Introduction to Photonics

Optical Spectra (1)

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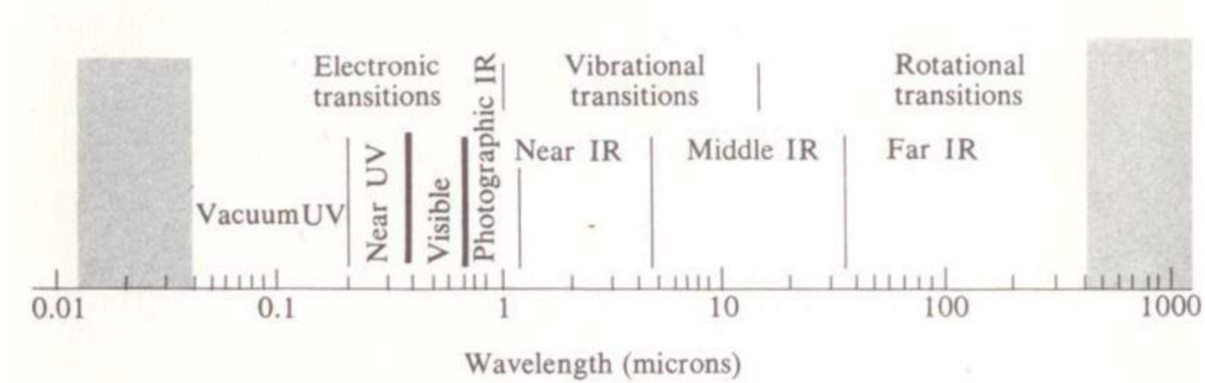
Optical Spectra

Spectrum:

→ An ordering of electromagnetic radiation according to frequency or wavelength



ı. R. Fowles, Introduction to Modern Optics, 1975.

Type of spectrum:

→ Continuous or discrete

→ Thermal radiation and atomic spectra (emission & absorption)

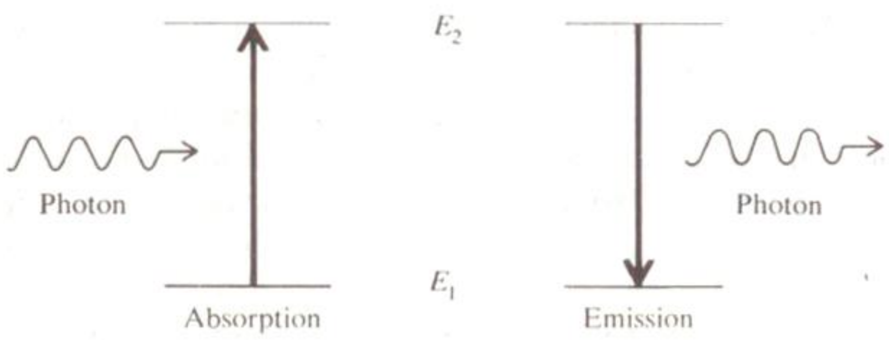
Elementary Theory of Atomic Spectra (1)

Bohr's two fundamental assumptions:

→ Discrete quantized states or orbits: These have different energies, and the one of lowest energy is the normal state of the atom, also known as the ground state.

→ When an electron undergoes a transition from one state to another, it can do so by emitting or absorbing radiation of ν:

$$\nu = \frac{\Delta E}{\lambda}$$



G. R. Fowles, Introduction to Modern Optics, 1975.

"Radical departure from the classical or Newtonian concept of the atom!"

Elementary Theory of Atomic Spectra (2)

The Bohr Atom and the Hydrogen Spectrum:

$\rightarrow$ Introduction of a fundamental postulate on angular momentum:

$$mur = \frac{nh}{2\pi} \quad (n = 1, 2, 3, \cdots)$$

$\rightarrow$ Quantization of the orbital angular momentum

Classical force equation:

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mu^2}{r}$$

Radii of the quantized orbits:

$$\rightarrow \ r = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2 = a_H n^2$$

$$\leftarrow \ a_H = 0.529 \ \mathrm{\mathring{A}}$$

Total energy of a given orbit:

$$E = \frac{1}{2} mu^2 - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r}$$

$$\rightarrow \ E_n = -\frac{R}{n^2} \quad \leftarrow \ R = \frac{me^4}{8\varepsilon_0^2 h^2}$$

