Introduction to Photonics

Amplification of Light: Lasers (1)

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Spontaneous Transitions between Atomic Levels

Atomic systems:

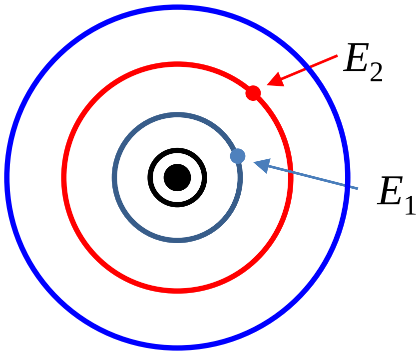
Predetermined set of energy states: Eigenstates

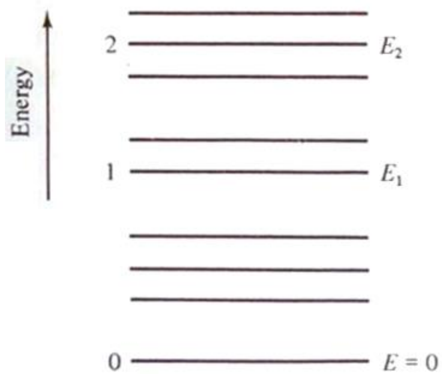
Spontaneous emission:

Photon emitting process without the inducement of a radiation field: $h\nu = E_2 - E_1$

Spontaneous emission rate:

$$\frac{dN_2}{dt} = -A_{21}N_2 \equiv -\frac{N_2}{\tau_{21,sp}}$$





A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Lineshape Function (1)

"Atomic" lineshape function $g(\nu)$:

Spectral distribution of emitted intensity vs. frequency

$$\int_{-\infty}^{+\infty} g(\nu)d\nu = 1$$

Homogeneous broadening: Uniform and identical frequency response

Incl. lifetime, collision, dipolar, and thermal broadening

$\rightarrow$ Lifetime broadening:

$$E(t) = E_0 e^{-t/2\tau} e^{i\omega_0 t} = E_0 e^{i(\omega_0 + i\sigma/2)t} \quad \leftarrow \sigma = \frac{1}{\tau} \quad \leftarrow t \geq 0$$

$$\rightarrow F(\omega) = \int_0^{+\infty} E(t) e^{-i\omega t} dt = \frac{-iE_0}{\omega - \omega_0 - i\sigma/2}$$

$$\rightarrow |F(\omega)|^2 \propto \frac{1}{(\omega - \omega_0)^2 + (\sigma/2)^2}$$

$$\rightarrow g(\nu) = \frac{\Delta\nu}{2\pi[(\nu - \nu_0)^2 + (\Delta\nu/2)^2]}$$ $\leftarrow$ Lorentzian lineshape function

$$\rightarrow \Delta\nu = \frac{\sigma}{2\pi} = \frac{1}{2\pi\tau}$$ $\leftarrow$ Linewidth (FWHM)

Lineshape Function (2)

Inhomogeneous broadening: Non-uniform and detuned frequency response

e.g. Doppler broadening, impurities

$\rightarrow$ Doppler broadening:

Doppler shift: $\rightarrow \nu = \nu_0 + \frac{v_x}{c}\nu_0$ $\leftarrow$ Non-relativistic collision

Maxwell velocity distribution of a gas:

$$f(v_x, v_y, v_z) = \left(\frac{M}{2\pi kT}\right)^{3/2} \exp\left[-\frac{M}{2kT}(v_x^2 + v_y^2 + v_z^2)\right]$$

$$\rightarrow g(\nu)d\nu = \left(\frac{M}{2\pi kT}\right)^{3/2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-M/2kT(v_y^2+v_z^2)} dv_y dv_z \times e^{-M/2kT(c^2/\nu_0^2)(\nu-\nu_0)^2} \left(\frac{c}{\nu_0}\right) d\nu$$

$$\rightarrow g(\nu) = \frac{c}{\nu_0}\left(\frac{M}{2\pi kT}\right)^{1/2} e^{-M/2kT(c^2/\nu_0^2)(\nu-\nu_0)^2}$$

