Introduction to Photonics

Ray Optics (1)

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

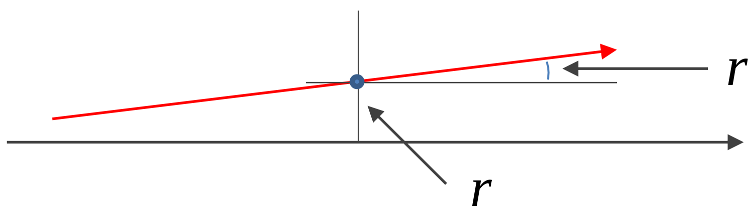
Email: yoonchan@snu.ac.kr

Paraxial Rays and Ray Matrix

Ray: A stream of light normal to the optical wavefront

Paraxial ray: Angular deviation from the reference longitudinal axis small enough $\rightarrow$ $\sin\theta \approx \theta, \ \tan\theta \approx \theta$

Ray distance from the axis and its slope:



$$r' = \frac{dr}{dz} \quad \rightarrow \begin{bmatrix} r \\ r' \end{bmatrix}$$

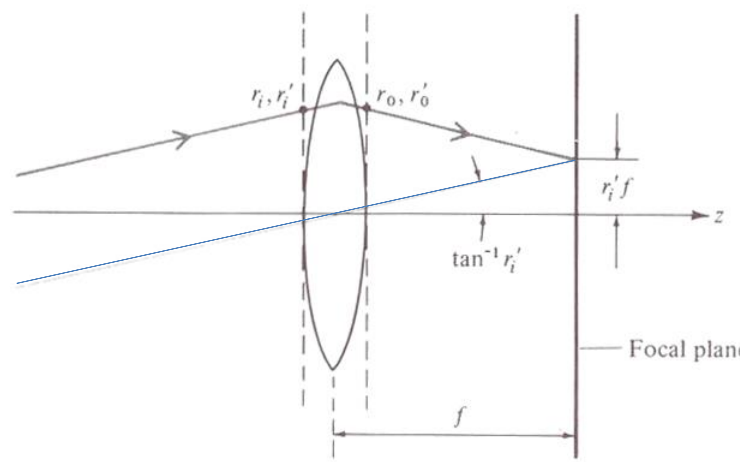
Deflection of a ray by a thin lens:

$$r_{out} = r_{in}$$

$$r'_{out} = r'_{in} - \frac{r_{in}}{f}$$

$$\rightarrow \begin{bmatrix} r_{out} \\ r'_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} r_{in} \\ r'_{in} \end{bmatrix}$$

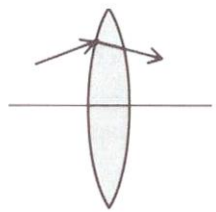
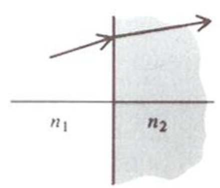
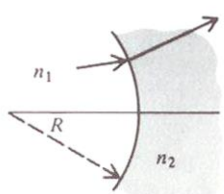
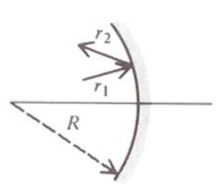
$\rightarrow$ Ray matrix



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Ray Matrices

Ray matrices for some common optical elements and media:

(1) Straight Section: Length d		$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$	$\leftarrow$ $r_2 = r_1 + r_1' d$ $r_2' = r_1'$
(2) Thin Lens: Focal length f ($f > 0$, converging; $f < 0$, diverging)		$\begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix}$	
(3) Dielectric Interface: Refractive indices n_1, n_2		$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$	$\leftarrow$ $r_2 = r_1$ $n_2 r_2' = n_1 r_1'$
(4) Spherical Dielectric Interface: Radius R		$\begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$	$\leftarrow$ $r_2 = r_1 \quad \rightarrow \quad r_1' = \theta_{r_1} - \theta_1$ $n_2\theta_2 = n_1\theta_1 \quad \rightarrow \quad r_2' = \theta_{r_1} - \theta_2$
(5) Spherical Mirror: Radius of curvature R		$\begin{bmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{bmatrix}$	$\leftarrow$ $r_2 = r_1 \quad \rightarrow \quad r_1' = \theta_{r_1} - \theta_1$ $\theta_2 = \theta_1 \quad \rightarrow \quad r_2' = -(\theta_{r_1} + \theta_2)$

