Introduction to Photonics

Optics of Solids (1)

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Macroscopic Fields and Maxwell's Equations

Maxwell's equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

Constitutive relations:

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad \leftarrow \quad \mathbf{P} = (\varepsilon - \varepsilon_0)\mathbf{E} = \chi \varepsilon_0 \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$

Electric susceptibility

Current equation (Ohm's law):

$$\mathbf{J} = \sigma \mathbf{E}$$

General Wave Equation

For nonmagnetic & electrically neutral media:

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t} + \mathbf{J}$$

$$\nabla \cdot \mathbf{E} = -\frac{1}{\varepsilon_0} \nabla \cdot \mathbf{P}$$

$$\nabla \cdot \mathbf{H} = 0$$

General wave equation:

Ohmic loss and metallic nature

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{J}}{\partial t}$$

Material characteristics:
Dispersion, absorption, birefringence, optical activity, etc.

Propagation of Light in Isotropic Dielectrics (1)

Macroscopic polarization:

$$\mathbf{P} = -Ne\mathbf{r}$$

For a static electric field:

$$-e\mathbf{E} = K\mathbf{r}$$ ← Restoring force for "bound" charges

Static polarization:

$$\rightarrow \mathbf{P} = \frac{Ne^2}{K}\mathbf{E}$$

Equation of motion:

$$m\frac{d^2\mathbf{r}}{\partial t^2} + m\gamma\frac{\partial \mathbf{r}}{\partial t} + K\mathbf{r} = -e\mathbf{E}$$

↑ Frictional damping force

Effective resonance frequency:

For a monochromatic electric field: $\rightarrow E \sim e^{-i\omega t}$

$$\leftarrow \omega_0 = \sqrt{\frac{K}{m}}$$

$$\rightarrow \mathbf{P} = \frac{Ne^2}{-m\omega^2 - i\omega m\gamma + K}\mathbf{E} = \frac{Ne^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega}\mathbf{E}$$

Propagation of Light in Isotropic Dielectrics (2)

Wave equation for linear dielectrics:

$$\nabla\times(\nabla\times\mathbf{E})+\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=-\mu_0\frac{\partial^2\mathbf{P}}{\partial t^2}-\cancel{\mu_0\frac{\partial\mathbf{J}}{\partial t}}$$

$$\cancel{\nabla(\nabla\cdot\mathbf{E})}-\nabla^2\mathbf{E}$$

$$\rightarrow\ \nabla^2\mathbf{E}=\frac{1}{c^2}\left(1+\frac{Ne^2}{m\varepsilon_0}\cdot\frac{1}{\omega_0^2-\omega^2-i\gamma\omega}\right)\frac{\partial^2\mathbf{E}}{\partial t^2}$$

Homogeneous plane harmonic wave solution:

$$\rightarrow\ \mathbf{E}=\mathbf{E}_0e^{i(\mathscr{K}z-\omega t)}$$

Dispersion relation:

$$\rightarrow\ \mathscr{K}^2=\frac{\omega^2}{c^2}\left(1+\frac{Ne^2}{m\varepsilon_0}\cdot\frac{1}{\omega_0^2-\omega^2-i\gamma\omega}\right)$$

$$\rightarrow\ \mathscr{K}=k+i\alpha=\frac{\omega}{c}\mathscr{N}$$

$$\leftarrow\ \mathscr{N}=n+i\kappa\ \leftarrow\ \alpha=\frac{\omega}{c}\kappa$$

$$\rightarrow\ \mathbf{E}=\mathbf{E}_0e^{-\alpha z}e^{i(kz-\omega t)}$$

Propagation of Light in Isotropic Dielectrics (3)

