Introduction to Photonics

Diffraction (2)

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Fresnel Diffraction Patterns (1)

Fresnel Zones: $\leftarrow U_p = -\frac{ikU_0 e^{-i\omega t}}{4\pi} \iint \frac{e^{ik(r+r')}}{rr'} [\cos(\mathbf{n},\mathbf{r}) - \cos(\mathbf{n},\mathbf{r}')] d\mathscr{A}$

$$r + r' = (h^2 + R^2)^{1/2} + (h'^2 + R^2)^{1/2}$$

$$= h + h' + \frac{1}{2} R^2 \left(\frac{1}{h} + \frac{1}{h'} \right) + \cdots$$

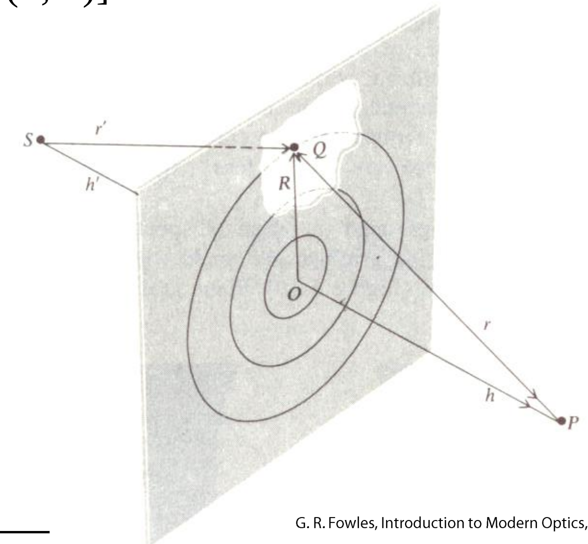
$$\rightarrow R = constant \rightarrow L = \left(\frac{1}{h} + \frac{1}{h'} \right)^{-1}$$

$$\rightarrow \tfrac{1}{2} R^2 / L = \tfrac{1}{2} n\lambda \quad (n = 1, 2, 3, \ldots)$$

$$\rightarrow R_1 = \sqrt{\lambda L}, R_2 = \sqrt{2\lambda L}, \ldots, R_n = \sqrt{n\lambda L}$$

$$\rightarrow \pi R_{n+1}^2 - \pi R_n^2 = \pi R_1^2$$

$\rightarrow$ The areas of the complete zones are all equal.



G. R. Fowles, Introduction to Modern Optics, 1975.

Optical disturbance at P:

$$|U_p| = |U_1| - |U_2| + |U_3| - \cdots$$

$\leftarrow$ Contributions from the various Fresnel zones

Consideration of the obliquity factor and the radial distance factor in the Fresnel-Kirchhoff formula:

$$\rightarrow |U_p| = \tfrac{1}{2}|U_1| + (\tfrac{1}{2}|U_1| - |U_2| + \tfrac{1}{2}|U_3|) + (\tfrac{1}{2}|U_3| - |U_4| + \tfrac{1}{2}|U_5|) + \cdots$$

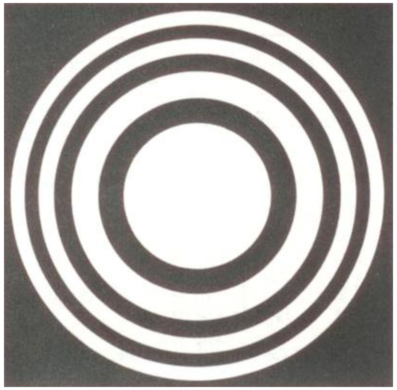
Fresnel Diffraction Patterns (2)

Zone plate:

$$|U_p| = |U_1| + |U_3| + |U_5| \cdots$$ $\leftarrow$ If the even-numbered zones are blocked

$\rightarrow$ This functions like a lens.

$$\rightarrow \; L = \frac{R_1^2}{\lambda}$$ $\leftarrow$ Equivalent focal length