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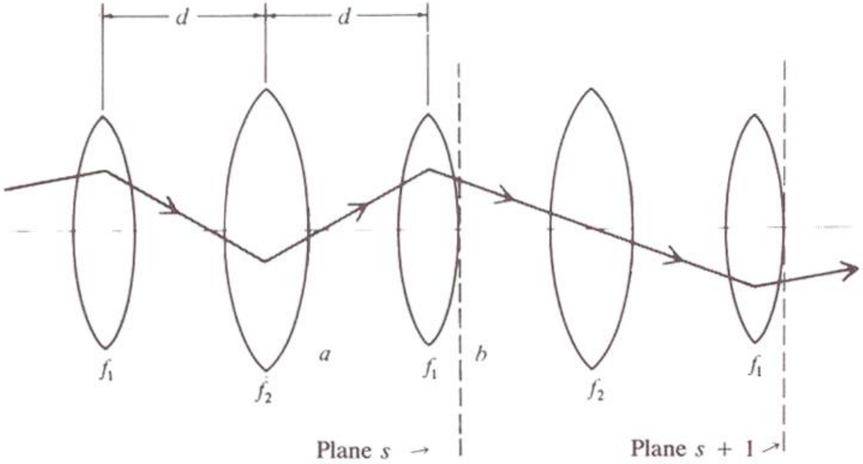
Lens Waveguide (1)

Propagation of an optical ray:

$$\begin{bmatrix} r_{out} \\ r'_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_{in} \\ r'_{in} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d \\ -1/f & 1-d/f \end{bmatrix} \begin{bmatrix} r_{in} \\ r'_{in} \end{bmatrix}$$



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$$\rightarrow \begin{bmatrix} r_{s+1} \\ r'_{s+1} \end{bmatrix} = \begin{bmatrix} 1 & d \\ -1/f_1 & 1-d/f_1 \end{bmatrix} \begin{bmatrix} 1 & d \\ -1/f_2 & 1-d/f_2 \end{bmatrix} \begin{bmatrix} r_s \\ r'_s \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_s \\ r'_s \end{bmatrix} \rightarrow \quad A = 1 - \frac{d}{f_2}, \quad B = d\left(2 - \frac{d}{f_2}\right)$$

$$C = -\left[\frac{1}{f_1} + \frac{1}{f_2}\left(1 - \frac{d}{f_1}\right)\right], \quad D = -\left[\frac{d}{f_1} - \left(1 - \frac{d}{f_1}\right)\left(1 - \frac{d}{f_2}\right)\right]$$

Recurrence relation:

$$r'_s = \frac{1}{B}(r_{s+1} - Ar_s) \quad \rightarrow \quad r'_{s+1} = \frac{1}{B}(r_{s+2} - Ar_{s+1}) = Cr_s + Dr'_s$$

$$\rightarrow \quad r_{s+2} - (A+D)r_{s+1} + (AD - BC)r_s = 0$$

Lens Waveguide (2)

Recurrence relation:

$$r_{s+2} - (A+D)r_{s+1} + (AD-BC)r_s = 0 \quad \leftarrow \quad AD-BC=1$$

$$\rightarrow \; r_{s+2} - 2br_{s+1} + r_s = 0 \leftarrow \; b = \frac{1}{2}(A+D) = 1 - \frac{d}{f_2} - \frac{d}{f_1} + \frac{d^2}{2f_1 f_2}$$

$$\leftarrow \; r_s = r_0 e^{isq}$$ $\rightarrow$ Trial solution

$$\rightarrow \; e^{2iq} - 2be^{iq} + 1 = 0 \;\; \rightarrow \; e^{iq} = b \pm i\sqrt{1-b^2} \equiv e^{\pm i\theta} \;\; \rightarrow \; \cos\theta = b$$

General solution:

$$r_s = r_{\max} \sin(s\theta + \alpha) \;\; \leftarrow \;\; r_{\max} = r_0 / \sin\alpha$$

Condition for a stable ray:

$$|b| \le 1$$

$$\rightarrow \; -1 \le 1 - \frac{d}{f_2} - \frac{d}{f_1} + \frac{d^2}{2f_1 f_2} \le 1$$

$$\rightarrow \; 0 \le \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right) \le 1$$

