Introduction to Photonics

Relativistic Optics (2)

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Length Contraction

Lorentz transform and inverse Lorentz transform:

$$x_0' = \gamma(x_0 - \beta x_1) \qquad x_0 = \gamma(x_0' + \beta x_1')$$
$$x_1' = \gamma(x_1 - \beta x_0) \qquad x_1 = \gamma(x_1' + \beta x_0')$$
$$x_2' = x_2 \qquad x_2 = x_2'$$
$$x_3' = x_3 \qquad x_3 = x_3'$$

Length measured in K at $t = 0$

Length measured in K' at $t' = 0$:

$$x_1 = \gamma(x_1' + \beta x_0') \rightarrow x_1 = \gamma x_1' \rightarrow L' = \frac{L}{\gamma}$$ $\leftarrow$ Length contraction

Event in K' at $t = 0$:

$$x_0' = -\gamma\beta x_1$$
$$x_1' = \gamma x_1$$

$\rightarrow$ Object that started from location γx_1 and at time $-\gamma\beta x_1$

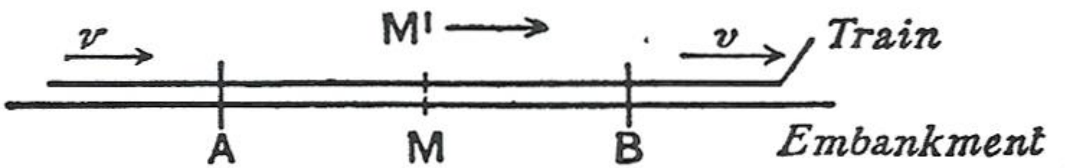
Distance to the object at time $x_0' = 0$:

$$L' = \gamma x_1 - \frac{v}{c}(\gamma\beta x_1) = \gamma x_1(1-\beta^2) = \frac{x_1}{\gamma} = \frac{L}{\gamma}$$ $\leftarrow$ Length contraction

Relativity of Simultaneity

Simultaneous events with reference to the railway embankment:

$\rightarrow$ Two simultaneous strokes of lighting A and B



$-x_1$ 0 x_1

A. Einstein, *Relativity: The Special and General Theory*, Crown Publishers, 1931.

$\rightarrow$ Are these two events also simultaneous relatively to the train?

Reference to the railway embankment:

$$ct_B + vt_B = x_1 \;\rightarrow\; t_B = \frac{x_1}{c+v} = \frac{x_1}{c}\frac{1}{1+\beta}$$

$$ct_A - vt_A = x_1 \;\rightarrow\; t_A = \frac{x_1}{c-v} = \frac{x_1}{c}\frac{1}{1-\beta}$$

Distance in K'

Time delay in K'

Reference to the train:

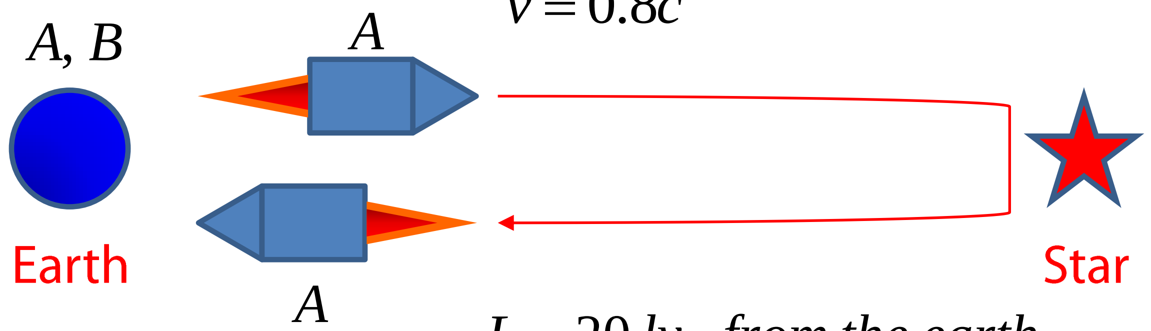
$$t'_B = \frac{t_B}{\gamma} = \frac{x_1}{\gamma c}\frac{1}{1+\beta} = \frac{\gamma x_1}{c}(1-\beta) = \frac{\gamma x_1}{c} - \frac{\gamma\beta x_1}{c}$$ $\leftarrow$ "Earlier event"

$$t'_A = \frac{t_A}{\gamma} = \frac{x_1}{\gamma c}\frac{1}{1-\beta} = \frac{\gamma x_1}{c}(1+\beta) = \frac{\gamma x_1}{c} + \frac{\gamma\beta x_1}{c}$$

Twin Paradox?

