Introduction to Photonics

Thermal Radiation and Light Quanta (2)

Yoonchan Jeong
School of Electrical Engineering, Seoul National University
Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Photon Statistics: Planck's Formula (1)

Most probable distribution for photons:

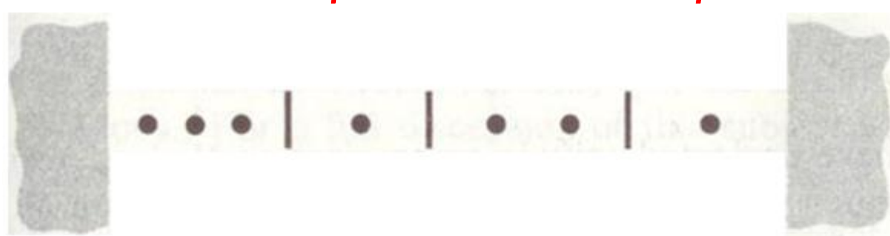
→ The largest value of W (total number of ways of the arrangement of photons)

Occupation index:

$$\langle n_\nu \rangle = \left(\frac{N_\nu}{g_\nu} \right)_{\max}$$

Number of photons per unit volume per unit frequency

Number of available quantum states (modes) per unit volume per unit frequency



$N_\nu = 7$ $g_\nu = 4$

G. R. Fowles, Introduction to Modern Optics, 1975.

Total number of ways:

$$\rightarrow W_\nu = \frac{(N_\nu + g_\nu - 1)!}{N_\nu!(g_\nu - 1)!}$$

← *Per unit frequency range centered at* ν

$$\rightarrow W = \prod_\nu W_\nu = \prod_\nu \frac{(N_\nu + g_\nu - 1)!}{N_\nu!(g_\nu - 1)!}$$

Photon Statistics: Planck's Formula (2)

Stirling's approximation:

$$\ln x! \cong x \ln x - x$$

Total number of ways:

$$\rightarrow \ln W = \sum_{\nu} [(N_\nu + g_\nu - 1)\ln(N_\nu + g_\nu - 1) - N_\nu \ln N_\nu - (g_\nu - 1)\ln(g_\nu - 1)]$$

$$\rightarrow \delta(\ln W) = \sum_{\nu} [\ln(N_\nu + g_\nu) - \ln N_\nu]\delta N_\nu = 0$$ ← *To maximize W*

$$\rightarrow E = h\nu N_\nu = const.$$

$$\rightarrow \delta E = \sum_{\nu} h\nu \delta N_\nu = 0$$

To be satisfied simultaneously

$$\rightarrow \delta(\ln W) - \beta \delta E = 0$$ ← *Lagrange's method of undetermined multipliers*

$$\rightarrow \sum_{\nu} [\ln(N_\nu + g_\nu) - \ln N_\nu - \beta h\nu]\delta N_\nu = 0$$

$$\rightarrow \ln(N_\nu + g_\nu) - \ln N_\nu - \beta h\nu = 0$$

Bose-Einstein distribution law for photons: → *cf. Fermi-Dirac statics*

$$\rightarrow \langle n_\nu \rangle = \left(\frac{N_\nu}{g_\nu}\right)_{\max} = \frac{1}{e^{\beta h\nu} - 1}$$ ← *Subject to E = const.*

Photon Statistics: Planck's Formula (3)

Spectral radiation function for blackbody radiation:

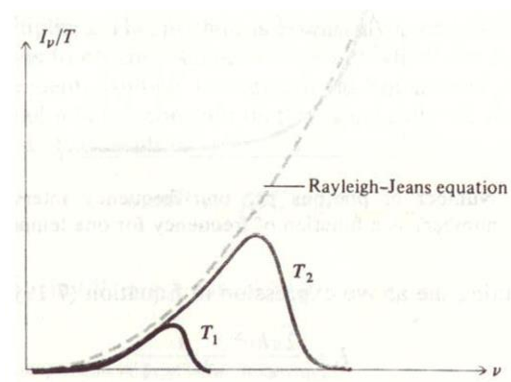
$$\rightarrow\ I_\nu = \frac{2\pi h\nu^3}{c^2}\langle n_\nu\rangle = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{\beta h\nu}-1}$$

For small frequencies ($\beta h\nu << 1$):