Otherwise: $\rightarrow$ *The ray will diverge!*

Identical-Lens Waveguide

In case: $f_1 = f_2 = f$

$$\rightarrow \begin{bmatrix} r_{s+1} \\ r'_{s+1} \end{bmatrix} = \begin{bmatrix} 1 & d \\ -1/f & 1-d/f \end{bmatrix} \begin{bmatrix} r_s \\ r'_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_s \\ r'_s \end{bmatrix}$$

$$\rightarrow b = \frac{1}{2}(A+D) = 1 - \frac{d}{2f}$$

Stability condition:

$$|b| \leq 1 \quad \rightarrow -1 \leq 1 - \frac{d}{2f} \leq 1$$

$$\rightarrow 0 \leq d \leq 4f$$

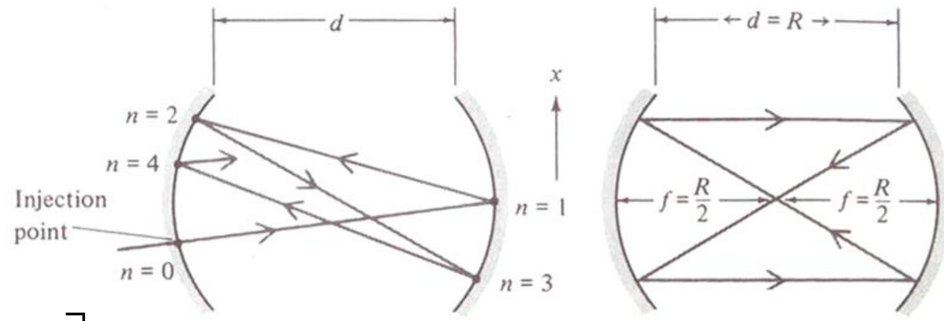
$$\rightarrow r_n = r_{\max} \sin(n\theta + \alpha) \leftarrow r_{\max} = r_0 / \sin\alpha$$

$$\rightarrow b = \cos\theta = 1 - \frac{d}{2f}$$

Propagation of Rays between Mirrors

Ray matrices:

$\rightarrow$ Thin lens: $\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$



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$\rightarrow$ Spherical mirror: $\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$

Equivalence in the formalism with the lens waveguide:

$$x_n = x_{\max} \sin(n\theta + \alpha_x)$$
$$y_n = y_{\max} \sin(n\theta + \alpha_y)$$

Re-entrant rays:

$\rightarrow 2v\theta = 2l\pi$ $\leftarrow$ v & l: any two integers

$$\rightarrow v = 2 \;\rightarrow \theta = \frac{\pi}{2} \;\rightarrow\; b = \cos\theta = 1 - \frac{d}{2f} = 0$$

$$\rightarrow\; d = 2f = R$$

$\rightarrow$ Symmetric confocal

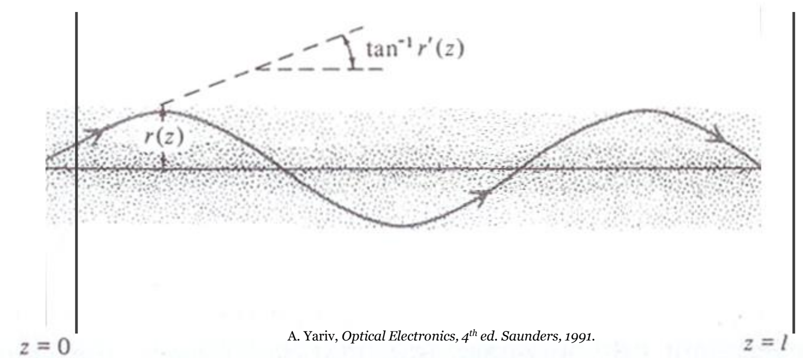
Rays in Lenslike Media (1)

Ideal thin lens:

$$r' = -\frac{r}{f} \quad \rightarrow \quad E_R(x,y) = E_L(x,y)\exp\left(+ik\frac{x^2+y^2}{2f}\right)$$

Lenslike medium: ← Phase delay: Why?