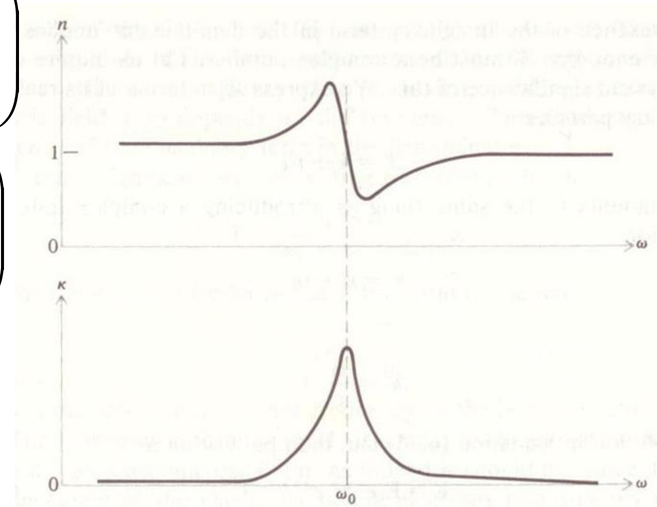
Complex index of refraction:

$$\mathcal{N}^2 = (n + i\kappa)^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)$$

$$\rightarrow\ n^2 - \kappa^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\left(\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right)$$

$$\rightarrow\ 2n\kappa = \frac{Ne^2}{m\varepsilon_0}\left(\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right)$$

$\rightarrow$ Kramers-Kronig relations (H.W.)



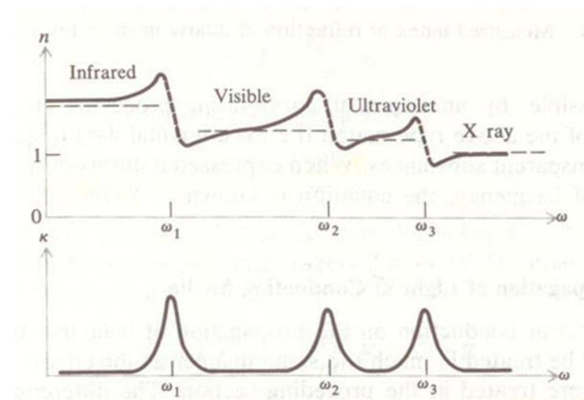
G. R. Fowles, Introduction to Modern Optics, 1975.

Complex index of refraction in general:

$$\rightarrow\ \mathcal{N}^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\sum_j\left(\frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}\right)$$

$$\rightarrow\ n^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\sum_j\left(\frac{f_j}{\omega_j^2 - \omega^2}\right)$$

$\rightarrow$ *c.f.* Sellmeier's formula



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Propagation of Light in Conducting Media (1)

Equation of motion:

$$m\frac{d\mathbf{v}}{\partial t}+m\tau^{-1}\mathbf{v}=-e\mathbf{E} \qquad \leftarrow \text{No restoring force}$$

$$\leftarrow \quad \mathbf{J}=-Ne\mathbf{v}$$

$$\rightarrow \quad \frac{d\mathbf{J}}{\partial t}+\tau^{-1}\mathbf{J}=\frac{Ne^2}{m}\mathbf{E}$$

For a static electric field:

$$\rightarrow \mathbf{J}=\frac{Ne^2}{m}\tau\,\mathbf{E}=\sigma\mathbf{E}$$

For a monochromatic electric field:

$$\rightarrow \mathbf{J}=\frac{\sigma}{1-i\omega\tau}\mathbf{E}$$

Wave equation for linear conducting media:

$$\nabla\times(\nabla\times\mathbf{E})+\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=-\mu_0\cancel{\frac{\partial^2\mathbf{P}}{\partial t^2}}-\mu_0\frac{\partial\mathbf{J}}{\partial t}$$

$$\rightarrow \quad \nabla^2\mathbf{E}=\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{\mu_0\sigma}{1-i\omega\tau}\frac{\partial\mathbf{E}}{\partial t}$$

Propagation of Light in Conducting Media (2)