Time dilation or length contraction for the space traveller:

$$v = 0.8c$$



$$L = 20\ ly \quad \textit{from the earth}$$

Time measured by B:

$$T_B = 2 \times \frac{20}{0.8} = 50\ yrs$$

Time measured by A:

$$L' = \frac{L}{\gamma} = L\sqrt{1-\beta^2} = 20 \times 0.6 = 12\ ly$$

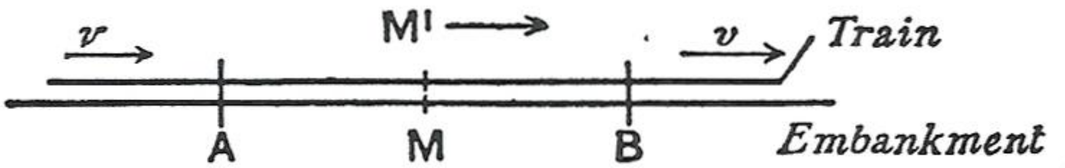
$$T_A = 2 \times \frac{12}{0.8} = 30\ yrs$$

What if A thinks that the earth and star are in motion?

Is this symmetric? $\rightarrow$ General theory of relativity

Special and General Principle of Relativity

Relative motion:



A. Einstein, *Relativity: The Special and General Theory*, Crown Publishers, 1931.

(a) The carriage is in motion relative to the embankment:
→ The embankment as reference-body

(b) The embankment is in motion relative to the carriage:
→ The carriage as reference-body

Which is correct? → *Only "experience" can decide!*

Special theory of relativity: → Only applicable to a uniform motion

For a non-uniform motion: Linked to acceleration

→ General theory of relativity

Gravitational Field

"If we pick up an apple and then let it go, why does it fall to the ground?"

$\rightarrow$ "Because it is attracted by earth: Newton's law of universal gravitation."

$$F = G\frac{m_1 m_2}{r^2}$$

Coulomb's law: $$F = k\frac{q_1 q_2}{r^2}$$ $\leftarrow$ Electric "field" involved

Einstein's thought: Why not "gravitational field"?

Newton's law of motion:

$$(\text{Force}) = (\text{intertial mass}) \times (\text{acceleration})$$

Under gravitational field:

$$(\text{Force}) = (\text{gravitati onal mass}) \times (\text{intensity of the gravitatio nal field})$$

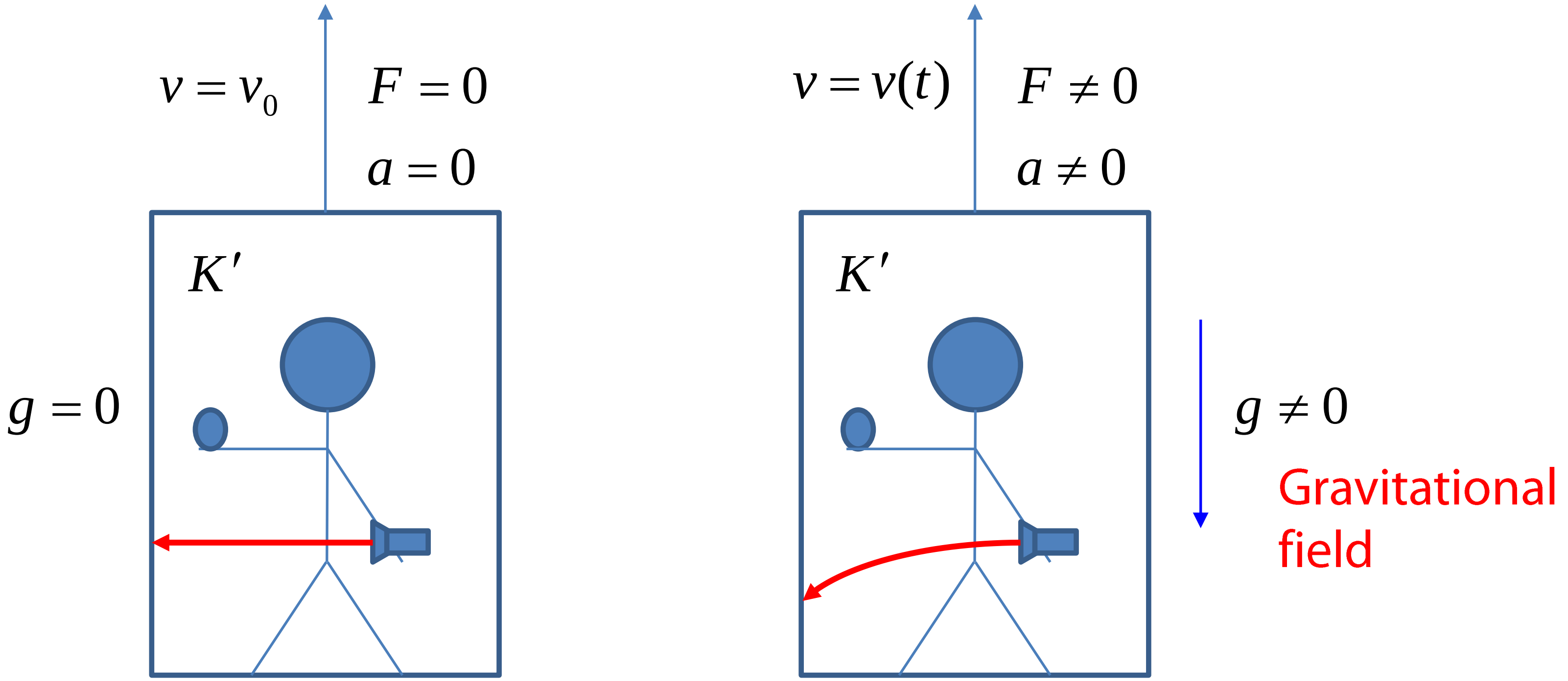
$$\rightarrow\ (\text{acceleration}) = \cancel{\frac{(\text{gravitational mass})}{(\text{inertial mass})}} \times (\text{intensity of the gravitational field})$$