$$n(x,y) = n_0\left[1 - \frac{k_2}{2k}(x^2+y^2)\right]$$



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Wave equation:

$$\nabla^2\Psi - \frac{n^2}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0$$

$$\rightarrow \; \Psi = A(\mathbf{r})e^{i[\omega t - \phi(\mathbf{r})]}$$

$$\rightarrow \; \nabla^2 A - 2i\nabla A\cdot\nabla\phi - A|\nabla\phi|^2 - iA\nabla^2\phi + \frac{n^2\omega^2}{c}A = 0$$

Real part: $\rightarrow \; \cancel{\nabla^2 A} - A|\nabla\phi|^2 + (\frac{2\pi}{\lambda}n)^2 A = 0$

← Slowly varying amplitude

Eikonal equation: $|\nabla\phi|^2 = (\frac{2\pi}{\lambda}n)^2$

Rays in Lenslike Media (2)

Deflection of a ray normal to an ideal thin lens:

$$\left|\nabla\phi\right|^2 = (\frac{2\pi}{\lambda}n)^2 \quad \rightarrow \nabla\phi$$: Normal to the wavefront

$$\rightarrow \nabla\phi = \frac{2\pi}{\lambda} n \frac{d\mathbf{r}}{ds}$$

Unit vector in the direction of $\nabla\phi$

$$\rightarrow \phi(\mathbf{r}) = \frac{2\pi}{\lambda}\int n ds$$

Ray trajectory:

$$\rightarrow \frac{d}{ds}(\nabla\phi) = \frac{d\mathbf{r}}{ds}\cdot\nabla(\nabla\phi) = \left(\frac{2\pi}{\lambda}\right)^{-1}\frac{1}{n}\nabla\phi\cdot\nabla(\nabla\phi)$$

$$= \left(\frac{2\pi}{\lambda}\right)^{-1}\frac{1}{2n}\nabla\left[\left|\nabla\phi\right|^2\right] = \left(\frac{2\pi}{\lambda}\right)^{-1}\frac{1}{2n}\nabla\left[\left(\frac{2\pi}{\lambda}n\right)^2\right]$$

$$= \left(\frac{2\pi}{\lambda}\right)\nabla n$$

$$\rightarrow \frac{d}{ds}(n\frac{d\mathbf{r}}{ds}) = \nabla n$$

Rays in Lenslike Media (3)

For paraxial rays: $\frac{d}{ds} \rightarrow \frac{d}{dz}$

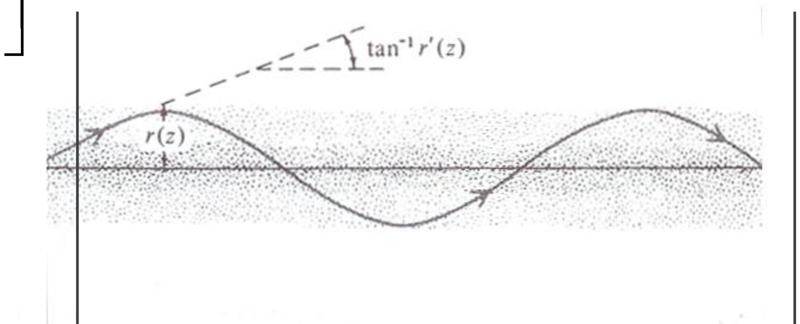
$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n \;\rightarrow\; \frac{d}{dz}\left(n\frac{d\mathbf{r}}{dz}\right) = \nabla n$$

Lenslike medium:

$$n(x,y) = n_0\left[1 - \frac{k_2}{2k}(x^2+y^2)\right]$$

$$\rightarrow\; n\frac{d^2 r}{dz^2} = -2n_0\frac{k_2}{2k}r$$

$$\rightarrow\; \frac{d^2 r}{dz^2} + \left(\frac{k_2}{k}\right)r = 0$$



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Ray trajectory vector:

$$r(z) = \cos\left(\sqrt{\frac{k_2}{k}}z\right)r_0 + \sqrt{\frac{k}{k_2}}\sin\left(\sqrt{\frac{k_2}{k}}z\right)r_0'$$

$$r'(z) = -\sqrt{\frac{k_2}{k}}\sin\left(\sqrt{\frac{k_2}{k}}z\right)r_0 + \cos\left(\sqrt{\frac{k_2}{k}}z\right)r_0'$$