Homogeneous plane harmonic wave solution:

$$\rightarrow\ \mathbf{E} = \mathbf{E}_0 e^{i(\mathscr{K} z - \omega t)}$$

Dispersion relation:

$$\rightarrow\ \mathscr{K}^2 = \frac{\omega^2}{c^2} + \frac{i\omega\mu_0\sigma}{1 - i\omega\tau}$$

For very low frequencies:

$$\rightarrow\ \mathscr{K}^2 \approx i\omega\mu_0\sigma$$

$$\rightarrow\ \mathscr{K} \approx \sqrt{i\omega\mu_0\sigma} = (1+i)\sqrt{\omega\mu_0\sigma/2} = k + i\alpha$$

$$\rightarrow\ k \approx \alpha \approx \sqrt{\frac{\omega\mu_0\sigma}{2}} \quad \rightarrow\ \delta \approx \frac{1}{\alpha} \approx \sqrt{\frac{2}{\omega\mu_0\sigma}}$$ ← Skin depth

$$\rightarrow\ n \approx \kappa \approx \sqrt{\frac{\sigma}{2\omega\varepsilon_0}}$$

Propagation of Light in Conducting Media (3)

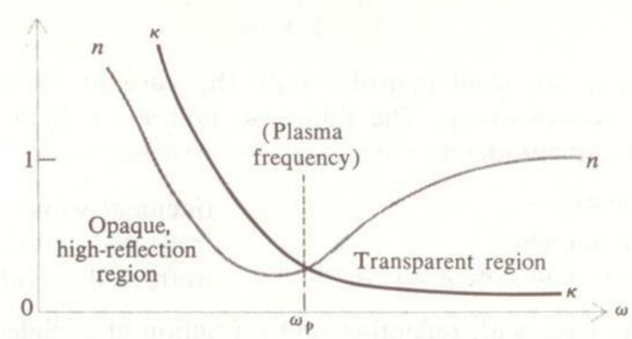
Complex index of refraction in general:

$$\rightarrow \mathcal{N}^2 = 1 - \frac{\omega_p^2}{\omega + i\omega\tau^{-1}} \quad \leftarrow \quad \omega_p = \sqrt{\frac{\mu_0 \sigma c^2}{\tau}} = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

← Plasma frequency

$$\rightarrow n^2 - \kappa^2 = 1 - \frac{\omega_p^2}{\omega^2 + \tau^{-2}}$$

$$\rightarrow 2n\kappa = \frac{\omega_p^2}{\omega^2 + \tau^{-2}}\left(\frac{1}{\omega\tau}\right)$$



G. R. Fowles, Introduction to Modern Optics, 1975.

For poor conductors and semiconductors:

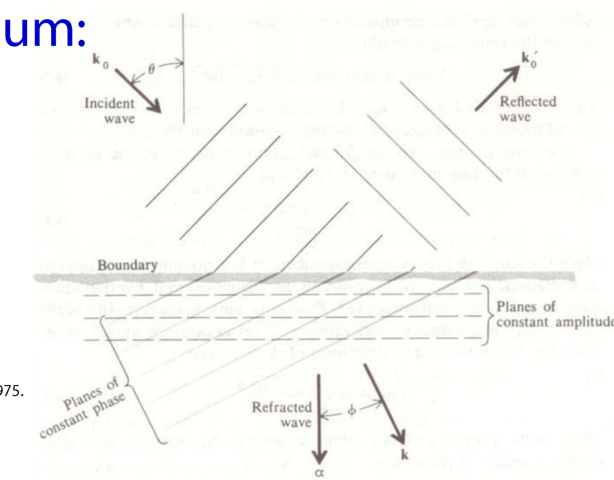
$$\rightarrow \mathcal{N}^2 = 1 - \frac{\omega_p^2}{\omega + i\omega\tau^{-1}} + \frac{Ne^2}{m\varepsilon_0}\sum_j \left(\frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}\right)$$

Reflection and Refraction: Absorbing Media (1)

