Introduction to Photonics

Thermal Radiation and Light Quanta (1)

Yoonchan Jeong
School of Electrical Engineering, Seoul National University
Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Thermal Radiation

Thermal radiation:

$\rightarrow$ Electromagnetic energy emitted from the surface of a heated body

Spectral distribution:

$\rightarrow$ Existence of a definite frequency at which the radiated power is maximum

$\rightarrow$ Varying in direct proportion to the absolute temperature: *Wien's law*

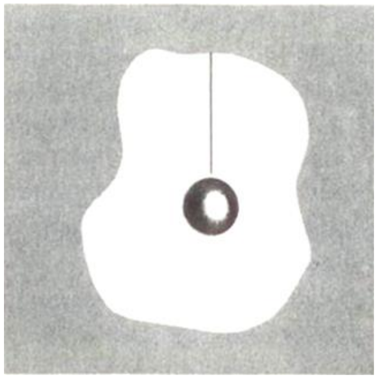
Total power:

$\rightarrow$ Increasing as the fourth power of the absolute temperature:

Stefan-Boltzmann law

Kirchhoff's Law: Blackbody Radiation (1)

Consider a hypothetical situation: A body contained inside a hollow cavity, being thermally insulated from the cavity



G. R. Fowles, Introduction to Modern Optics, 1975.

In equilibrium:

$$H = bI$$

Irradiance of the thermal radiation (per unit area)

Fraction of incident power that the body absorbs

Radiance that the body emits (per unit area)

Kirchhoff's law:

$$I = \frac{H_1}{b_1} = \frac{H_2}{b_2} = \cdots$$

Good absorbers are also good emitters, and vice versa.

Blackbody:

$$\rightarrow\ b = 1 \ \rightarrow\ H_{\max} = I$$

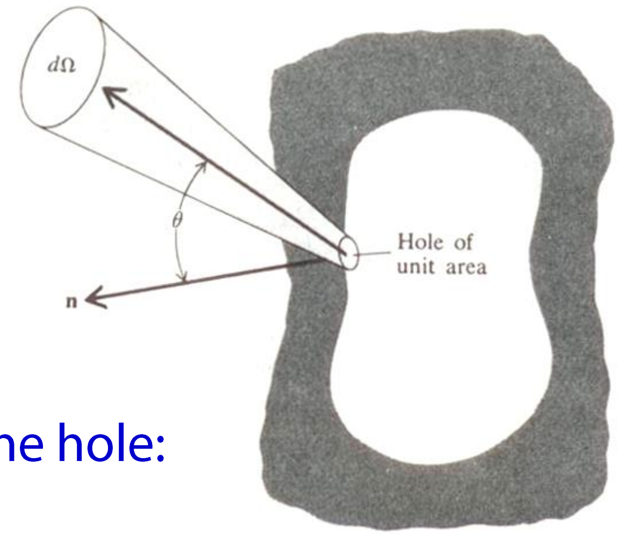
Kirchhoff's Law: Blackbody Radiation (2)

Energy density and spectral energy density:

$$u = \int_0^\infty u_\nu d\nu$$

Amount of energy per unit time through a hole of unit area within $d\Omega$:

$$\rightarrow\ uc\cos\theta\ d\Omega / 4\pi$$



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Total amount of energy per unit time through the hole:

$$\rightarrow\ \int_0^{2\pi}\int_0^{\pi/2} uc\cos\theta\sin\theta\,\frac{d\theta d\phi}{4\pi} = \frac{uc}{4}$$

Total radiation emitted per unit time per unit area:

$$\rightarrow\ I = \frac{uc}{4}$$

Spectral radiation:

$$\rightarrow\ I_\nu = \frac{u_\nu c}{4}$$

Modes of Electromagentic Radiation in a Cavity

Consider the fundamental wave function for a cavity of rectangular shape:

$$e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = e^{ik_x x} e^{ik_y y} e^{ik_z z} e^{-i\omega t}$$