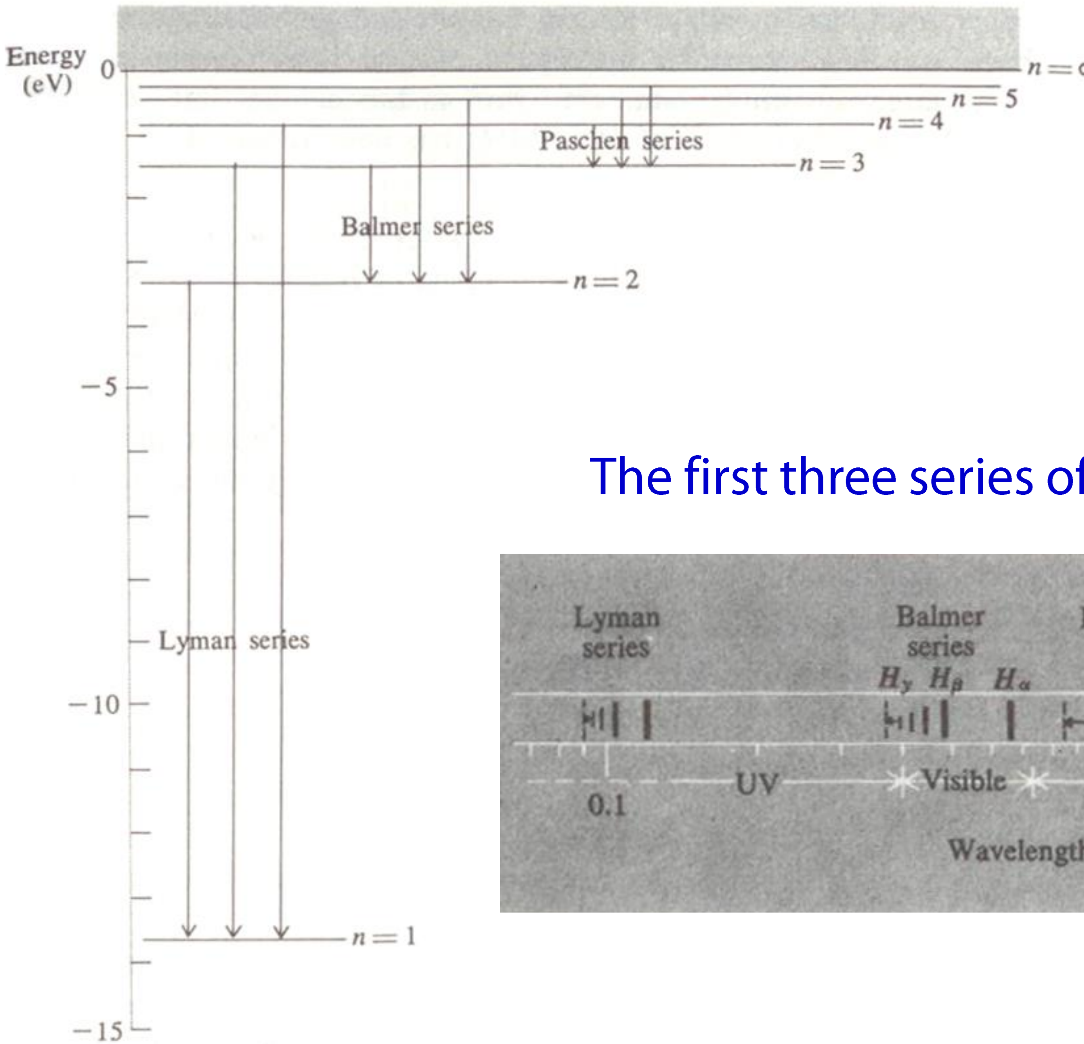
$\leftarrow$ *Rydberg constant (Empirical)*

Hydrogen spectrum:

$$\rightarrow \ \nu = \frac{E_2 - E_1}{h} = \frac{R}{h}\left(\frac{1}{n_1^2} - \frac{1}{n_{12}^2}\right)$$