Complex index of refraction of an absorbing medium:

$$\mathscr{N} = n + i\kappa$$

$$\to \mathscr{K} = \mathbf{k} + i\boldsymbol{\alpha}$$



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Boundary conditions:

$$\to \mathbf{k}_0 \cdot \mathbf{r} = \mathbf{k}_0' \cdot \mathbf{r}$$

$$\to \mathbf{k}_0 \cdot \mathbf{r} = \mathscr{K} \cdot \mathbf{r} = (\mathbf{k} + i\boldsymbol{\alpha}) \cdot \mathbf{r} \;\to\; \mathbf{k}_0 \cdot \mathbf{r} = \mathbf{k} \cdot \mathbf{r} \to k_0 \sin\theta = k \sin\phi$$

$$\to \boldsymbol{\alpha} \cdot \mathbf{r} = 0$$

Wave equation:

$$\to \nabla^2 \mathbf{E} = \frac{\mathscr{N}^2}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \to (\mathbf{k} + i\boldsymbol{\alpha}) \cdot (\mathbf{k} + i\boldsymbol{\alpha}) = (n + i\kappa)^2 k_0^2$$

$$\to k^2 - \alpha^2 = (n^2 - \kappa^2) k_0^2$$

$$\to \mathbf{k} \cdot \boldsymbol{\alpha} = k\alpha \cos\phi = n\kappa k_0^2$$

Complex index of refraction:

$$\to k\cos\phi + i\alpha = k_0 \sqrt{\mathscr{N}^2 - \sin^2\theta}$$

$$\to \mathscr{N} \sin\varphi = \sin\theta \to \cos\varphi = \sqrt{1 - \frac{\sin\theta}{\mathscr{N}^2}} \quad \to \mathscr{N} = \frac{k\cos\phi + i\alpha}{k_0 \cos\varphi}$$

→ c.f. Snell's law

Reflection and Refraction: Absorbing Media (2)

Electric and magnetic fields:

$$\mathbf{E},\ \mathbf{H} = \frac{1}{\mu_0 \omega}\mathbf{k}_0 \times \mathbf{E} \qquad \leftarrow \text{Incident}$$

$$\mathbf{E}',\ \mathbf{H}' = \frac{1}{\mu_0 \omega}\mathbf{k}_0' \times \mathbf{E}' \qquad \leftarrow \text{Reflected}$$

$$\mathbf{E}'',\ \mathbf{H}'' = \frac{1}{\mu_0 \omega}\mathscr{K} \times \mathbf{E}'' = \frac{1}{\mu_0 \omega}(\mathbf{k} \times \mathbf{E}'' + i\boldsymbol{\alpha} \times \mathbf{E}'') \qquad \leftarrow \text{Refracted}$$

Boundary conditions (TE):

$$E + E' = E''$$

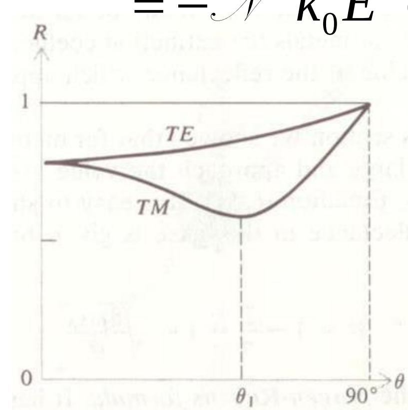
$$H_t + H_t' = H_t'' \ \rightarrow\ -k_0 E\cos\theta + k_0 E'\cos\theta = -kE''\cos\phi + i\alpha E''$$

$$= -\mathcal{N} k_0 E''\cos\varphi$$

Reflection coefficients:

$$\rightarrow\ r_s = \frac{\cos\theta - \mathcal{N}\cos\varphi}{\cos\theta + \mathcal{N}\cos\varphi} \quad (TE)$$

$$\rightarrow\ r_p = \frac{-\mathcal{N}\cos\theta + \cos\varphi}{\mathcal{N}\cos\theta + \cos\varphi} \quad (TM)$$



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