$\leftarrow$ Gaussian lineshape function

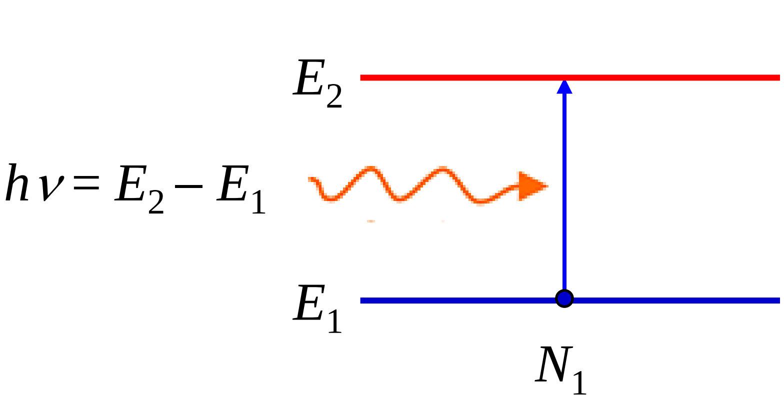
$$\rightarrow \Delta\nu_D = 2\nu_0\sqrt{\frac{2kT}{Mc^2}\ln 2}$$

$\leftarrow$ Linewidth (FWHM)

Induced Transitions (1)

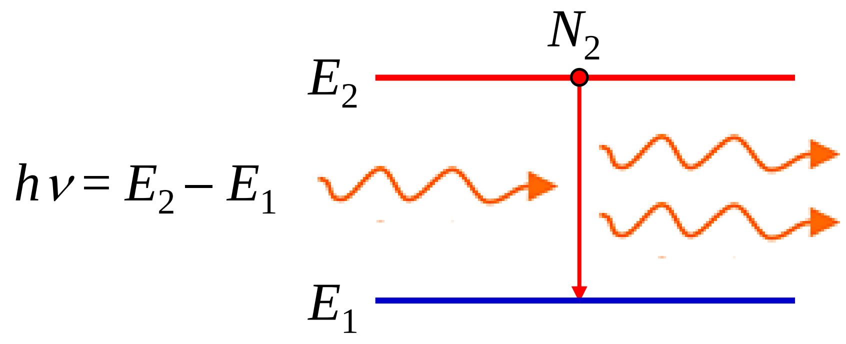
Induced transitions and their rates per atom:

Absorption



$$(W'_{12})_{induced} = B_{12} u_\nu$$

Stimulated emission



$$(W'_{21})_{induced} = B_{21} u_\nu$$

Energy density per unit frequency (u_ν)

Total upward (1→2) transition rate:

$$W'_{12} = (W'_{12})_{induced} = B_{12} u_\nu$$

Total downward (2→1) transition rate:

$$W'_{21} = B_{21} u_\nu + A_{21}$$

Induced Transitions (2)

For thermal equilibrium:

$$N_2 W'_{21} = N_1 W'_{12} \rightarrow N_2[B_{21}u_\nu + A_{21}] = N_1 B_{12} u_\nu$$

$$\rightarrow u_\nu = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

Boltzmann distribution:
(with no degeneracy)

$$\frac{N_i}{N} = \frac{e^{-E_i/kT}}{\sum_i e^{-E_i/kT}} \rightarrow \frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-h\nu/kT}$$

$$\rightarrow u_\nu = \frac{A_{21}}{B_{21}} \frac{1}{(B_{12}/B_{21})e^{h\nu/KT} - 1} = \frac{8\pi n^3 h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

Energy density for thermal (blackbody) radiation

Einstein's A and B coefficients:

$$\rightarrow B_{12} = B_{21} \qquad \rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$$

$$\rightarrow W'_i = \frac{A_{21}c^3}{8\pi n^3 h\nu^3} u_\nu = \frac{c^3}{8\pi n^3 h\nu^3 \tau_{sp}} u_\nu$$

Induced Transitions (3)

Monochromatic transition rate:

$$W_i(\nu_k) = \frac{c^3 u_{\nu_k}}{8\pi n^3 h \nu_k^3 \tau_{sp}} g(\nu_k)$$

Atomic lineshape function

"Probability of the transition tuned at ν_k"

Total transition rate:

$$W_i' = \sum_{\nu_k} W_i(\nu_k) = \frac{c^3}{8\pi n^3 h \tau_{sp}} \sum_{\nu_k} \frac{u_{\nu_k}}{\nu_k^3} g(\nu_k)$$

$$\rightarrow W_i' = \frac{c^3}{8\pi n^3 h \tau_{sp}} \int_{-\infty}^{+\infty} \frac{u_\nu}{\nu^3} g(\nu) d\nu \quad \leftarrow \int_{-\infty}^{+\infty} g(\nu) d\nu = 1$$

Transition rate in terms of the intensity:

$$I_\nu = \frac{c}{n} u_\nu$$

$$\rightarrow W_i(\nu) = \frac{c^2 I_\nu}{8\pi n^2 h \nu^3 \tau_{sp}} g(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{sp}} g(\nu)$$