← Rayleigh-Jeans formula

$$\rightarrow\ I_\nu = \frac{2\pi\nu^2}{c^2}\frac{1}{\beta} \qquad \leftarrow\ I_\nu = \frac{2\pi\nu^2 kT}{c^2}$$

$$\rightarrow\ \beta = 1/kT$$



Planck's formula:

$$\rightarrow\ I_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

G. R. Fowles, Introduction to Modern Optics, 1975.

Agreement with Wien's law and Stefan-Boltzmann law:

$$\rightarrow\ x = \frac{h\nu}{kT} \qquad \rightarrow\ I_\nu = \frac{2\pi k^3T^3}{c^2h^2}\frac{x^3}{e^x-1} \qquad \rightarrow\ \nu_{\max} = \frac{2.82kT}{h}$$

← Wien's law

$$\rightarrow\ I = \int_0^\infty I_\nu d\nu = \frac{2\pi k^4T^4}{c^2h^3}\int_0^\infty \frac{x^3dx}{e^x-1} = \frac{2\pi^5k^4}{15c^2h^3}T^4 = \sigma T^4$$

← Stefan-Boltzmann law

Photoelectric Effect

Photoelectrons:

$\rightarrow$ *Emitted from a metal surface when a beam of light illuminates it*

Maximum kinetic energy of a photoelectron:

$$E_{\max} = h\nu - e\phi$$

Work function

$\rightarrow$ *The energy does not depend on the intensity of the incident light but on the frequency.*

$\rightarrow$ *The photocurrent varies with the intensity of the light.*

$\rightarrow$ *Possible to detect individual photons!*

Momentum of a Photon

Radiation pressure based on classical EM theory:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \;\rightarrow\; P = \frac{I}{c}$$

Quantum description of light:

$$\rightarrow\; h\nu = mc^2 \qquad \leftarrow \textit{Energy}$$

$$\rightarrow\; p = mc = \frac{h\nu}{c} = \frac{h}{\lambda} \qquad \leftarrow \textit{Linear momentum}$$

Illumination of a stream of photons on a perfectly absorbing medium:

$$\rightarrow\; P = \frac{Nh\nu}{c} \qquad \leftarrow \textit{Pressure (Force per unit area)}$$

Number of photons per unit area per unit time

$$\leftarrow\; I = Nh\nu \qquad \leftarrow \textit{Irradiance (Power per unit area)}$$

$$\rightarrow\; P = \frac{I}{c}$$

For a perfectly reflecting medium:

$$\rightarrow\; P = \frac{2I}{c}$$

Angular Momentum of a Photon

Torque per unit area based on classical EM theory:

$$\mathbf{T} = q(\mathbf{r} \times \mathbf{E}) \;\rightarrow v = \omega r \;\rightarrow \mathscr{T} = \frac{I}{\omega} = \frac{I}{2\pi\nu}$$

Quantum description of light:

$$\rightarrow \mathscr{T} = \frac{Nh}{2\pi}$$

Spin of a photon (circularly polarized):

$$\rightarrow s = \pm\frac{h}{2\pi}$$ $\leftarrow$ *Right or left circularly polarized light*

Wavelength of a Material Particle

de Broglie's Hypothesis:

$$\rightarrow \quad p = h/\lambda$$ $\leftarrow$ *Photon's momentum*

$$\rightarrow \quad p = mu$$ $\leftarrow$ *Particle's momentum*

Matter wave:

$$\rightarrow \quad \lambda = \frac{h}{p} = \frac{h}{mu}$$

Experiment by Davisson and Germer in 1927:

$$\rightarrow \quad n\lambda = 2d\cos\theta$$

$\rightarrow$ *An electron beam also obeys the optical grating formula!*

$\rightarrow$ *"Wave-particle duality"*

Heisenberg's Uncertainty Principle

Uncertainty principle by Heisenberg in 1927:

$$\Delta P \Delta Q \approx h$$

Two conjugate quantities,
e.g., energy and time, position and momentum, etc.

Lower limit of uncertainty: Fourier resolution of a pulse

$$\rightarrow \ \Delta \nu \Delta t \approx 1 \ \rightarrow \ \Delta (h\nu) \Delta t \approx h$$

$$\rightarrow \ \Delta E \Delta t \approx h$$ *← For energy and time*

$$\rightarrow \ \Delta x = c \Delta t = \frac{c}{\Delta \nu}$$

$$\rightarrow \ \Delta p = \frac{h \Delta \nu}{c}$$

$$\rightarrow \ \Delta x \Delta p \approx h$$ *← For position and momentum*