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Rectangular aperture:

$$\rightarrow \; r + r' = h + h' + \frac{1}{2L}(x^2 + y^2)$$

Slowly varying

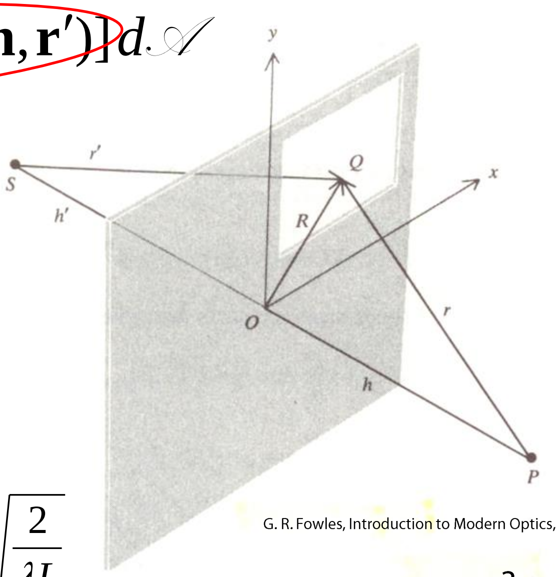
$$\rightarrow \; U_p = -\frac{ikU_0 e^{-i\omega t}}{4\pi} \iint \frac{e^{ik(r+r')}}{rr'} [\cos(\mathbf{n},\mathbf{r}) - \cos(\mathbf{n},\mathbf{r}')] d\mathscr{A}$$

$$\rightarrow \; U_p = C_1 \int_{x_1}^{x_2} \int_{y_1}^{y_2} e^{ik(x^2+y^2)/2L} dxdy$$

$$= C_1 \int_{x_1}^{x_2} e^{ikx^2/2L} dx \int_{y_1}^{y_2} e^{iky^2/2L} dy$$

$$= U_1 \int_{u_1}^{u_2} e^{i\pi u^2/2} du \int_{v_1}^{v_2} e^{i\pi v^2/2} dv$$

$$\leftarrow \; u = x\sqrt{\frac{k}{\pi L}} = x\sqrt{\frac{2}{\lambda L}}, \quad v = y\sqrt{\frac{k}{\pi L}} = y\sqrt{\frac{2}{\lambda L}}$$



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Fresnel Diffraction Patterns (3)

Fresnel integrals:

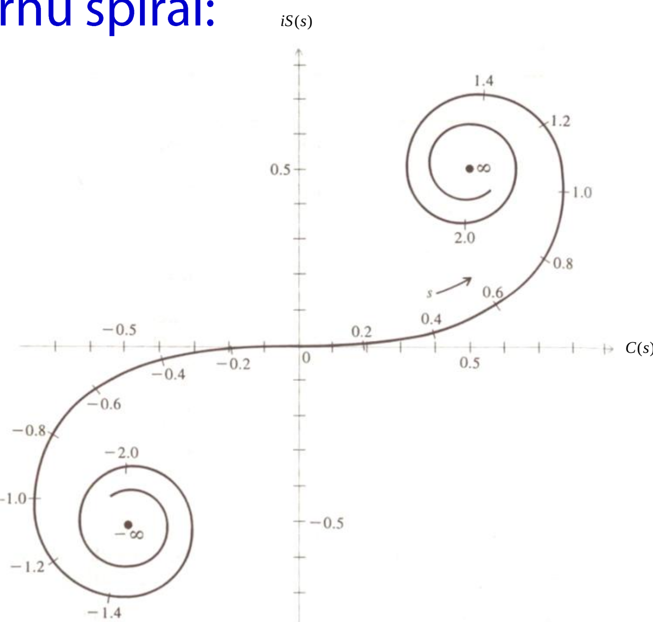
$$\int_0^s e^{i\pi w^2/2} dw = C(s) + iS(s) \quad \leftarrow \quad \begin{aligned} C(s) &= \int_0^s \cos(\pi w^2/2)\,dw \\ S(s) &= \int_0^s \sin(\pi w^2/2)\,dw \end{aligned}$$

Table 5.2. FRESNEL INTEGRALS

s	$C(s)$	$S(s)$
0.0	0.000	0.000
0.2	0.200	0.004
0.4	0.398	0.033
0.6	0.581	0.111
0.8	0.723	0.249
1.0	0.780	0.438
1.2	0.715	0.623
1.4	0.543	0.714
1.6	0.366	0.638
1.8	0.334	0.451
2.0	0.488	0.343
2.5	0.457	0.619
3.0	0.606	0.496
3.5	0.533	0.415
4.0	0.498	0.420
∞	0.500	0.500

→ H.W.

