omega}{\delta\omega} = \frac{\nu}{\delta\nu} = \frac{\lambda}{\left|\delta\lambda\right|} \quad \leftarrow \delta\omega = \left|\omega - \omega'\right| \approx \frac{2c}{d}F^{-1/2}$$

$$\rightarrow \mathrm{RP} = N\mathscr{F} = N\pi\left(\frac{\sqrt{R}}{1-R}\right)$$

"Compromise between FSR and RP"