"Equivalent"

Equivalence Principle

Imagine a chest with an observer inside that is being pulled upwards:

$\rightarrow$ Without external gravitation



$\rightarrow$ *The acceleration of the object towards the floor of the chest is always of the same magnitude, whatever kind of body the person may happen to use for the experiment.*

$\rightarrow$ *Light bends toward the direction of the gravitational field.*

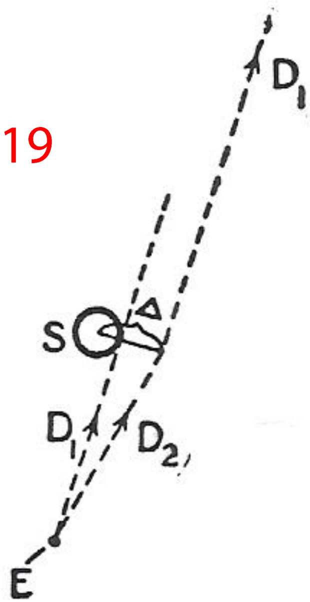
General Principle of Relativity

Deflection of light in gravitational field:

→ Measured during the solar eclipse of 29th May, 1919

Time dilation by gravitational field:

→ Time is slowed down in gravitational field.



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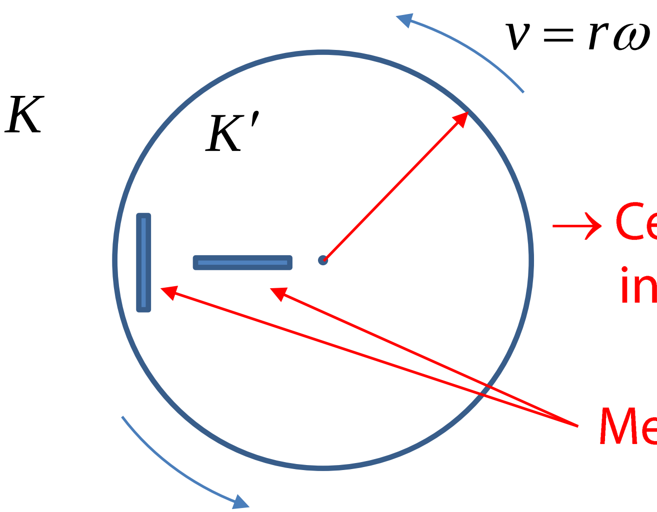
Rotating body of reference:



→ Centrifugal force (acceleration) outwards in a radial direction: $a = \dfrac{v^2}{r}$

Measuring-rod

$\rightarrow C' \neq 2\pi R$

→ Non-Euclidean geometry: → Curved

→ Finite and yet unbounded universe: → In doubt ← Hubble's law