Cornu spiral:



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Normalized form:

$$\rightarrow U_p = \frac{U_0}{(1+i)^2}\left[C(u)+iS(u)\right]_{u_1}^{u_2}\left[C(v)+iS(v)\right]_{v_1}^{v_2}$$

Fresnel Diffraction Patterns (4)

Slit: $\rightarrow u_1 = -\infty, u_2 = +\infty$

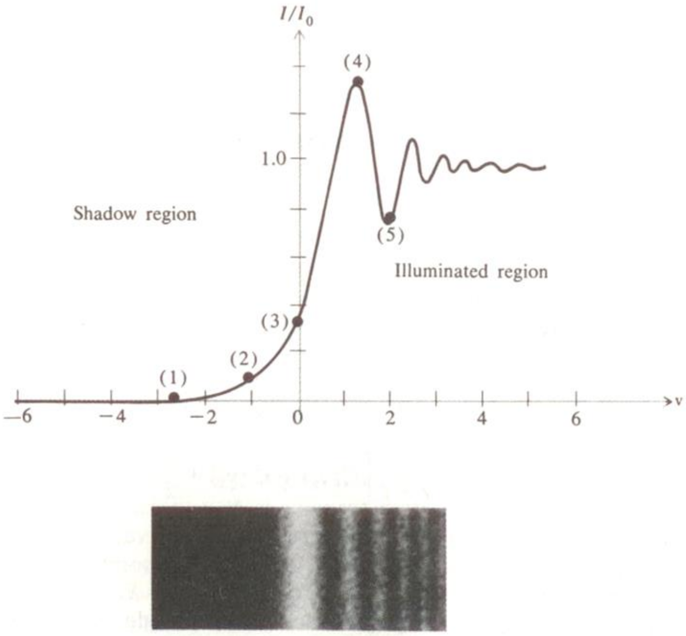
$$\rightarrow U_p = \frac{U_0}{1+i}[C(v) + iS(v)]_{v_1}^{v_2}$$

Straightedge:

$$\rightarrow u_1 = -\infty, u_2 = +\infty, v_1 = -\infty$$

$$\rightarrow U_p = \frac{U_0}{1+i}[C(v) + iS(v)]_{-\infty}^{v_2}$$

$$= \frac{U_0}{1+i}[C(v_2) + iS(v_2) + \frac{1}{2} + \frac{1}{2}i]$$



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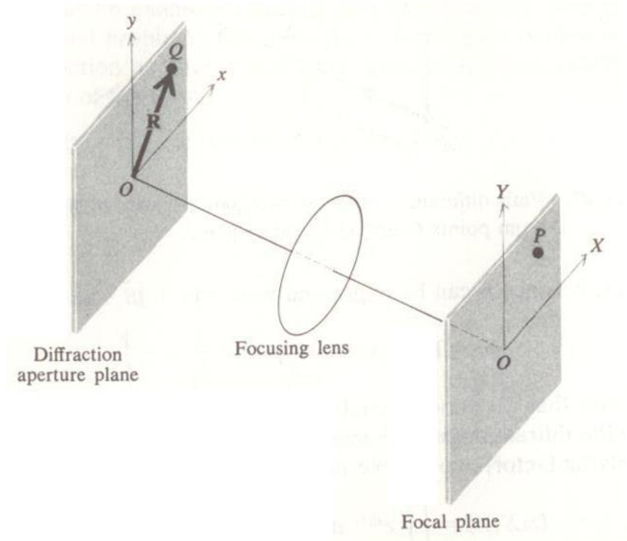
Applications of the FT to Diffraction (1)

Fraunhofer diffraction:

→ Diffracting aperture in the xy plane

→ Diffraction pattern in the XY plane



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All rays in a given direction:

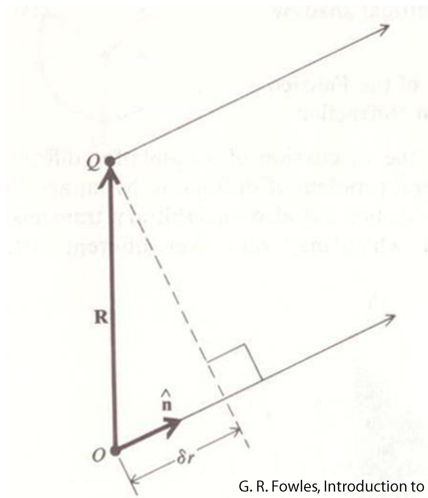
$$\rightarrow\ \hat{\mathbf{n}} = \hat{\mathbf{i}}\alpha + \hat{\mathbf{j}}\beta + \hat{\mathbf{k}}\gamma$$

$$\rightarrow\ X \approx L\alpha, Y = L\beta$$

$$\leftarrow\ \alpha \approx \tan\alpha,\ \beta \approx \tan\beta,\ \gamma \approx 1$$

Path difference:

$$\rightarrow\ \delta r = \mathbf{R}\cdot\hat{\mathbf{n}} = x\alpha + y\beta = x\frac{X}{L} + y\frac{Y}{L}$$



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For a uniform aperture:

$$\rightarrow\ U(X,Y) = \iint e^{ik\delta r}\,d\mathscr{A} = \iint e^{ik(xX+yY)/L}\,dxdy$$

For a nonuniform aperture:

$$\rightarrow\ U(X,Y) = \iint g(x,y)e^{ik(xX+yY)/L}\,dxdy$$

$$\rightarrow\ U(\mu,\nu) = \iint g(x,y)e^{i(\mu x+\nu y)}\,dxdy \ \leftarrow\ \mu = \frac{kX}{L}, \nu = \frac{kY}{L}$$

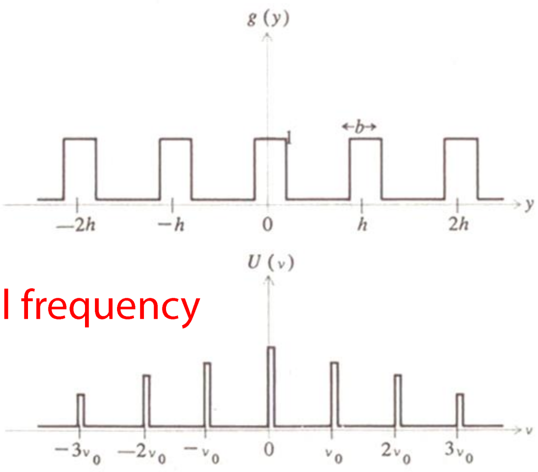
Applications of the FT to Diffraction (2)

One-dimensional grating:

$$g(y) = g_0 + g_1 \cos(\nu_0 y) + g_1 \cos(2\nu_0 y) + \cdots$$

$$\leftarrow \ \nu_0 = \frac{2\pi}{h}$$

→ Fundamental spatial frequency



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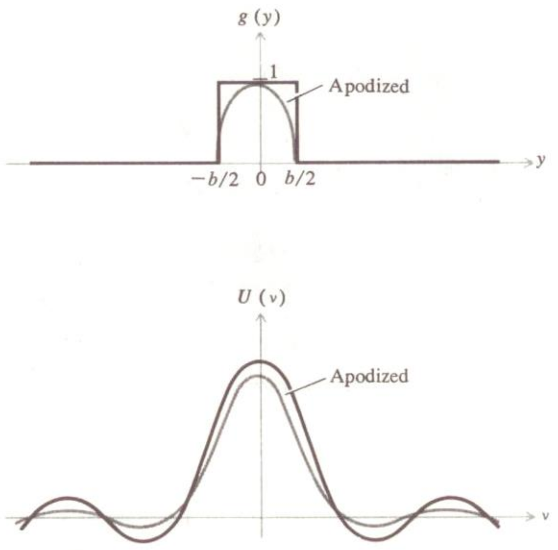
Apodization:



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$$\rightarrow U(\nu) = \int_{-b/2}^{+b/2} e^{i\nu y}\,dy = b\,\frac{\sin(\frac{1}{2}\nu b)}{\frac{1}{2}\nu b}$$

$$\rightarrow U(\nu) = \int_{-b/2}^{+b/2} \cos\left(\frac{\pi y}{b}\right) e^{i\nu y}\,dy$$

$$= \cos(\nu b/2)\left(\frac{1}{\nu - \pi/b} - \frac{1}{\nu + \pi/b}\right)$$

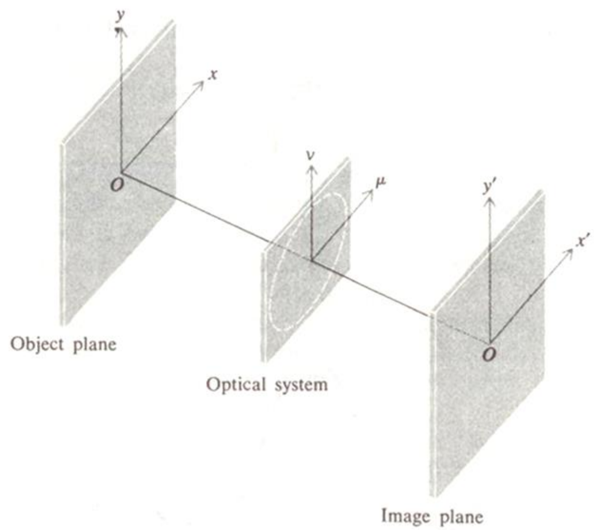
Applications of the FT to Diffraction (3)

Spatial filtering:

$$U'(\mu,\nu) = T(\mu,\nu)U(\mu,\nu)$$

Transfer function of the optical system

$$\rightarrow \ g'(x',y') = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} T(\mu,\nu)U(\mu,\nu)e^{i(\mu x'+\nu y')}d\mu d\nu$$



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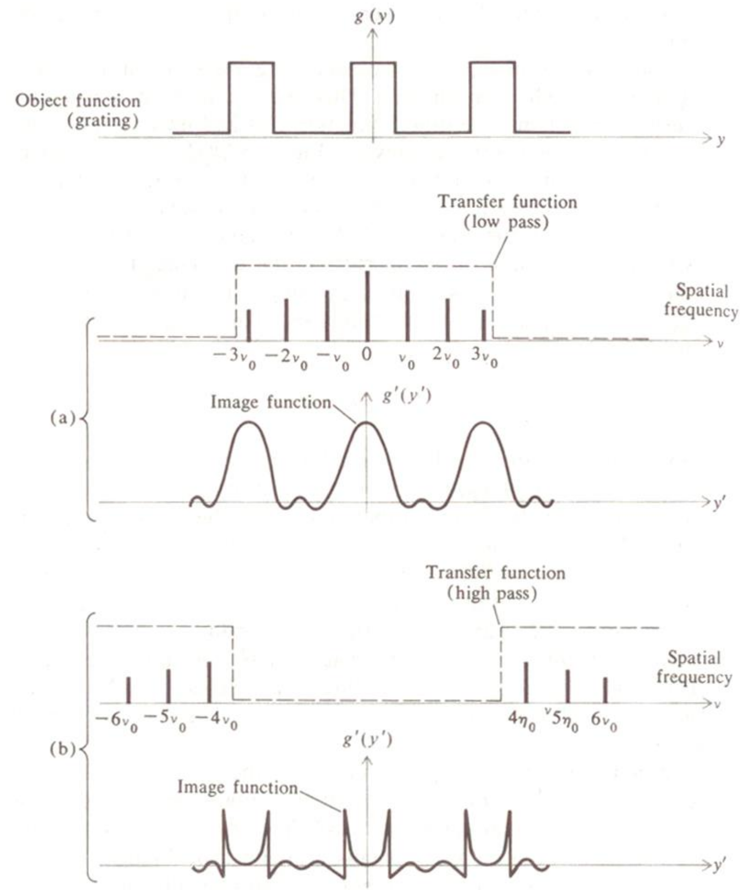