Absorption and Amplification

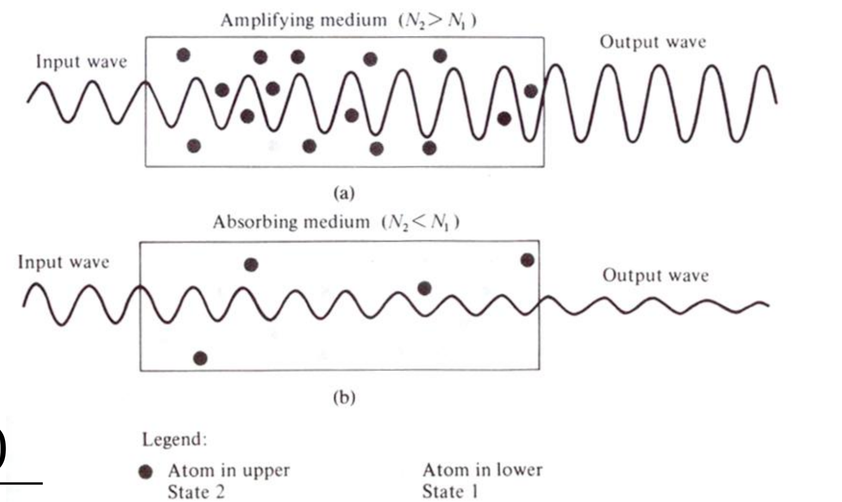
Net power generated per unit volume:

$$\frac{P_\nu}{Volume} = (N_2 - N_1) W_i h\nu$$

$$\rightarrow \frac{dI_\nu}{dz} = (N_2 - N_1)\frac{c^2 g(\nu)}{8\pi n^2 \nu^2 \tau_{sp}} I_\nu$$

$$\rightarrow I_\nu(z) = I_\nu(0) e^{\gamma(\nu) z}$$

$$\leftarrow \gamma(\nu) = (N_2 - N_1)\frac{c^2 g(\nu)}{8\pi n^2 \nu^2 \tau_{sp}}$$



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Absorption:

$$\frac{N_2}{N_1} = e^{-h\nu/kT} \rightarrow N_2 < N_1 \leftarrow \text{Thermal equilibrium}$$

Amplification: $N_2 > N_1 \leftarrow$ Population inversion via a metastable state

$$\tau_{sp} >> 0$$

Emission and Absorption Crosssections (1)

Boltzmann distribution:

$$\frac{N_i}{N} = \frac{e^{-E_i/kT}}{\sum_i e^{-E_i/kT}} \rightarrow \frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-h\nu/kT}$$ $\leftarrow$ With no degeneracy

With degeneracy:

$$\rightarrow \frac{N_i}{N} = \frac{g_i e^{-E_i/kT}}{\sum_i g_i e^{-E_i/kT}} \rightarrow \frac{N_2}{N_1} = \frac{g_2 e^{-E_2/kT}}{g_1 e^{-E_1/kT}} = \frac{g_2}{g_1} e^{-h\nu/kT}$$

Einstein's A and B coefficients:

$$\rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3} \rightarrow B_{21} = \frac{g_1}{g_2} B_{12}$$

$$\rightarrow W'_{i,e} = \frac{A_{21}c^3}{8\pi n^3 h\nu^3} u_\nu \qquad \rightarrow W'_{i,a} = \frac{g_2}{g_1} \frac{A_{21}c^3}{8\pi n^3 h\nu^3} u_\nu$$

Transition rate in terms of the intensity:

$$\rightarrow W_{i,e}(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h\nu\tau_{sp}} g(\nu) \rightarrow W_{i,a}(\nu) = \frac{g_2}{g_1} \frac{\lambda^2 I_\nu}{8\pi n^2 h\nu\tau_{sp}} g(\nu)$$

Emission and Absorption Crosssections (2)

Transition rate in terms of the intensity:

$$\rightarrow W_{i,e}(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h\nu\tau_{sp}} g(\nu) \quad \rightarrow W_{i,a}(\nu) = \frac{g_2}{g_1}\frac{\lambda^2 I_\nu}{8\pi n^2 h\nu\tau_{sp}} g(\nu)$$

Emission and absorption crosssections:

$$\rightarrow \sigma_e(\nu) = \frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu) \quad \rightarrow \sigma_a(\nu) = \frac{g_2}{g_1}\frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu) = \frac{g_2}{g_1}\sigma_e$$

$$\rightarrow W_{i,e}(\nu) = \frac{\sigma_e I_\nu}{h\nu} \quad \rightarrow W_{i,a}(\nu) = \frac{\sigma_a I_\nu}{h\nu}$$

Gain constant with no degeneracy:

$$\gamma(\nu) = (N_2 - N_1)\frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu)$$

With degeneracy:

$$\rightarrow \gamma(\nu) = (N_2 - \frac{\sigma_a}{\sigma_e} N_1)\sigma_e$$

"Effective population inversion"