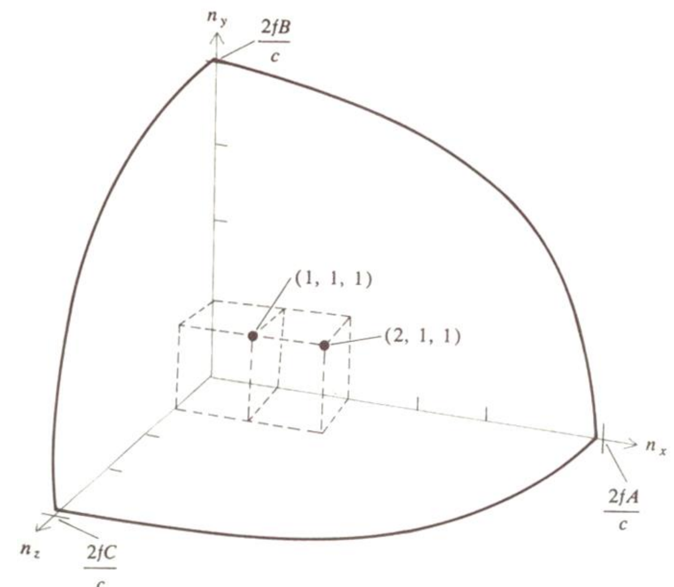
Conditions for a mode (stationary pattern):

$$\rightarrow \; k_x A = \pi n_x, \quad k_y B = \pi n_y, \quad k_z C = \pi n_z$$

$$\rightarrow \; k^2 = k_x^2 + k_y^2 + k_z^2$$

$$\rightarrow \; k^2 = \frac{\omega^2}{c^2} = \pi^2 \left(\frac{n_x^2}{A^2} + \frac{n_y^2}{B^2} + \frac{n_z^2}{C^2} \right)$$

$$\rightarrow \; \frac{4\nu^2}{c^2} = \frac{n_x^2}{A^2} + \frac{n_y^2}{B^2} + \frac{n_z^2}{C^2}$$



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Number of modes for all $f \leq \nu$:

$$\rightarrow \; \frac{1}{8} \frac{4\pi}{3} \frac{2\nu A}{c} \frac{2\nu B}{c} \frac{2\nu C}{c} = \frac{4\pi\nu^3 ABC}{3c^3} = \frac{4\pi\nu^3}{3c^3} V$$

Number of modes per unit volume for all $f \leq \nu$:

Two orthogonal polarizations

$$\rightarrow \; g = \frac{8\pi\nu^3}{3c^3} \qquad \rightarrow \; dg = \frac{8\pi\nu^2}{c^3} d\nu \qquad \rightarrow \; g_\nu = \frac{8\pi\nu^2}{c^3}$$

Classical Theory of Blackbody Radiation

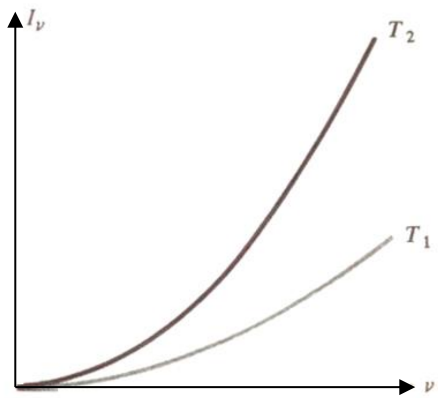
Principle of equipartition of energy for a gas:

$\frac{1}{2}kT$ $\leftarrow$ *Average energy associated with each degree of freedom of a molecule*

Rayleigh-Jeans formula:

$$\rightarrow \; u_\nu = g_\nu kT = \frac{8\pi\nu^2 kT}{c^3}$$

$$\rightarrow \; I_\nu = \frac{2\pi\nu^2 kT}{c^2}$$



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Ultraviolet catastrophe:

$\rightarrow$ *Rayleigh-Jeans formula: Found to agree well with experimental data for low frequencies, but not for high frequencies*

$\rightarrow$ *Fundamental error in the classical approach!*

Quantization of Cavity Radiation

Quantization of electromagnetic radiation:

$\rightarrow$ *Introduced by Planck in 1901*

$\rightarrow$ *Postulated:*
The EM energy exists only in integral multiples of some lowest amount of quantum.
The quantum is proportional to the frequency of the radiation.

$$\rightarrow \; E_\nu = h\nu\langle n_\nu\rangle \longleftarrow$$ *Average number of photons per mode (occupation index)*

Constant of proportionality

$$\rightarrow \; u_\nu = g_\nu h\nu\langle n_\nu\rangle = \frac{8\pi h\nu^3}{c^3}\langle n_\nu\rangle$$

Spectral radiation function for blackbody radiation:

$$\rightarrow \; I_\nu = \frac{1}{4} c g_\nu h\nu\langle n_\nu\rangle = \frac{2\pi h\nu^3}{c^2}\langle n_\nu\rangle$$