Applications of the FT to Diffraction (4)

Phase gratings:

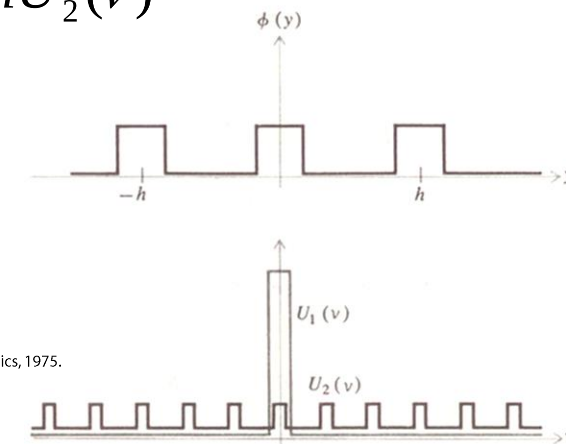
$$g(y) = e^{i\phi(y)}$$ ← Transparent film with index modulation

$$\rightarrow \quad g(y) = 1 + i\phi(y)$$

$$\rightarrow \quad U(\nu) = \int_{-\infty}^{+\infty} [1 + i\phi(y)] e^{i\nu y} dy = U_1(\nu) + iU_2(\nu)$$

$\pi/2$ out of phase

→ No interference



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Insertion of a phase plate ($\lambda/4$):

$$U_1(\nu) + iU_2(\nu) \;\rightarrow\; U_1(\nu) + U_2(\nu)$$

$$\rightarrow \quad g'(y') = \int U_1(\nu) e^{i\nu y'} d\nu + \int U_2(\nu) e^{i\nu y'} d\nu$$

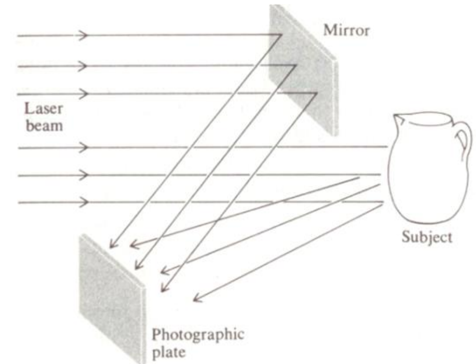
$$= g_1(y') + g_2(y')$$

→ Fringe pattern rendered from the phase grating

Reconstruction of the Wave Front by Diffraction

Wave-front reconstruction proposed by Gabor in 1947:

$\rightarrow$ Use of a special diffraction screen: Hologram



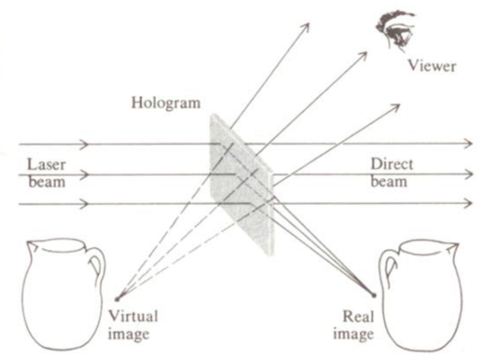
Object beam:

$$U(x,y) = a(x,y)e^{i\phi(x,y)}$$

Reference beam:

$$U_0(x,y) = a_0 e^{i(\mu x + \nu y)}$$

$$\leftarrow \quad \mu = k\sin\alpha,\ \mu = k\sin\beta$$



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Irradiance recorded by the photographic film:

$$\rightarrow \quad I(x,y) = |U + U_0|^2$$

$$= a^2 + a_0^2 + aa_0 e^{i[\phi(x,y) - \mu x - \nu y]} + aa_0 e^{-i[\phi(x,y) - \mu x - \nu y]}$$

$$= a^2 + a_0^2 + 2aa_0 \cos[\phi(x,y) - \mu x - \nu y]$$

Reconstructed beam:

$$\rightarrow \quad U_T(x,y) = U_0 I$$

Virtual image

$$= (a^2 + a_0^2)U_0 + a_0^2 U + U^* U_0^2$$

$\leftarrow$ Real image