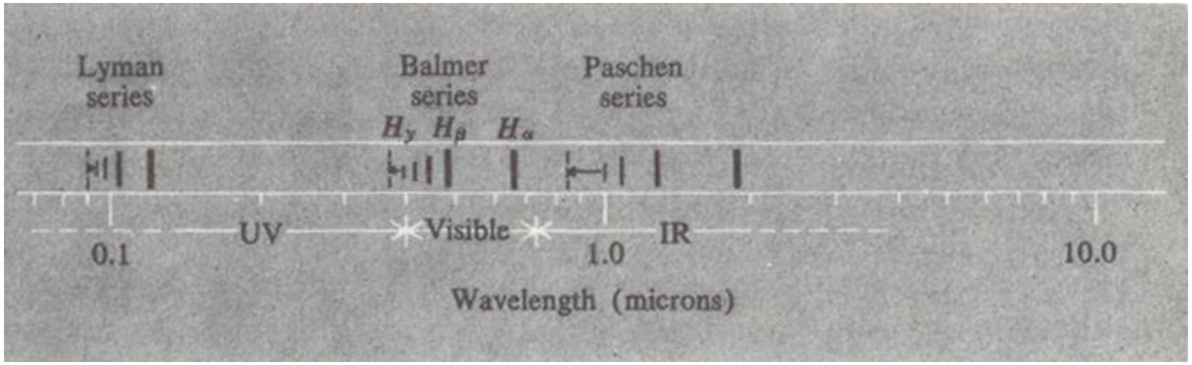
Elementary Theory of Atomic Spectra (3)

Energy levels of atomic hydrogen:



G. R. Fowles, Introduction to Modern Optics, 1975.

The first three series of atomic hydrogen:



G. R. Fowles, Introduction to Modern Optics, 1975.

Elementary Theory of Atomic Spectra (4)

Effect of a finite nuclear mass:

$\rightarrow$ Consider for the center of mass

$$\mu = \frac{mM}{m+M} \quad \leftarrow \text{Reduced mass of the electron}$$

$$\rightarrow \quad R_H = \frac{\mu e^4}{8\varepsilon_0^2 h^2}$$

Limitation of Bohr model:

$\rightarrow$ Unable to account for the fact that the electron does not radiate while traveling in its orbit in the ground state

$\rightarrow$ Too difficult to apply to more complicated atoms or molecules

$\rightarrow$ Superseded by the modern quantum theory of atom

Spectra of the Alkali metals:

$$\rightarrow \quad \nu = \frac{R}{h}\left(\frac{1}{(n_1-\delta_1)^2} - \frac{1}{(n_2-\delta_2)^2}\right) \quad \leftarrow \text{Empirical formula}$$

Quantum defects (δ_1, δ_2)

Quantum Mechanics (1)

Pioneered by Schrödinger, Heisenberg, and others in 1920s:

$\rightarrow$ Wave mechanics & matrix mechanics: Completely equivalent

Basic postulates:

$\rightarrow$ Wave function or state function: $\rightarrow \Psi$

$\rightarrow$ Probability density: $\rightarrow |\Psi|^2$

$\rightarrow$ Probability: $\rightarrow \Psi^*(x, y, z, t)\Psi(x, y, z, t)\Delta x \Delta y \Delta z$

$\rightarrow$ Quadratically integrable:

$$\rightarrow \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Psi^*\Psi\, dxdydz = 1$$

Quantum Mechanics (2)

Stationary states: $\rightarrow$ Characteristic states or eigenstates

$$\rightarrow \ \Psi_n(x,y,z,t) = \psi_n(x,y,z)e^{-iE_n t/\hbar}$$

$$\rightarrow \ \Psi_n^*\Psi_n = \psi_n^* e^{iE_n t/\hbar}\psi_n e^{-iE_n t/\hbar} = \psi_n^*\psi_n$$ $\leftarrow$ Stationary (no radiation)

Coherent states:

$$\rightarrow \ \Psi = c_1\psi_1 e^{-iE_1 t/\hbar} + c_2\psi_2 e^{-iE_2 t/\hbar}$$ $\leftarrow$ Sinusoidal oscillation (radiation)

$$\rightarrow \ \Psi^*\Psi = c_1^*c_1\psi_1^*\psi_1 + c_2^*c_2\psi_2^*\psi_2 + c_1^*c_2\psi_1^*\psi_2 e^{i\omega t} + c_2^*c_1\psi_2^*\psi_1 e^{-i\omega t}$$

$$\leftarrow \ \omega = \frac{E_1 - E_2}{\hbar} \quad \leftarrow \ \nu = \frac{E_1 - E_2}{h}$$

Schrödinger equation:

$$\rightarrow \ \nabla^2\psi + \left(\frac{2\pi}{\lambda}\right)^2\psi = 0 \quad \leftarrow \ \lambda = h/p$$ $\leftarrow$ de Broglie's hypothesis

$$\rightarrow \ \nabla^2\psi + \left(\frac{2\pi p}{h}\right)^2\psi = 0 \quad \leftarrow \ E = \tfrac{1}{2}mu^2 + V,\ p = mu$$

$$\rightarrow \ \nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$